Financing Higher Education in an Imperfect World

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Abstract

This paper explains why under laissez-faire the financing of higher education is both inefficient and inequitable. It is argued that a government-run scheme of income contingent loans (ICLs) for higher education would achieve superior outcomes. We advocate a refinement of existing ICLs schemes. Following Apps, Long and Rees (2014), the paper proposes a piecewise-linear repayment schedule that serves both equity and efficiency objectives.

Keywords: Higher education financing; Income-contingent loans; Efficiency; Equity.

1 Introduction

Higher education is an essential engine of economic growth. However, there is a lack of consensus as to how higher education should be financed. In most European countries, higher education is largely free. In some other advanced economies, such as the USA, the UK, Australia, and New Zealand, students must pay substantial fees to attend universities or colleges. Facing such fees, many young people must rely on loans to finance their own education, and some must give up hopes of attending a tertiary institution. While some can borrow from parents or relatives, others resort to loans offered by financial institutions (with or without guarantee from the government). In some countries, the government operates an income contingent loans (ICLs) scheme whereby the amounts the individuals are required to repay in any period is dependent on their income in that period. A striking example of success is Australia's ICLs scheme, originally known as the Higher Education Contribution Scheme (HECS). HECS was used for the payment of tuition fees for all undergraduate university courses.[1] The success of the Australian ICLs system for higher education financing has been praised internationally.[2] In this paper, I argue that ICLs operated by the government is an efficient and equitable way of financing higher education.[3] In addition, I will present some arguments for further refinements of the current ICLs schemes so as to improve the outcomes.

According to Stiglitz (2014, p. 31), income contingent loans "represent an efficient way of implementing equity contracts for human capital." When repayments for loans are contingent on the income of the borrowers, the loans contracts practically become equity contracts: a government that provides ICLs is effectively sharing the risks with those who make investments in their human capital. The main reason why these risks should be shared by the government rather by private investors is that the moral hazard and adverse selection problems can be more efficiently mitigated by the government, thanks to

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[1]Bruce Chapman, the original architect of Australia's HECS (Chapman, 1988), has argued that, in addition to higher education financing, ICL have many other potential applications that are welfare improving. See Chapman (2006, 2014).

[2]See for example the article in the New York Times, titled "America Can Fix Its Student Loan Crisis. Just Ask Australia" by Susan Dynarski (2016). She argues out that, in terms of economic efficiency, the Australian system of student loans dominates both the standard American plan and the income-based repayment plan recently made available in the U.S.

[3]In the U.S., the federal program allows most federal student loan borrowers to enroll in income-based payment plans, but there are serious shortcomings, e.g., private companies that operate the program often fail to explain recertification or to notify people of deadlines. See the New York Times' editorial article titled "Unfairly squeezing Student Borrowers" (Feb. 4, 2017).

its superior ability to collect information about individuals' income. The market system, by itself, would not provide efficient ICLs schemes, because of high informational and enforcement costs that private lenders face.

This paper provides a theoretical justification for ICLs to support higher education. The plan of the paper is as follows. In Section 2, I review the arguments that in some areas of economic activities the free market does not ensure an efficient outcome. In Section 3, a simple model of ICLs is presented, and its efficiency properties are discussed. In Section 4, I propose a piecewise linear repayment scheme as an improvement over the linear scheme. Section 5 concludes.

2   The virtues and limitations of market allocations

Economists generally praise the many virtues of the market system. However, with very few exceptions, they do not offer unconditional support for the laissez-faire regime. They point to various causes of market failure, and to the possibility of improving upon the market outcomes by using appropriate corrective measures. This section offers a brief review of this literature in order to identify the raison d'être of ICLs for the financing of higher education.

2.1   Adam Smith's nuanced view of the invisible hand

One of the central messages of Adam Smith's The Wealth of Nations is that it is the self-interest of economic agents that lies behind prosperity:"It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love..."(Smith, 1776, Vol.I, p. 13).

However, Smith's enthusiasm about the virtue of the market system was not unqualified. Indeed, he was well aware of the need to regulate the banking system, the evil of monopoly, and the moral hazard problem in economic relationships.[4] Smith's nuanced

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[4]Furthermore, the price system alone is not sufficient to ensure the smooth functioning of the economy. Indeed, Smith maintained that no society can survive if individuals fail to respect moral norms. He wrote that "upon the tolerable observance of these duties, depends the very existence of human socuiety, which would crumble into nothing if mankind were not generally impressed with a reverence for those important rules of conduct." (Smith, 1970, Part III, Ch. V, p. 190).

view of the price mechanism has led his followers to identify market failures and propose remedies for them. The formalization of the theoretical apparatus for analyzing market failure was carried out by Pigou, the father of the theory of public finance. What is the role of the government in a market economy? Under what conditions does the market system fail to achieve an efficient allocation of resources? What kind of actions should a government take to mitigate the market failures? These questions have been the motivating force behind much of economics. In the Preface of his book, *The Economics of Welfare* (1920), Pigou wrote what we would call today a 'mission statement' of economics:

> "*The complicated analyses which economists endeavour to carry through are not mere gymnastic. They are instruments for the bettering of human life. The misery and squalor that surround us (...) are evils too plain to be ignored. By the knowledge that our science seeks it is possible that they may be restrained. Out of the darkness light! To search for it is the task, to find it, perhaps, the prize, which the 'dismal science of Political Economy' offers to those who face its discipline.*"

For the market outcome to coincide with an efficient allocation of resources, a number of conditions must be satisfied. These conditions were precisely identified by Arrow and Debreu, and further elaborated in Greenwald and Stiglitz (1986). These include:

1) Firms do not collude to wield market powers at the expense of consumers.

2) There are no powerful groups that engage in grabbing and rent-seeking.[5]

3) A perfect insurance market exists, such that individuals can purchase actuarially fair insurance against risks.[6]

4) A perfect credit market exists, such that individuals can borrow against their future income.

These conditions are highly restrictive. To be anywhere near the efficient frontier that an idealized market system could achieve, policies must be designed to correct for market

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[5]In the language of the Arrow-Debreu model, this is reflected in the assumptions that each firm has an inviolable production set and each consumer has an inviolable consumption set. See Long (1994) for a demonstration that when firms' production sets are insecure, the market outcome is typically inefficient. Long (2013) argued that rent-seeking behavior can be viewed as efforts to modify consumption and production opportunities by non-market means.

[6]Conditions (3) and (4) are reflected in the assumptions that there is a full range of Arrow-Debreu securities. Greenwald and Stiglitz (1986) show that under asymmetric information, competitive equilibrium is in general not efficient.

imperfections.[7] Thus, in most economies, there are anti-trust legislations to ensure that firms are reasonably competitive, anti-corruption laws, regulations on lobbying, and so on, to deal with items (1) and (2) above. The challenges that arise in connection with items (3) and (4) are also quite formidable, because of the pervasiness of informational asymmetry in the insurance and credit markets. Let us elaborate on the problems created by asymmetric information in these markets.

