Private versus public risk sharing: Should governments provide reinsurance?

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Abstract

The paper examines alternative arrangements for intergenerational risk sharing in a small open economy subject to macroeconomic disturbances. Under certain conditions, private pension funds can provide substantial risk sharing across generations. Private risk sharing alleviates the burden on governments to provide insurance, but it is limited by mobility in the labor market and by the ability of corporate plan sponsors to default. Government has a role in correcting these limitations by providing reinsurance and it can enter insurance arrangements on behalf of future generations. Optimal reinsurance includes bonds indexed to longevity and to productivity.

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Technical Appendix: A Model of Optimal Risk Sharing

This appendix provides a more formal treatment of the optimal risk sharing problems discussed in the paper and of the conditions for optimal reinsurance.

A.1. The Economic Environment

Time is indexed by discrete periods $t$. Each period, a vector of shocks $s_t$ is realized. A history $h_t$ encodes the sequence of shocks up to period $t$ and is defined recursively by $h_t=(s_t,h_{t-1})$.

Individuals live for three periods, youth, working age, and retirement. Retirement is a fractional period of length $\lambda_t=\lambda(h_t)$. Assume generation $t$ reaches working age in period $t$, and without loss of generality, assume a single representative domestic agent is born each period. Generation $t$ maximizes the expectation of

$$U_t = u_w(c_{wt}) + \lambda_{t+1}u_r(c_{rt+1})$$ (A1)

where $u_w$ and $u_r$ are increasing and concave utilities over working-age consumption $c_{wt}$ and retirement consumption $c_{rt+1}$. Individuals have no income or consumption in youth, but they may enter financial markets. Individuals in working age (workers) supply a unit of labor, which has an exogenous marginal product $w_t=w(h_t)$, they pay lump-sum taxes $T(h_t)$, and they may save, either individually and/or by contributing to a pension fund. Individuals in retirement age (retirees) receive exogenous government transfers $B_{t+1}(h_{t+1})$, while living, and returns from working-age savings and/or a pension. Let pensions for generation $t$ be defined generally by a state-contingent benefit profile $X(h_{t+1})\geq 0$ and contributions $x(h_t)\geq 0$.

Because savings can be viewed as purchases of state contingent claims, individuals maximize utility subject to

$$\lambda_{t+1}c_{rt+1}(h_{t+1}) = a_r(h_{t+1}) + X(h_{t+1}) + \lambda_{t+1}B_{t+1}(h_{t+1})$$ (A2)

$$c_{wt}(h_t) + \sum_{h_{t+1}} \xi(h_{t+1} \mid h_t) a_r(h_{t+1}) = w_t(h_t) - T(h_t) - x(h_t)$$ (A3)

by choice of state-contingent retirement savings $a_r(h_{t+1})$ for given prices $\xi(h_t \mid h_0)$. For reference below, let state prices be written as product of conditional probabilities, denoted $\pi(h_t \mid h_0)$, and the pricing kernel $\xi(h_t \mid h_0) = \pi(h_t \mid h_0)m(h_t \mid h_0)$. Then the present value of
assets, \( \sum_{h_{t_i}} \xi(h_{t_i} | h_t) a_t(h_{t_i}) = E[m(h_{t+1} | h_t)a_t(h_{t+1}) | h_t] \) can be interpreted as conditional expectation weighted by the pricing kernel.

The government maximizes a welfare function \( \bar{U}_0 \) that weights the ex ante utility of generation \( t \) by \( \omega_t > 0 \),
\[
\bar{U}_0 = E\left[u_r(c_r) + \sum_{t \geq 0} \omega_t U_t \mid h_0\right],
\]
subject to the intertemporal budget constraint
\[
NW_0 + \sum_{t \geq 0} \sum_{h_t} \xi(h_t | h_0) w(h_t) = \sum_{t \geq 0} \sum_{h_t} \xi(h_t | h_0)(c_{w_t}(h_t) + \lambda(h_t)c_{r_t}(h_t))
\]
where \( NW_0 \) is the country’s initial net worth. Cohort size is normalized to one. Assume utility is strictly concave, the l.h.s. of (A4) is finite, and the world economy is dynamically efficient.