2.2   The insurance market and the credit market

The raison d'être of the insurance market and the credit market can be summarized in one word: consumption-smoothing. Economists explain the need for insurance in terms of the natural desire for *consumption-smoothing across the states of nature* in any given period. And to satisfy their desire for *consumption-smoothing across periods*, individuals must turn to the credit market.

In an ideal world, risk sharing could eliminate fluctuations in income caused by random shocks that occur to individuals. In the real world, the market for risk sharing is not perfect, because contracting parties do not have the same access to relevant information. Economists have pointed out that asymmetric information hampers the operation of risk-sharing arrangements. A fully insured individual would not have an incentive to make the required level effort to reduce risks, if effort levels are not verifiable. This is called the "moral hazard" problem.[8] In the case of income-contingent loans for education, a typical moral hazard is that after signing a loan contract, when a graduate joins the workforce, she might have an incentive to choose an occupation with low wage and high leisure. Another type of informational asymmetry problem is "adverse selection." It refers to situations where individuals misrepresent themselves to take advantage of contractual arrangements intended for others. For example, a student who do not intend to join the workforce upon graduation (or who know that she cannot repay her debt in full) may pretend that she will be able to repay in order to obtain a loan. Alternatively, an unemployed graduate who has the ability to re-tool his skills may pretend that he is unable to do so, because he prefers to receive long-term unemployment insurance payouts that are intended to help those who lack the ability to re-tool. Insurance contracts that are intended for one type

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[7]See Stiglitz (2003, pp. 73-74) for a brief exposition of conditions under which the invisible hand fails to work.

[8]Moral hazard has also been described as ex-post opportunistic behavior (or hidden actions).

of individuals may be chosen by individuals of a different type. The 'selection' made by the latter has 'adverse' consequences for the functionning of both private and social insurance.[9] Interestingly, in a recent paper, Palacios (2014) argues that in the case of ICLs for financing higher education, the moral hazards and adverse selections are less severe than one might think. He points out that students are likely to have less information about their future prospects than their lenders, and that the risk of shirking is likely to be small as a graduate would know that working hard earlier on in her career is a good form of investment.[10]

Because of moral hazard and adverse selection, the insurance market and the credit market are imperfect, and as a result, the laissez-faire outcome does not achieve an efficient allocation of resources. This by itself does not necessarily mean that a government can always achieve a better outcome, unless there are reasons to believe that the government has access to better information, or has the ability to enforce participation in a market (as in the case of universal health insurance).

2.3 ICLs and Social Insurance

As a general rule, the government should not interfere in the normal operations of the market. However, when an insurance market does not exist or functions very poorly, and the risks that uninsured individuals face are substantial, there are compelling reasons for the government to provide insurance. In fact, the most important lottery in a person's life is the lottery of birth: some people are lucky to be borne healthy and endowed with capabilities to function well, while others are severely disadvantaged. Since individuals cannot buy an insurance contract that would compensate them for the bad luck surrounding the circumstances of their birth, there is a strong case for social insurance. While it is not possible, nor desirable, to iron out all the differences in life circumstances, the expected life-time welfare of an (as yet unborn) individual should be maximized by a system of social insurance. With this perspective in mind, it is interesting to note that social security, which has been perceived as serving an equity objective, may also be viewed as an efficiency-inducing measure to achieve ex-ante efficiency, since one can argue that all

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[9]Adverse selection is often associated with ex-ante opportunistic behavior (sometimes described as hidden characteristics). It has been shown that, in some cases, the adverse selection problem can be so severe that a market fails to exists.

[10]Palacios wrote: "I had very little knowledge on what my future income was going to be. I had a vague idea of how much it could be, but the standard deviation was huge." (Palacios, 2014, p. 208).

individuals are ex-ante identical.

The provision of ICLs for higher education is a form of social insurance. The most obvious reason for advocating government-run ICLs is that there are substantial economies of scope in debt collection thanks to the income tax system. These economies of scope imply that government-operated ICLs are more cost-efficient than any alternative scheme operated by the private market. The second reason is that unlike the private market, in modern democracies, the government, chosen by the electorate, has an interest in maximizing a social welfare function. Socially provided insurance typically involves forced pooling, which is desirable from the point of view of equity, as it enhances equality of opportunities. In addition, reducing the risk of not being able to repay education loans will lead to more investment in education, and this yields benefits to the government in the forms of a more enlightened electorate and a higher tax revenue from a well educated workforce. A private lender would not take these benefits into account.

Income contingent loans, whereby repayment rates are made dependent on the income level of the debtors, have been increasingly accepted as a great idea for improving efficiency and equity in situations where the operations of the private credit and insurance markets are severely affected by asymmetric information giving rise to moral hazard and adverse selection problems (Chapman 2006, 2010; Barr and Johnston, 2010; Jacob and van der Ploeg, 2006; Gary-Bobo and Trannoy, 2013; Findeisen and Sachs, 2016; Stantcheva, 2017). The success of Australia's income contingent loans for financing university education has been acclaimed as a case where a combination of judicious economic analysis, practical ideas, and a good administrative system for collecting repayments, can mitigate the failure of the market.

The present paper outlines an approach that provides a theoretical basis for providing income contingent loans with piecewise linear repayment rates. The approach combines efficiency and equity considerations, while keeping in mind that policy rules should be simple.

The efficient allocation of resources is a major objective of economic policy designs. The dominant criterion for efficiency is Pareto efficiency. In general there exists many Pareto efficient allocations. Equity considerations can be imposed as constraints that narrow down the range of acceptable efficient allocations. The optimization of a social welfare function without equity constraints may fail to deal with the equity issues in a

satisfactory way.[11]

3 A simple model of ICLs

This section shows how ICLs can improve welfare in a context of imperfect credit market. Let us begin with a simple model of education financing for a cohort of students. In period $t = 0$, they all borrow the same amount money to pay for their education, which costs D_1 dollars per person. The students take the same course of study. In this sense, the education decision is exogenous. I assume that a fraction of unsuccessful students will default on the their debts in period 1. There is a constant and risk-free interest rate r, but since lending to students is risky, a higher interest rate, denoted by r^*, is charged on student loans. As the financial institutions must break even, the interest payments received from the non-defaulting students must be sufficient to recover the opportunity cost of fund. Let π be the fraction of students who default. Then the break-even condition is that r^* must satisfy the condition

$$(1 + r^*)\pi D_1 = (1 + r)D_1$$

That is,

$$r^* = \left(\frac{1 + r}{\pi}\right) - 1$$

To enable the borrower to smooth out consumption, the financial institutions could spread out repayments in T' equal annual amounts, $\overline{R}$, where the payment is to be made at the end of each period, $t = 1, 2, ..., T'$. Here, we assume that T' is a fixed number, and $T' < T$. If there were no risk of default, $\overline{R}$ would be chosen such that

$$\left(a + a^2 + ... + a^{T-1} + a^{T'}\right)\overline{R} = (1 + r^*)K$$

i.e.

$$a\left(\frac{1 - a^{T'}}{1 - a}\right)\overline{R} = (1 + r^*)K$$

where $a = 1/(1 + r)$.

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[11]Unless the objective function itself embodies a mixture of welfare and rights, as proposed by Long and Martinet (2018).

We consider two sources of heterogeneity among graduates. First, they differ in innate ability, which we represent by the symbol ω, a real number that can take any value in the real interval $[\underline{\omega}, \overline{\omega}] \equiv \Omega$, where $0 < \underline{\omega} < \overline{\omega}$. Individuals know their innate ability, but this is private information. The planner only knows the distribution function $F(\omega)$, where $F(\underline{\omega}) = 0$ and $F(\overline{\omega}) = 1$. We assume that a graduate's labor productivity is $\omega_i \theta_{it}$, where θ_{it} represents a random shock that occurs in period t, and is specific to individual i. Without loss, assume that θ_{it} has a density function $f(\theta_{it})$ that is strictly positive over the interval $[\underline{\theta}, \overline{\theta}]$ and its mean is $E\theta_{it} = 1$. Adverse productivity shocks, for example due to ill health, are represented by a fall in θ_{it}. Our model admits the possibility of future defaults by successful students, especially by those who are adversely affected by a sequence of unexpectedly bad personal productivity shocks. To deal with such default risks, the financial institutions could require that each period's repayment be at least equal to some minimum repayment $\widetilde{R}$, and specify that the interest rate on the balance of the loan, D_t, for $t = 1, 2..., T'$ is set at some level $\widetilde{r} > r$, such that the balance evolves according to the difference equation.

$$D_{t+1} - D_t = \widetilde{r}D_t - R_t, \ R_t \geq \widetilde{R}$$

Here $\widetilde{R}$ is specified such that if $R_t = \widetilde{R}$ for all $t = 1, 2, ..., T'$, then $D_{T'+1} = 0$. To be precise, $\widetilde{R}$ is the solution of the equation

$$b \left(\frac{1 - b^{T'}}{1 - b} \right) \widetilde{R} = (1 + r^*)K,$$

where $b = 1/(1 + \widetilde{r})$. (It follows that $\widetilde{R} > \overline{R}$, which in intuitively plausible.)

In what follows, we will show that the inflexibility of the requirement that $R_t \geq \widetilde{R}$ causes a welfare loss to workers who are adversely affected by negative productivity shocks. It would be better to have a system where repayments are made dependent on current earnings. To do this, we consider the case where the individual is not obliged to pay back a minimum amount $\widetilde{R}$ per period. The only requirement is that the debt is eventually paid back before the individual retires from the work force. We show that in this case, the individual's time profile of optimal debt repayments over her working life will be positively correlated with her earnings: in years where she suffers from an adverse productivity shock, her repayment is typically low, and in good years, she increases her repayment.

This shows that an arrangement whereby debt repayment is contingent on earnings is economically efficient.

Let us specify the choices that individuals face concerning consumption, labor supply, and asset accumulation.

Assume that individual i's earnings at time t, denoted by Y_{it}, is a function of her labor supply, $L_{it} \in [0, \overline{L}]$, and her productivity level in period t, denoted by $\omega_i \theta_{it}$, such that

$$Y_{it} = \omega_i \theta_{it} L_{it}$$

The individual's type is her private information and is assumed to be constant across periods. At the beginning of each period t, the individual observes the realisation of the random variable θ_{it} before she makes her labor supply and consumption decision. The realised value θ_{it} is also private information. We assume that $E_t[\omega_i \theta_{it}] \overline{L} > \widetilde{R}$. This means that if the individual's labor supply is at the upper bound $\overline{L}$ in all periods of her working life, her expected earnings will be more than sufficient to pay off her debts. However, it may not be in her interest to supply the maximum labour, $\overline{L}$, because the marginal disutility of working at L near $\overline{L}$ may be very high.

In the remainder of this section, we will omit the subscript i for simplicity. We consider three phases in the life of a representative graduate who does not default. Phase 1 begins in period 1 (after graduation) and ends at an endogenously determined date T' (the date at which the debt balance becomes zero). During this phase the individual works, consumes, and and repays her debts. Phase 2 begins as soon as the stock of debt falls to zero, and during this phase she works, consumes, and accumulates financial assets. This phase ends at time T (exogenous retirement date). In Phase 3, she is a retiree and finances her consumption for the remaining part of her life from past savings. We assume Phase 3 begins at time T (exogenous) and ends at time $T + x$, where x is also exogenous.

In reality individuals also have the option of defaulting on their debts by declaring bankruptcy. If an economic agent has been hit by a sequence of adverse productivity shocks, it is rational for them to exercise this option, even though declaring bankruptcy does involve some costs (e.g. the inability to raise loans for the purchase of durable goods due to bad credit rating). In the remaining part of this section, we focus on the case of no default. At the end of section 3, we discuss how the decision to default can be modelled.

Note that in any period t, the individual can choose to accumulate a stock of financial

assets $A_t \geq 0$ that earns the risk-free rate of interest r. However, it will become clear that since $r < \tilde{r}$, she will find it optimal to repay her debt before starting to accumulate financial assets. Let I_t denote the additional investment in financial assets. Then

$$A_{t+1} - A_t = rA_t + I_t \tag{1}$$

where I_t equals her earnings, $\omega\theta_t L_t$, minus her debt repayment R_t and her consumption, C_t:

$$\omega\theta_t L_t - C_t - R_t - I_t = 0 \tag{2}$$

In principle, I_t can be positive or negative. Since we assume that the individual cannot borrow against her future income, given any positive current asset level, $A_t \geq 0$, we require that her next period's A_{t+1} be non-negative. In view of equation (1), this requirement is equivalent to the requirement that

$$I_t + (1+r)A_t \geq 0 \tag{3}$$

It is straightforward to check that the individual would want to pay off his debt first, before accumulating her asset stock A_t. Thus, her optimal portfolio decision implies that there is no period t such that both D_t and A_t are strictly positive.