Let \( \Lambda \) denotes the Lagrange multiplier on (A4). Then Pareto optimality requires
\[
\pi(h_t | h_0) \omega_t u_r'(c_{w_t}(h_t)) = \xi(h_t | h_0) \cdot \Lambda \tag{A5}
\]
\[
\omega_{t-1} \pi(h_t | h_0) \lambda(h_t) u_r'(c_{r_t}(h_t)) = \xi(h_t | h_0) \lambda(h_t) \cdot \Lambda \tag{A6}
\]
By concavity, (A4-A6) define a unique allocation. In the following, optimal values are highlighted by superscript star (*). Implied are consumption values
\[
c_{w_t}^*(h_t) = (u_r)^{-1}\left(m(h_t | h_0) \cdot \frac{\lambda}{\omega_r}\right) \quad \text{and} \quad c_{r_t}^*(h_t) = (u_r)^{-1}\left(m(h_t | h_0) \cdot \frac{\lambda}{\omega_r}\right). \tag{A7}
\]
where and a path for national net worth
\[
NW_t^*(h_t) = \sum_{t \geq 0} \sum_{h_{t_i}} \xi(h_{t_i} | h_t)(c_{w_{t_i}}^*(h_{t_i}) + \lambda(h_{t_i})c_{r_{t_i}}^*(h_{t_i}) - w(h_{t_i})).
\]
I will call an allocation efficient if it is optimal for some set of welfare weights \( \{\omega_t\} \).

Define the generational accounts of generation \( t \) in working-age and youth by
\[
GA_t(h_t) = T(h_t) - \sum_{h_{t+1}} \xi(h_{t+1} | h_t) B(h_{t+1})
\]
and
\[
\overline{GA}_t(h_{t-1}) = \sum_{h_{t-1}} \xi(h_t | h_{t-1}) G_t(h_t).
\]
Then unique optimal account balances
\[
GA_{t}^*(h_t) \equiv w(h_t) - c_{w_t}^*(h_t) - \sum_{h_{t+1}} \xi(h_{t+1} | h_t) \lambda(h_{t+1}) c_{r_{t+1}}^*(h_{t+1})
\]
and
\[
\overline{GA}_{t}^*(h_{t-1}) = \sum_{h_{t-1}} \xi(h_t | h_{t-1}) G_t^*(h_t)
\]
are implied by the optimal allocation.
A.2. Implementing Efficient Allocations

An optimal allocation for given \( \{ \omega_t \} \) can be decentralized in several equivalent ways. The numbering below matches the cells in Table 1. In each case, conditions for efficiency are noted, even if unrealistic. The sequence is driven by analytical convenience:

**Setting #1A: Optimal public pensions**: Set \( B(h_t) = \lambda(h_t)c_{rt}^*(h_t) \) for all \( h_t \neq h_0 \), \( T(h_t) = w(h_t) - c_{wt}^*(h_t) \) for all \( h_t \), and assume \( B(h_0) = \lambda(h_0)c_{r0}^*(h_0) - a_{r0} \) at \( t=0 \), where \( a_{r0} \) are assets held by the old.\(^{23} \) Assume the government invests or borrows funds abroad in a portfolio that returns \( NW_t^*(h_t) \) in state \( h_t \). Then the allocation is optimal with zero savings and no need for pensions.

**Setting #1B: Optimal mandatory private pensions**. Suppose taxes and government benefits are arbitrary sequences \( T(h_t) < w(h_t) - c_{wt}(h_t) \) and \( B(h_t) \leq \lambda(h_t)c_{rt}(h_t) \) that satisfy the intertemporal budget constraint:

\[
\sum_{i \geq 0} \sum_{h_{t+i}} \xi(h_{t+i} \mid h_t) (T(h_{t+i}) - \lambda(h_{t+i})B(h_{t+i})) = D(h_t). \tag{A8}
\]

where \( D(h_t) \) is government debt (potentially negative).