Consumption c_t yields a utility level $u(C_t)$ while labor supply L_t yields a disutility $\phi(L_t)$. We assume that $u(.)$ is increasing and strictly concave, and $\phi(.)$ is increasing and convex, with

$$\lim_{L \to 0} \phi'(L) = 0.$$

The individual's net utility in period t during her working life is denoted by U_t, where

$$U_t \equiv u(C_t) - \phi(\dot{L_t}) = u(\omega\theta_t L_t - R_t - I_t) - \phi(L_t). \tag{4}$$

After retirement, the individual no longer works, i.e. $L_t = 0$, and she finances her consumption by withdrawing from her bank account, i.e., $I_t < 0$, and $C_t = -I_t > 0$, for $t > T + 1$.

The individual seeks to maximize her expected life-time welfare, defined as

$$E \sum_{t=1}^{T+x} \beta^{t-1} U_t \tag{5}$$

where β is her utility discount factor. It is related to her rate of time preference $\rho > 0$ by the equation

$$\beta = \frac{1}{1+\rho} < 1.$$

To simplify the analysis, we adopt the standard assumption that the rate of time preference ρ is equal to the riskless rate of interest r.[12] We state this as Assumption A1.

Assumption A1: The rate of time preference ρ is equal to the riskless rate of interest r :

$$\beta(1+r) = 1. \tag{6}$$

For the moment, for simplicity, we abstract from default after graduation, by assuming that the cost of declaring bankruptcy is so high that the individual prefers to supply enough labour time per period to payback the debt before time T. The analysis that follows gives us the following result:

Proposition 1: *For an individual that does not default, the optimal time path of consumption, labor supply, debt repayment, and asset accumulation satisfies the following properties. There are three phases. In Phase 1, the individual works to earn enough income to reduce her debt burden to zero. In this phase, her expected labor supply is high, and it gradually falls over time, while her expected consumption rises over time. In periods with high realization of the productivity shock θ_t, she supplies more labor, earns more income, and increases her repayments. Phase 2 begins immediately after the debt has been fully repaid. During Phase 2, planned consumption may initially rise, but eventually it reaches a plateau. In Phase 3, the individual lives in retirement. Consumption is constant during this phase, and the individual's stock of financial asset declines gradually.*

The proof of Proposition 1 proceeds as follows. Using the backward solution method, we first consider the post-retirement phase, Phase 3, and characterize the optimal consumption decision after the retirement date $T + 1$, for any given asset level $A_{T+1} > 0$. In this phase, since the individual does not work, the optimization problem is deterministic.

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[12] This is a standard assumption. See for example Obstfeld and Rogoff (1996).

The individual chooses a sequence of consumptions $C_{T+1}, C_{T+2}, ..., C_{T+x}$ to maximize the utility over the remaining life time:

$$V_{T+1}(A_{T+1}) = \max \sum_{j=1}^{x} \beta^{j-1} u(C_{T+j})$$

subject to

$$A_{T+j+1} - A_{T+j} = rA_{T+j} - C_{T+j}, \text{ where } j = 1, 2, ..., x, \text{ and } A_{T+1} \text{ given,}$$

and

$$A_{T+x+1} = 0.$$

Since $\beta(1 + r) = 0$, we obtain the result that her optimal consumption level is a constant $\overline{C} > 0$ during the retirement phase. Then, using the consumer's intertemporal budget constraint over the retirement phase, we can solve for $\overline{C}$:

$$\overline{C} = \left(\frac{1-a}{1-a^x}\right) A_{T+1}$$

where $a = 1/(1 + r)$. Thus the individual's welfare over the retirement phase is

$$V_{T+1}(A_{T+1}) = \left(\frac{1-a^x}{1-a}\right) u\left(\left(\frac{1-a}{1-a^x}\right) A_{T+1}\right)$$

For example, suppose that $u(C) = (1 - \frac{1}{\sigma})^{-1} C^{1-\frac{1}{\sigma}}$, where $\sigma > 0$ is the elasticity of intertemporal substitution.[13] Then

$$V_{T+1}(A_{T+1}) = \left(\frac{1-a}{1-a^x}\right)^{-\frac{1}{\sigma}} \left(1 - \frac{1}{\sigma}\right)^{-1} (A_{T+1})^{1-\frac{1}{\sigma}}$$

Now, we turn to Phase 2 of the consumer's working life, which begins at the beginning of period $T' + 1$, such that $D_{T'+1} = 0$ and $A_{T'} = 0$. It is possible that $A_{T'+1}$ is strictly positive, because in period T', the individual may earn more than sufficient to finance her

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[13]This functional form is commonly used in empirical applications, both in microeconomics and in macroeconomics. Macroeconomic estimates of the elasticity of intertemporal substitution tend to be below unity. However, recently Barro (2009) pointed out that such estimates are biased downwards, and proposed that correct estimates should be around $\sigma = 2$.

planned consumption $C_{T'}$ and to pay back her debt, $(1+\widetilde{r})D_{T'}$. Then, in that case, since $A_{T'} = 0$, we have $A_{T'+1} = \theta_{T'}L_{T'} - (1+\widetilde{r})D_{T'} - C_{T'} > 0$.

In Phase 2, given $A_{T'+1}$, and given the observed realization $\theta_{T'+1}$, the consumer chooses the sequence of labor supply L_t (where $t = T'+1, T'+2, ..., T$), and investment I_t and hence, consumption, $C_t = \omega\theta_t L_t - I_t$, to maximize her expected life-time wellbeing over the time horizon $[T'+1, T+1+x]$

$$\max E_{T+1} \sum_{j=1}^{T-T'} \beta^{j-1} \left\{ u(\omega\theta_{T'+j}L_{T'+j} - I_{T'+j}) - \phi(L_{T'+j}) \right\} + \beta^{T-T'}V_{T+1}(A_{T+1})$$

subject to the transition equation

$$A_{T'+j+1} = (1+r)A_{T'+j} + I_{T'+j}$$

where $j = 1, 2, ..., T - T'$, and $A_{T'+1}$ and $\theta_{T'+1}$ are given. Also, one should take into account the constraint (3).

For any time $T'+j$ where $j = 1, 2, ..., T - T'$, let $V_{T'+j}(A_{T'+j}; \omega\theta_{T'+j})$ denote the value function, given the her knowledge of $A_{T'+1}$ and $\theta_{T'+1}$. The value function must satisfy the Bellman equation:

$$V_{T'+j} = \max_{L,I} \left\{ u(\omega\theta_{T'+j}L_{T'+j} - I_{T'+j}) - \phi(L_{T'+j}) + E_{T'+j}\beta V_{T'+j+1} \right\} \tag{7}$$

This yields the first order conditions

$$\omega\theta_{T'+j}u'(\omega\theta_{T'+j}L_{T'+j} - I_{T'+j}) = \phi'(L_{T'+j}) \tag{8}$$

$$-u'(\omega\theta_{T'+j}L_{T'+j} - I_{T'+j}) + E_{T'+j}\beta V'_{T'+j+1} \leq 0 \ (\ = 0 \text{ if } A_{T'+j+1} > 0) \tag{9}$$

where $V'_{T'+j+1}$ denotes the derivative of the value function $V_{T'+j}(A_{T'+j}; \omega\theta_{T'+j})$ with respect to $A_{T'+j}$.

Furthermore, applying the envelope theorem to equation (7), we have

$$\frac{\partial V_{T'+j}(A_{T'+j}; \omega\theta_{T'+j})}{\partial A_{T'+j}} = E_{T'+j}\beta(1+r)\frac{\partial V_{T'+j+1}(A_{T'+j+1}; \omega\theta_{T'+j+1})}{\partial A_{T'+j+1}} \tag{10}$$

It follows from Assumption A1 and equation (10) that the asset is accumulated (or

decumulated) so that its expected marginal value is the same in every period. Further-more, if $A_{T'+j}$ is strictly positive for all $j = 1, 2, ..., T - T'$ then, from equation (9), the expected marginal utility of consumption is constant in all periods over Phase 2. This is the consumption smoothing property of the optimal plan. Note, however, that if there are periods where the constraint (3) is binding, so that $A_{T'+j}$ is zero (indicating that perhaps because of adverse productivity shocks, the consumer would like to borrow against future income, but cannot), then the individual must consume less in those periods.

Finally, we turn to Phase 1, over which the individual aims to pay off the student loan debts, and she has no financial assets. The individual chooses a sequence of debt repayment R_t and labor supply L_t to maximize her life-time welfare:

$$E_1 \sum_{t=1}^{T'} \beta^{t-1} \left\{ u(\omega\theta_t L_t - R_t) - \phi(L_t) \right\} + \beta^{T'-1} E_1 V_{T'+1}(A_{T'+1}, \omega\theta_{T'+j})$$

subject to the transition equation

$$D_{t+1} = (1 + \widetilde{r})D_t - R_t$$

where D_1 is given. In this phase, the state variables are the debt level, D_t, and the productivity level $\omega\theta_t$. We denote the value function in Phase 1 by $V_t^1(D_t; \omega\theta_t)$. The Bellman equation is

$$V_t^1(D_t; \omega\theta_t) = \max_{L,R} \left\{ u(\omega\theta_t L_t - R_t) - \phi(L_t) + \beta E_t V_{t+1}^1(D_{t+1}; \omega\theta_{t+1}) \right\}$$

This yields the first order conditions

$$\omega\theta_t u'(\omega\theta_t L_t - R_t) = \phi'(L_t) \tag{11}$$

$$-(1 + \widetilde{r})\beta E_t \frac{\partial V_{t+1}^1(D_{t+1}; \omega\theta_{t+1})}{\partial D_{t+1}} = u'(\omega\theta_t L_t - R_t) \tag{12}$$

Since $u' > 0$, this equation implies that $E_t \frac{\partial V_{t+1}^1(D_{t+1}; \omega\theta_{t+1})}{\partial D_{t+1}} < 0$. We define the "shadow price of debt" to be the positive number $-E_t \frac{\partial V_{t+1}^1(D_{t+1}; \omega\theta_{t+1})}{\partial D_{t+1}}$.

Using the envelope theorem, we have

$$-\frac{\partial V_t^1(D_t; \omega\theta_t)}{\partial D_t} = -(1+\widetilde{r})\beta E_t \frac{\partial V_{t+1}^1(D_{t+1}; \omega\theta_{t+1})}{\partial D_{t+1}}$$

or

$$\frac{-\frac{\partial V_t^1(D_t; \omega\theta_t)}{\partial D_t}}{-E_t \frac{\partial V_{t+1}^1(D_{t+1}; \omega\theta_{t+1})}{\partial D_{t+1}}} = (1+\widetilde{r})\beta \tag{13}$$

Since $\widetilde{r} > r$, we have $(1+\widetilde{r})\beta > 1$. Thus equation (13) indicates that the shadow price of debt is decreasing over time, as debt is being reduced gradually. This in turn implies, via equation (12), that expected marginal utility of consumption is falling over time, i.e., consumption is expected to increase over time during Phase 1. To illustrate, suppose that $\omega\theta_t$ is constant over this phase. Then the rising consumption implies, via equation (11), that the individual labor supply falls over time. This is a plausible result: the individual makes labor supply decision to earn enough income to reduce the debt burden, and as the debt is falling, the individual can afford to reduce gradually the number of working hours per period.

Remark 1: So far we have considered the case where the individual is not required to pay back a minimum amount $\widetilde{R}$ per period. It is clear that introducing such a requirement would reduce the individual's welfare: Adding a constraint to an optimization problem will necessarily reduce the value of the objective function, if the constraint is expected to be binding in some periods. Thus, we can state**:** *Imposing a lower bound on the individual's repayment per period will reduce her welfare.*

Finally, let us discuss briefly the case where the bankruptcy cost is low enough such that agents who have suffered a series of very adverse productivity shocks will find it optimal to declare bankruptcy. This can be modelled as an optimal stopping problem, and the analysis would be very similar to models of sovereign debt defaults in the international economics, as in e.g. Arellano (2008), and Bianchi (2011). The major difference is that in the latter literature, the time horizon is infinite, while in the student loans model, the time horizon is finite. Basically, one must determine the conditions under which it becomes optimal for an individual to declare bankruptcy and incur the penalty associated with default. In our model, at the beginning of any given period, the agent has observed the "state of the system", which consists of three variables: her current level of debt,

her current productivity, and and how many periods remain at her disposal before her retirement. Let $(D_t, \omega\theta_t, z_t)$ denote this observation at time t. Her strategy is to choose a decision rule that maximizes her expected utility. As is usual in the optimal stopping literature, there exists two regions in the $(D_t, \omega\theta_t, z_t)$ space, called the "continuation region" and the "stopping region." If $(D_t, \omega\theta_t, z_t)$ lies in the stopping region, she declares bankruptcy; otherwise, she continues with the regime of making installments to pay back her debt. Clearly, if D_t is large (because of past inability to make sufficient repayments, due to a sequence of bad productivity shocks) and z_t is small (i.e., there remains very few periods before her retirement) then (D_t, z_t) belongs to the stopping region, and it is individually optimal to file for bankruptcy.[14] The boundary of the stopping region depends on factors such as how big the penalty is, and how likely the agent will encounter a future stream of good productivity shocks that would enable her to earn more than sufficient for debt repayment. While it is possible to map out the stopping region if enough specific assumptions are made about the distribution of the productivity shocks and about the utility function, we do not attempt to carry out that task here due to lack of space and negligible value added.[15]

4 A refinement of ICLs: piecewise linear repayment schedule

In the preceding section, we considered the optimization problem of a representative individual, and showed that because of the universal desire to smooth consumption across periods and across states of nature, the individual is better off if her debt repayment is permitted to be low in periods when her earning is low, even if she is required to eventually pay off her debt in full. The model can be extended to allow for the possibility that individuals who are adversely affected by a series of bad shock are not required to repay their debts in full. For the scheme to break even, this implies that individuals who are

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[14]The boundary of the stopping region depends on whether the discharge of student loan debt in bankrupcy is allowed (as in Australia) or not (as in the US). Ionescu (2009, p. 206) reported that recent legislations in the US, which make student loans non-dischargeable under Chapter 13, have deterred students from declaring bankruptcy.