Suppose there is a member-owned (employer-independent) pension fund with asset \( F_t = F(h_t) \) in period \( t \). It faces the intertemporal budget constraint:

\[
F(h_t) = \sum_{i \geq 0} \sum_{h_{t+i}} \xi(h_{t+i} \mid h_t) (X(h_{t+i}) - x(h_{t+i})) = \]

\[
= X(h_t) + \sum_{i \geq 0} \sum_{h_{t+i}} \xi(h_{t+i} \mid h_t) (\xi(h_{t+i+1} \mid h_{t+i})X(h_{t+i+1}) - x(h_{t+i})) \tag{A9}
\]

where \( \hat{x}(h_t) = x(h_t) - \sum_{h_{t+i+1}} \xi(h_{t+i+1} \mid h_t)X(h_{t+i+1}) \) is the gap between pension contributions and the present value of benefits (interpretable as payment for insurance against shocks realized before working age). If assets in period \( t \) are \( F(h_t) = NW_t^*(h_t) + D(h_t) \) and plan managers maximize welfare \( U_0 \), the plan will pay \( X(h_t) = \lambda(h_t)c_{rt}^*(h_t) - B(h_t) \) and collect \( x(h_t) = w(h_t) - T(h_t) - c_{wt}^*(h_t) \); the resulting allocation is optimal.///

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\(^{23} \) In all settings, retiree benefits in period zero require special treatment if initial conditions differ from “normal” conditions in the respective setting; to avoid distractions, assume a suitable initial tax or transfer.
Remark: The inequality restrictions on taxes and benefits ensure that pension payments and receipts are non-negative; that is, government must not be “too big.” This is plausibly satisfied in the Netherlands, where 1st pillar benefits are low.

Setting #3A: Optimal individual savings for retirement. Suppose there is no pension system, so each generation must save for retirement. Assume the government sets taxes and transfers so that generational accounts $GA_t(h_t) = GA_t^*(h_t)$ are optimal, and assume markets are complete. Then it is straightforward to verify that $c_{wt}(h_t)$ and $c_{rt+1}(h_{t+1})$ maximize utility, so the allocation is efficient (analogous to Bohn 2009).

Remark: Without loss of generality, one may assume that the government sets benefits arbitrarily (say, in a simple way) and sets taxes

$$T(h_t) = GA_t^*(h_t) + \sum_{h_{t+1}} \xi(h_{t+1} | h_t)B(h_{t+1}).$$

Note that returns to retirement savings are not taxed in this setting. Hence savings are best interpreted as tax sheltered, e.g. in a DC plan. Also note that optimality generally requires access to state contingent claims markets; otherwise government benefits would have to be optimally designed (as in Setting #1A) and not arbitrary. Let $a_r(h_{t+1})$ denote optimal private savings in this setting.

Setting #2A: Optimal individual savings and optimal insurance in youth. Suppose there is no pension system, so each generation must save. However, assume now that youths can commit, so generation-t workers can enter financial market in period $t-1$ and buy/sell state contingent claims $a_w(h_t)$. Government policy sets taxes and transfers so that generational accounts $\overline{GA_t}(h_{t-1}) = \overline{GA_t^*}(h_{t-1})$ are optimal for all generations. Because youths have no income, their utility maximization is subject to (A2),

$$\sum_{h_t} \xi(h_t | h_{t-1})a_w(h_t) = 0,$$

and

$$c_{wt}(h_t) + \sum_{h_{t+1}} \xi(h_{t+1} | h_t)a_r(h_{t+1}) = w_t(h_t) + a_w(h_t) - T(h_t) - x(h_t),$$

which replaces (A3). It is straightforward to verify that if $\overline{GA_t}(h_{t-1}) = \overline{GA_t^*}(h_{t-1})$, then $c_{wt}(h_t)$ and $c_{rt+1}(h_{t+1})$ maximize utility. Hence the allocation is efficient. Moreover, $a_r(h_{t+1}) = a_r^*(h_{t+1})$ and $a_w(h_t) = GA_t(h_t) - GA_t^*(h_t)$.
Remark: Without loss of generality, one may assume that $GA_t(h_t)$ is designed to simplify other policy choices. Of particular interest here is debt management: Government may issue debt with arbitrary return distribution $R(h_t | h_{t-1})$. To see this, define end of period t-1 debt by