[15]The issue of bankruptcy in student loans has received a very detailed treatment in Ionescu (2011). The paper simulates bankruptcy characteristics of the student loan market and compares alternative bankruptcy regimes.

luckier than average will end up paying more than the average person does. This is of course the essence of social insurance. Ex post, there is cross subsidization, while ex ante all individuals are equally treated.

There is a parallel between income contingent loans and redistributive taxation that maximizes social welfare. A wealthy person pays more tax than average. He has no ground to complain if he realizes that he could have been born into a poor family. It is morally compelling to take the view that individuals ought to make their choice among alternative social security systems behind the veil of ignorance. One's circumstances at birth are random events that everyone is subject to. Ex ante, we are identical, but ex post, we are heterogeneous. It becomes important to treat individuals with different life circumstances in different ways, while preserving the principle of anonymity.

In this section, we argue that social welfare can be improved by replacing a linear income-contingent repayment schedule with a piecewise linear one. The idea is related to the literature on optimal income taxation. The approach pioneered by Mirrlees (1971) is a useful framework to address the question of social insurance and redistribution. Individuals differ in their earning capacity and face different shocks. Society would like help out people who are less advantaged than others. This is partially a reflection of the view that if we do not know our ability and health, we would like to be insured by society. Behind the veil of ignorance, we would all want to seek insurance.

A well known difficulty with the provision of of insurance and redistribution is that abilities may be private information, and under these circumstances, individuals who are not in need may pretend to be those to whom redistribution is intended. Thus, some able individuals that face a progressive tax system may decide to work less hard. The optimal tax scheme or insurance program must balance the welfare gains from redistribution with the efficiency loss of reduced work incentives. The Mirrlees framework has recently been generalized to a dynamic setting. Contributions to this "New Dynamic Public Finance" (NDPF) theory include Golosov, Kocherlakota and Tsyvinski (2003), Kocherlakota (2005), Golosov and Tsyvinski (2006, 2007), Golosov, Troshkin and Tsyvinski (2016), and Golosov, Tsyvinski and Werquin (2016).[16]Concerning the insurance market, Golosov and Tsyvinski (2007) have shown that while the private market can provide a significant

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[16]Golosov et al. (2003) show that if individuals face serially correlated productivity shocks, the optimal tax scheme requires a positive implicit tax on capital. Thus the standard result that capital income tax should be zero is no longer applicable when there are heterogeneous individuals with a stochastic process of private information.

amount of insurance, it fails to internalize pecuniary externalities, and consequently the government can improve welfare by additional taxation measures. Building on the work of Piketty (1997) and Saez (2001) that deals with static redistribution, Golosov, Tsyvinski and Werquin (2016) have developed a method to optimize with respect to the tax function in a multi-period context.

In a recent paper, Findeisen and Sachs (2016) apply the NDPE to the design of education finance and tax system. They consider a two-period model. In the first period, agents study, and in period 2 they join the workforce. There are two types of agents: those with high inate ability, and those with low inate ability (these are private information). In period 1, agents make binary education decisions: 'high' or 'low' level of education expenditure. In period 2, they enter the labor market, and draw their labor market ability from a continuous cumulative distribution function, which is conditional on both their innate ability and education expenditure. The authors consider a sequential mechanism: in period 1, agents report their type, and in period 2, they report their labor market ability. The planner, upon receiving the first period reports, assigns period-1 consumption level and education expenditure. Individuals are offered a menu of second period utility. Education, consumption, and income are observable. Dynamic (two-period) incentive compatibility is imposed. The authors consider a second-best allocation scheme, and find that an incentive-compatible allocation can be implemented by a compulsory loan menu and a loan-repayment menu. Using simulation, it is found that there is a case for loan repayment that increases with income, at least for low and intermediate income levels. The authors find that the computed welfare gains from ICLs is increasing in the degree of risk aversion.

A related paper is Stantcheva (2017), where human capital accumulation is modelled as taking place over the individual's entire working life. The aim was to determine whether human capital expenses should be fully tax deductable. She finds that if the wage elasticity with respect to ability is increasing in human capital, then the optimal tax deductibility is less than full. The optimum can be implemented by a scheme of income-contingent loans or a deferred tax deductiblity scheme, both of which are age-dependent. For ICLs, the optimal loan repayment schedules are contingent on the whole history of earnings and human capital investments. Being built on the NDPE framework, Stantcheva's results are, not surprisingly, too complicated for practical application. For example, it is required that tax deductions must depend in a complicated way on the age of the tax payers.

Our view is that, for practical policy purposes, one should heed the advice of Diamond and Saez (2001) that policy prescriptions should be fairly simple and should not go against commonly held normative view. Along this line of thought, a model of optimal piece-wise linear income tax has been proposed by Apps, Long and Rees (2014). In this section, we take this practical approach and apply it to the design of income-contingent loans.

The model of Apps, Long and Rees (2014) was formulated in a static framework: individuals live for only one period. In our model, loan repayment is a dynamic process, as it takes place over many periods. Our approach is a generalization of the model of Apps et al. (2011) to account for the dynamics of debt repayments, as will become clear in equations (14)-(22).

Recall that in Section 3, we posit that individuals are heterogeneous with respect to their inate ability ω and they are subjected to idiosyncratic productivity shocks θ_{it}. The social planner would want to affect transfers from high-productivity individuals to low-productivity ones. If ω and θ_{it} were observable, then the redistribution problem would be relatively easy to solve, though, of course, there are still disagreements on what would be an appropriate social welfare function: one could choose the Rawlsian maximin, or one of the various versions of utilitarianism, or some other criteria, such as the Rights and Welfare Index proposed by Long and Martinet (2018). In this paper, we take the view that social preferences display inequality aversion.[17] This means that the social welfare function is a weighted sum of individual levels of wellbeing, where worse off individuals receive a greater weight. This is in line with the optimal income taxation approach advocated by Mirrlees (1971), where the government maximizes the sum of a concave transformation of individual utilities. It is as though society attaches greater weights to the utilities of worse off individuals. To make the discussion concrete, let us consider the following simple two-period example. Assume that in period 1, the graduates enter the workforce with an education debt $D_1 > 0$. Let $\omega_i \theta_{it} > 0$ denote individual i's labor productivity in period t. Here, $\omega_i > 0$ is the individual's type (which is constant across periods), and $\theta_{it} > 0$ is a random variable representing her productivity shock. Let her labor supply (effort level) in period t be denoted by L_{it}. Her output is $Q_{it} = \omega_i \theta_{it} L_{it}$. Following Mirrlees (1971), we assume that Q_{it} are observable and verifiable, while ω_i, θ_{it} and L_{it} are private information. For concreteness, let us suppose that the utility function

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[17]Inequality aversion is well documented in behavioral economics, see for example Fehr and Schmidt (1999), and Bolton and Ockenfels (2000).

takes the following form:

$$U_{it} = \ln C_{it} - \frac{L_{it}^2}{2} \qquad (14)$$

where C_{it} is the consumption level, and L_{it} is the labor supply. For simplicity, we assume that the income tax rate is constant, denoted by $\tau > 0$. We assume that the ICLs planner must take the income tax rate as given. The individual's after-tax income is

$$Y_{it} = (1 - \tau)Q_{it}.$$

While it is feasible for the individual to pay back all her debt (principal plus interest) in period 1, this can be very painful if $\omega_i \theta_{i1}$ is low. The objective of the government is to design a relatively simple debt repayment scheme that maximizes a social welfare function which is the sum of expected life-time utilities of a cohort of heterogenous individuals.

Recall that all individuals have the same initial debt, D_1. For any invididual i if her repayment in period 1(made at the end of period 1) is R_{i1}, her debt balance at the beginning of period 2 is

$$D_{i2} = (1 + r)D_1 - R_{i1}$$

If the debt D_{i2} must be cleared, the individual would have to pay the amount $R_{i2} = (1 + r)D_{i2}$ at the end of period 2. An income-contingent repayment scheme, however, allows for the possibility that individuals that are struck by a very adverse productivity shock (i.e., a very low θ_{i2}) are required to pay back only a fraction of her debt. Consider the following period 2 scheme. In period 2, individuals whose after-tax income Y_2 is below the threshold level $\widehat{Y}$ are required to pay back only an amount equal to bY, where $0 < b < 1$ is the marginal repayment rate set by the planner. Of course, if $(1 + r)D_2$ is smaller than bY, then the required repayment is equal to $(1 + r)D_2$. Individuals whose after-tax income Y_2 exceeds $\widehat{Y}$ must pay $b\widehat{Y} + b_H(Y - \widehat{Y})$, where b_H is the marginal repayment rate for high income earners. Under a progressive repayment scheme, we have $1 > b_H > b$. If $b\widehat{Y} + b_H(Y - \widehat{Y}) > (1 + r)D_2$, then the required repayment is $(1 + r)D_2$. A similar scheme operates in period 1.

At the beginning of period 1, an individual i with private information $\omega_i \theta_{i1}$ must decide on her effort level L_{i1} and her effort level in period 2 (conditional on her information θ_{i2} that will be available at the beginning of period 2).

Let us solve the individual's optimization problem by backward induction. At the

beginning of period 2, the shock θ_{i2} is revealed to the individual. Thus, given θ_{i2} and D_{i2}, her optimization problem in period 2 is to choose her effort (labor supply) to maximize period 2 utility. Noting that there is a unique linear relationship between L_{i2} and Y_{i2}, we find it convenient to use Y_{i2} as the choice variable. Then, after substituting $\frac{Y_{i2}^2}{2(\omega_i\theta_{i2})^2}$ for L_2 in eq. (14) we consider the problem

$$\max_{Y_{i2}} \left\{ \ln C_{i2} - \frac{Y_{i2}^2}{2(\omega_i\theta_{i2})^2} \right\} \tag{15}$$

subject to

$$C_{i2} = Y_{i2} - R_{i2} \tag{16}$$

where

$$R_{i2} = \begin{cases} \min\left[bY_{i2}, (1+r)D_{i2}\right] & \text{if } Y_{i2} < \widehat{Y} \\ \min\left[b\widehat{Y} + b_H(Y_{i2} - \widehat{Y}), (1+r)D_{i2}\right] & \text{if } Y_{i2} \geq \widehat{Y} \end{cases} \equiv R_{i2}(Y_{i2}; D_{i2}) \tag{17}$$

The first order condition for an interior maximum for problem (15) is[18]

$$\frac{1}{Y_{i2} - R_{i2}} \frac{d\left[Y_{i2} - R_{i2}\right]}{dY_{i2}} = \frac{Y_{i2}}{(\omega_i\theta_{i2})^2} \tag{18}$$

This equation determines her optimally determined after-tax income level, Y_{i2}^* (and hence her effort level, L_{i2}^*). The left-hand side is the marginal benefit (reflecting the increase in consumption) of an increase in after-tax income brought about by an increase in effort. The right-hand side reflects the marginal effort cost required by an increase in after-tax income. It is clear from the first order condition (18) that if the productivity $\omega_i\theta_{i2}$ is low, the individual will choose to end up with a low income, and thus effectively default (being unable to pay back all her debt). We can calculate her expected end-of-period stock of debt, $D_{i3} = (1+r)D_{i2} - R_{i2}$ as a function D_{i2}.

Note that, from (18) and (17), Y_{i2}^* is a function of D_{i2} and $\omega_i\theta_{i2}$, as well as of the parameters $\widehat{Y}$, b and b_H. From $Y_{i2}^*(D_{i2}, \omega_i\theta_{i2}, \widehat{Y}, b, b_H)$, we can deduce her optimal consumption, $C_{i2}^* = Y_{i2}^* - R_{i2}(Y_{i2}^*; D_{i2}) = C_{i2}^*(D_{i2}, \omega_i\theta_{i2}, \widehat{Y}, b, b_H)$. Then, after substituting these optimal values into the objective function of the individual, we obtain the individual's

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[18]Whenever the derivative of R_{i2} with respect to Y_{i2} does not exist, the FOC (18) must be modified.

maximized utility level in period 2, and denote it by the function $V_2(\omega_i \theta_{i2}, D_{i2}; \widehat{Y}, b, b_H)$.