$$\tilde{D}(h_{t-1}) = D(h_{t-1}) - T(h_{t-1}) + \lambda(h_{t-1})B(h_{t-1})$$

and assume the government issues debt with returns $R(h_t | h_{t-1})$, which are subject only to the arbitrage condition $\sum_{h_t} \xi(h_t | h_{t-1})R(h_t | h_{t-1}) = 1$. Then debt at the start of period t is

$$D(h_t) = R(h_t | h_{t-1})\tilde{D}(h_{t-1}).$$

From (A9) and the definition of generational accounts, debt must also satisfy

$$D(h_t) = -\lambda(h_t)B(h_t) + GA_t(h_t) + \sum_{i \geq 1} \sum_{h_{t+i-1}} \xi(h_{t+i} | h_t)GA_{t+i}^*(h_{t+i-1}).$$

Optimal consumption for other generations requires that generation t’s account must be

$$GA_t(h_t) = R(h_t | h_{t-1})\tilde{D}(h_{t-1}) + \lambda(h_t)B(h_t) - \sum_{i \geq 1} \sum_{h_{t+i-1}} \xi(h_{t+i} | h_t)GA_{t+i}^*(h_{t+i-1}).$$

Thus taxes on generation t absorb all variations in the return on government debt. One may assume that government debt is safe. ///

Setting #3B: Optimal savings with a voluntary DB pensions offered in working age. Assume again that $GA_t(h_t) = GA_t^*(h_t)$ and that markets are complete, but now assume there are competing pension plans. With competition, a plan that promises benefits $X(h_{t+1})$ must charge actuarially fair contributions $x(h_t) = \sum_{h_{t+i}} \xi(h_{t+i} | h_t)X(h_{t+i})$. Then individual can obtain $c_{w,t}(h_t)$ and $c_{r,t+1}^*(h_{t+1})$ for arbitrary benefits $X(h_{t+1})$ by saving $a_r(h_{t+1}) = a_r^*(h_{t+1}) - X(h_{t+1})$ on their own. Thus pension plans are unnecessary. Alternatively, one might assume that the pension plan sets $X(h_{t+1}) = a_r^*(h_{t+1})$, so individual savings become unnecessary. ///

Setting #2B: Optimal savings with voluntary DB pension offered in youth. Consider the same setting as in #3B. Suppose employers offer pensions with arbitrary benefits $X(h_{t+1})$ and assume contributions satisfy

$$\sum_{h_t} \xi(h_t | h_{t-1})x(h_t) = \sum_{h_{t+1}} \xi(h_{t+1} | h_{t-1})X(h_{t+1}).$$

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Then $c^*_w(h_t)$, $c^*_{t+1}(h_{t+1})$ again maximize generation t’s utility, now with

$$a_w(h_t) = \left[ GA(h_t) - GA^*(h_t) \right] + \left[ x(h_t) - \sum_{h_{t+1}} \xi(h_{t+1} \mid h_t) X(h_{t+1}) \right]$$

and $a_r(h_{t+1}) = a^*_r(h_{t+1}) - X(h_{t+1})$. Again, individual savings are unnecessary if the pension plan is designed appropriately, namely if $X(h_{t+1}) = a^*_r(h_{t+1})$ and

$$x(h_t) = x^*(h_t) = \sum_{h_{t+1}} \xi(h_{t+1} \mid h_t) a^*_r(h_{t+1}) - \left[ GA(h_t) - GA^*(h_t) \right].$$

As emphasized in the text, the link of insurance and employment is promising because the employment relationship serves as collateral for insurance. Hence setting #2B is more credible than #2A. ///

In summary, allocations in all six settings are equivalent in principle. For each allocation without pension plans (settings 1A, 2A, 3A) there is a corresponding allocation with private pension plan (1B, 2B, 3B), and each can be designed to make separate private savings unnecessary. If individuals lack access to complete markets on their own, only the settings with optimal pensions are efficient.

Comparing pairs #1A-B, #2A-B, and #3A-B, one finds that private pension plans simplify government policy. The virtue of simplicity is not modeled explicitly because the implied complications would be distracting. Given the equivalence of allocations in principle, differences in cost and commitment are potentially decisive for a choice of systems.

Note that the pension plans above do not require a plan sponsor, provided they have access to complete markets. Implicitly, pension contributions are invested in state contingent claims that perfectly match the plan’s obligations. That is, they are neither underfunded, which avoids default problems, nor overfunded, so there is no residual claimant.