Turning now to period 1, the individual's optimization problem is

$$\max_{Y_{i1}} \left\{ \left[\ln C_{i1} - \frac{Y_{i1}^2}{2(\omega_i \theta_{i1})^2} \right] + \beta E V_2(\omega_i \theta_{i2}, D_{i2}; \widehat{Y}, b, b_H) \right\} \tag{19}$$

where

$$C_{i1} = Y_{i1} - R_{i1} \tag{20}$$

$$D_{i2} = (1+r)D_{i1} - R_{i1} \tag{21}$$

$$R_{i1} = \begin{cases} \min\left[bY_{i1}, (1+r)D_{i1}\right] & \text{if } Y_{i1} < \widehat{Y} \\ \min\left[b\widehat{Y} + b_H(Y_{i1} - \widehat{Y}), (1+r)D_{i1}\right] & \text{if } Y_{i1} \geq \widehat{Y} \end{cases} \equiv R_{i2}(Y_{i1}; D_{i1}) \tag{22}$$

The first order condition is

$$\frac{1}{Y_{i1} - R_{i1}} \frac{d\left[Y_{i1} - R_{i1}\right]}{dY_{i1}} - \frac{Y_{i1}}{(\omega \theta_{i1})^2} + \beta \frac{\partial E V_2(\omega_i \theta_{i2}, D_{i2}; \widehat{Y}, b, b_H)}{\partial D_{i2}} \times \frac{dD_{i2}}{dY_{i1}} = 0 \tag{23}$$

This first order condition determines her period 1 after-tax income level, Y_{i1}^* (and hence her period 1 effort level, L_{i1}^*). It is dependent on her initial debt D_{i1} and her productivity $\omega_i \theta_{i1}$, as well as on $\widehat{Y}, b$, and b_H. Substituting for $Y_{i1}^*(D_{i1}, \omega_i \theta_{i1}, \widehat{Y}, b, b_H)$ into the objective function (19), we obtain the life-time welfare of individual i, denoted by $V_1(\omega_i \theta_{i1}, D_{i1}; \widehat{Y}, b, b_H)$. We can calculate her expected end-of-period stock of debt, $D_{i3} = (1+r)D_{i2}^* - R_{i2}^*$ as a function D_{i2}^*, where $D_{i2}^* = (1+r)D_{i1} - R_{i1}^*(\omega_i \theta_{i1}, D_{i1}; \widehat{Y}, b, b_H)$.

From the planner's point of view, θ_{i1} is unknown. The planner can form the expected life-time welfare of the representative individual of type ω

$$W(\omega, D_1; \widehat{Y}, b, b_H) \equiv E V_1(\omega_i \theta_{i1}, D_{i1}; \widehat{Y}, b, b_H) \tag{24}$$

Note that W is increasing in ω, because higher productivity types have higher income, on average. For earch type ω, the planner can also compute the expected end-of-period 2 outstanding debt, denoted by $ED_3(\omega, D_1; \widehat{Y}, b, b_H)$.

The planner knows the probability distribution of the productivity parameter ω. Following Mirrlees (1971), we assume that the planner is utilitarian and applies a monotone increasing and concave transformation $\Psi(W(\omega, D_1; \widehat{Y}, b, b_H))$ to individual utilities. Then, since there is a continuum of types distributed over a compact interval $[\omega_L, \omega_H] \equiv \Omega$ with

the density function $h(\omega)$, the "social welfare" is

$$S = \int_{\Omega} \Psi(W(\omega, D_1; \widehat{Y}, b, b_H)) h(\omega) d\omega \tag{25}$$

The planner's objective is to choose the repayment rates b and b_H and the threshold income level $\widehat{Y}$ to maximize S, subject to the constraint that the expected outstanding debt D_3 must not exceed a certain tolerable level $\overline{D}$:

$$\int_{\Omega} \left[ED_3(\omega, D_1; \widehat{Y}, b, b_H) \right] h(\omega) d\omega \leq \overline{D} \tag{26}$$

Let $\lambda \geq 0$ be the Kuhn-Tucker multiplier associated with the constraint (26). The optimal value of λ can be interpreted as the marginal social cost of public fund. Maximizing social welfare (subject to the constraint (26)) with respect to the repayment rates b and b_H and the threshold income level $\widehat{Y}$, we obtain the optimality conditions:

$$\int_{\Omega} \left\{ \Psi'(W) \frac{\partial W(\omega, D_1; \widehat{Y}, b, b_H)}{\partial \widehat{Y}} - \lambda \frac{\partial ED_3(\omega, D_1; \widehat{Y}, b, b_H)}{\partial \widehat{Y}} \right\} h(\omega) d\omega = 0 \tag{27}$$

$$\int_{\Omega} \left\{ \Psi'(W) \frac{\partial W(\omega, D_1; \widehat{Y}, b, b_H)}{\partial b} - \lambda \frac{\partial ED_3(\omega, D_1; \widehat{Y}, b, b_H)}{\partial b} \right\} h(\omega) d\omega = 0 \tag{28}$$

$$\int_{\Omega} \left\{ \Psi'(W) \frac{\partial W(\omega, D_1; \widehat{Y}, b, b_H)}{\partial b_H} - \lambda \frac{\partial ED_3(\omega, D_1; \widehat{Y}, b, b_H)}{\partial b_H} \right\} h(\omega) d\omega = 0 \tag{29}$$

and

$$\lambda \geq 0 \,, \; \overline{D} - \int_{\Omega} [ED_3(\omega)] \, h(\omega) d\omega \geq 0 \,, \; \lambda \left\{ \overline{D} - \int_{\Omega} [ED_3(\omega)] \, h(\omega) d\omega \right\} = 0 \tag{30}$$

Conditions (27) to (30) determine the optimal $b^*, b_H^*, \widehat{Y}^*$ and λ^*. These conditions have intuitive interpretations. For example, condition (27) states that the threshold income level $\widehat{Y}$ beyond which the marginal repayment rate takes on the higher value b_H must be chosen such that the socially weighted sum of marginal gain to the graduates for having a wider interval of low repayment rate b is equated to the marginal cost of public fund. Numerical values for these optimal repayment rates and income threshold can be obtained

once we have specified the probability distribution of ω and the transformation $\Psi(W)$.[19] It is expected that, for any plausible distribution of ω, if the concave transformation $\Psi(W)$ has a strong curvature (indicating that the inequality aversion is high) then the marginal repayment rate b (for the low income bracket) is very small relative to the marginal repayment rate b_H (for the high income bracket). The welfare gain by replacing a linear repayment scheme with an optimally chosen piece-wise linear repayment schemes is of course positive, because the former scheme belongs to the set of feasible piece-wise linear repayment schemes. The order of magnitude of the welfare gain is an empirical matter that is beyond the scope of this paper.

5 Conclusion

There is a clear case that the government has an important role to play in higher education financing. The market system does not provide an efficient loan scheme because of severe problems of moral hazard and adverse selection. While the government cannot eliminate these problems, it has considerable informational and enforcement advantages over the market. Moreover, ICLs provide a form of social insurance.

This paper proposes a refinement of existing ICLs schemes. Following Apps, Long and Rees (2014), we argue that a piecewise linear repayment schedule can improve the outcome in terms of both equity and efficiency. The model considers only the two bracket case, but clearly it can be extended to an arbitrary number of brackets.

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