For the following settings assume individuals and pension plans may have limited access to financial markets. Assume corporations are valued under the pricing kernel $m$ and the Modigliani-Miller theorem applies. Consider:

**Setting #2C: Optimal savings with employer-sponsored pensions** offered in young age.

Assume $GA_t(h_{t-1}) = GA^*_t(h_{t-1})$ and assume that corporations offer a pension plan to workers and entering young workers. Sponsoring a pension means the corporation contributes
\( z(h_t) \) in period \( t \) and is entitled to a return \( Z(h_{t+1}) \) in period \( t+1 \). The representative corporation earns a return \( R^k(h_{t+1}) \) on equity capital \( K_t \). Fund investments have a return distribution \( R^F(h_{t+1}) \), which is imperfectly controllable—say, exogenous for simplicity. The investments \( z(h_t) + x(h_t) \) are worth \( F(h_{t+1}) = R^F(h_{t+1})[z(h_t) + x(h_t)] \) in the next period.

The corporation is worth \( V_{t+1}(h_{t+1}) = R^k(h_{t+1})K_t + Z(h_{t+1}) \) in period \( t+1 \). From the Modigliani-Miller theorem, \( \sum_{h_{t+1}} ^\xi(h_{t+1} | h_t)V_{t+1}(h_{t+1}) = K_t + z(h_t) \), which means the corporation can afford actuarially fair contributions. With incomplete markets, individuals cannot generally attain \( c^*_w(h_t) \) and \( c^*_{rt+1}(h_{t+1}) \) on their own and would participate even in plans that are actuarially unfair. But competing employers must offer optimal pensions \( X(h_{t+1}) = a^*_r(h_{t+1}) \) and charge optimal contributions \( x(h_t) = x^*(h_t) \). The pension surplus in period \( t+1 \), \( Z(h_{t+1}) = F(h_{t+1}) - X(h_{t+1}) + \hat{x}(h_{t+1}) \) is accrues to the corporation. This is feasible provided the value of the corporation is non-negative in all state of nature, \( Z(h_{t+1}) \geq -R^k(h_{t+1})K_t \), or equivalently,

\[
R^F(h_{t+1})(z(h_t) + x(h_t)) = F(h_{t+1}) \geq X(h_{t+1}) - \hat{x}(h_{t+1}) - R^k(h_{t+1})K_t \forall h_{t+1}. \tag{A13}
\]

Provided \( R^F(h_{t+1}) > 0 \) (e.g. with some safe assets), this can be satisfied by setting \( z(h_t) \) high enough, i.e., by sufficient contributions in period \( t \). Thus efficient risk sharing can be attained even with incomplete markets. However, solvency in all states of nature will generally require a (large) funding buffer in (most or all) other states of nature.///

Setting #3C: Optimal savings with employer-sponsored pensions offered in working age. Consider a corporate plan as in #2C but only for workers. Assume that \( GA_t(h_t) = GA^*_t(h_t) \). Then competing employers must offer optimal pensions \( X(h_{t+1}) = a^*_r(h_{t+1}) \) and charge actuarially fair contributions \( x(h_t) = \sum_{h_{t+1}} ^\xi(h_{t+1} | h_t)X(h_{t+1}) \). The pension surplus or deficit in period \( t+1 \), \( Z(h_{t+1}) = F(h_{t+1}) - X(h_{t+1}) \), is assigned to the corporation. The corporation is solvent provided (A13) holds with \( \hat{x}(h_{t+1}) \). As in setting #3A, the resulting allocation attains \( c^*_w(h_t) \)

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24 Note that corporate contribution may be in cash or by paying employees a wage that differs from the marginal product of labor and having the employee contribute a corresponding amount. The return might be a reduced contribution. Thus measurement is not trivial.
and \( c_{rt+1}(h_{t+1}) \). Thus efficiency is feasible even with incomplete markets. But as in #3A-B, there is no intergenerational risk sharing except through the government.///

**Setting #1C.** For completeness, note the possibility of mandatory employer-sponsored pensions. Such a pension plan would operate under the intertemporal budget constraint
\[
F(h_t) = X(h_t) - \tilde{x}(h_t) - \sum_{i \geq 1} \tilde{\xi}(h_{t+i} | h_t) \tilde{x}(h_{t+i}) + \sum_{i \geq 0} \tilde{\xi}(h_{t+i} | h_t) (Z(h_{t+i}) - z(h_{t+i}))
\]
The mechanics would be a straightforward combination of #2C and #1B. However, without competitive entry, corporations would have no incentive to offer actuarially fair plans. Workers would be taxed for private profit. (Higher \( \tilde{x}(h_{t+i}) \) reduces \( z(h) \) in the budget constraint.) As this seems socially unacceptable, setting #1C is disregarded in the text.///

**A.3. Macroeconomic Shocks**

Consider Setting #2B, where pensions have a key role in risk sharing. The pricing kernel reflects aggregate risks. To be specific, assume world investors have power utility with risk aversion \( \gamma \), and assume their consumption depends positively on stochastic labor productivity (measured by the wage \( \overline{w}(h_t) \)), positively on a stochastic return to world capital \( \overline{R}^k(h_t) \) and negatively on longevity \( \overline{\lambda}(h_t) \). Then the pricing kernel can be written (approximately) as
\[
m(h_t | h_0) = \left( m_0 + m_R \overline{R}^k(h_t) + m_w \overline{w}(h_t) - m_{\overline{\lambda}} \overline{\lambda}(h_t) \right)^\gamma
\]
where \((m_0, m_R, m_w, m_{\overline{\lambda}})\) are positive parameters. Recall that government benefits and debt management do not have to provide insurance against current shocks: assume both are safe (non-contingent). Finally, assume domestic individual also have power utility. Then optimality consumption in (A7) can be written as
\[
c^{*}_{wt}(h_t) = \left( \Lambda / \alpha_t \right)^{-1/\gamma} \left( m_0 + m_R \overline{R}^k(h_t) + m_w \overline{w}(h_t) - m_{\overline{\lambda}} \overline{\lambda}(h_t) \right)
\]
\[
c^{*}_{rt+1}(h_{t+1}) = \left( \Lambda \beta / \alpha_t \right)^{-1/\gamma} \left( m_0 + m_R \overline{R}^k(h_{t+1}) + m_w \overline{w}(h_{t+1}) - m_{\overline{\lambda}} \overline{\lambda}(h_{t+1}) \right)
\]
where \( \beta \) denotes preference between working age and retirement. From (A2), optimal retirement benefits have the form
\[ X(h_{t+1}) = \lambda_{t+1} \left[ c^*_{rt+1}(h_{t+1}) - B_{t+1} \right] \]

\[ = \lambda(h_{t+1}) \left[ \left( \frac{\Lambda^0}{\omega^0} \right)^{-1/\gamma} \left( m_0 + m_R \bar{R}^k(h_{t+1}) + m_w \bar{w}(h_{t+1}) - m_\lambda \bar{\lambda}(h_{t+1}) \right) - B_{t+1} \right] \] (A15)

Thus pensions should be linked to the return to world capital and to world wages, fully indexed to domestic longevity, but less than fully indexed to world longevity. From (A2-3), optimal pension contributions have the form

\[ x(h_t) = w_t(h_t) - T_t(h_t) - c^*_{wl}(h_t) \]

\[ = w_t(h_t) - T_t(h_t) - \left( \frac{\Lambda}{\omega^0} \right)^{-1/\gamma} \left( m_0 + m_R \bar{R}^k(h_t) + m_w \bar{w}(h_t) - m_\lambda \bar{\lambda}(h_t) \right) \] (A16)

Ceteris paribus, pension contributions should be fully indexed to own wages; negatively linked to the return to world wages; negatively linked to the return to world capital; and positively linked to world longevity. Intuitively, full indexing to own wages and a negative link to world wages means that idiosyncratic wage shocks are fully hedged whereas shocks that impact both own wages and world wages are generally not fully hedged.

A.4. Frictions

Several frictions destroy efficiency unless policy responds appropriately.

(a) Mobility: Consider an optimal pension offered in youth (setting #2B), but now assume workers can exit the pension plan at a cost \( \chi_w > 0 \) and then save on their own (as in #3A) or join an employer offering a retirement plan to workers (as in #3B). To avoid exit, the original pension plan must operate under the mobility constraint

\[ x(h_t) - \sum_{h_{t+1}} \xi(h_{t+1} | h_t) X(h_{t+1}) \leq \chi_w \text{ for all } h_t. \] (A17)

If \( x(h_t) = x^*(h_t) \) in (A17) violates this constraint, private pensions cannot provide efficient risk sharing. One remedy is straightforward from the construction of \( x^*(h_t) \): The government can ensure that

\[ x^*(h_t) - \sum_{h_{t+1}} \xi(h_{t+1} | h_t) X(h_{t+1}) = -[GA(h_t) - GA^*(h_t)] \leq \chi_w, \]

by setting \( GA(h_t) \geq GA^*(h_t) - \chi_w \). That is, taxes on generation \( t \) must be high enough in states of nature where (A17) would otherwise be violated. From (A16), the mobility constraints

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constraint is most likely binding in states of nature with high wages, low returns on capital and high longevity. From (A12), $GA(h_t)$ is raised if the government issues bonds that promise a high return. Thus wage and longevity indexed government bonds relax (A17), which is an argument for issuing such securities.

(b) **Incomplete markets with funding costs:** Suppose securities are priced according to (A14), but the only generally available investment options for pension funds are government bonds and claims on the value of corporate assets.

One way to maintain efficiency is to use a corporation as residual claimant (plan sponsor), as noted above. However, to satisfy (A13) in all states of nature, $z(h_t)$ would have to be very high if $RF$ is near zero in some states of nature. In most other states of nature, there would be large surpluses, and potentially a return of funds to the corporation ($Z<0$). To make such “churning” of funds matter formally, one may assume that fund management is costly. A government supply of wage and longevity indexed bonds would reduce these imbalances and enhance efficiency, which is another argument for issuing such bonds.

(c) **Incumbent selfishness:** Consider the setting with mandatory pensions (setting #1B). Suppose pensions in period $t$ are increased marginally to $X^+(h_t) = \lambda(h_t)c_{rt}^*(h_t) - B(h_t) + \Delta X$ and contributions are reduced to $x^-(h_t) = w(h_t) - T(h_t) - c_{wt}^*(h_t) - \Delta x$. In the next period, contributions are increased to $x^+(h_{t+1}) = w(h_{t+1}) - T(h_{t+1}) - c_{wt}^*(h_{t+1}) + RF(h_{t+1})(\Delta x + \Delta X)$.

This change increases the utility of generations $t$ and $t-1$ at the expense of generation $t+1$. Hence a mandatory pension plan managed by incumbents who value own utility more than the social planner—or even place zero weight on future generations—would not implement the socially optimum.

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25 For example, suppose that pension returns are reduced by a management cost, so $\sum_{h_t} \xi(h_{t+1} | h_t)RF(h_{t+1}) = 1 - \chi_F$, where $0 < \chi_F < 1$. To avoid bias against private plans, one should then also recognize that taxes are distortionary. A formal treatment would be complicated and hence is not presented here.
Note that voluntary entry provides an effective constraint on such redistribution. First, if members enter in working age (as in setting #3B), competition between employers and plans ensures \( \hat{x}(h_t) \leq 0 \) for all \( t \), so (A9) reduces to \( X(h_t) \leq F(h_t) \). This means plan management can be delegated to plan members. To maximize own utility, each generation will provide optimally for their own retirement; the resulting allocation is the optimal allocation. Second, if members enter in youth (as in setting #2B), competition ensures
\[
\sum_{h_{t+1}} \xi(h_{t+1} | h_t) \hat{x}(h_{t+1}) \leq 0 \quad \text{for all } t
\]
so (A9) reduces to \( X(h_t) \leq F(h_t) + \hat{x}(h_t) \). Moreover, \( \hat{x}(h_t) \) is constrained because the plan must credibly commit to honor (A18) in \( t-1 \). Hence plan management can again be delegated to the retiring generation. Mandatory funding rules may also constrain redistribution, but they create distortions except with complete markets and under a rule that requires an exact matching of assets and obligations. Matching is needed not only to avoid underfunding but also to avoid excess funds that could be diverted by those who control the fund.