Test 2, Version A (3:30 lecture)
Econ 134A, Winter 2012
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(a) (4 points) Sophia the Second of Somwherealandia is wondering how to invest her great wealth. She wants to have a 6-year geometric average of 5% on her investments. She assumes that her rate of return each of the next 6 years will be 4%, 6%, 1%, 9%, X%, and X%. Find X so that her assumption leads to a 5% geometric average.

**Answer:**

\[
(1.04)(1.06)(1.01)(1.09)(1 + X\%)^2 = (1.05)^6
\]

\[
(1 + X\%)^2 = \frac{(1.05)^6}{(1.04)(1.06)(1.01)(1.09)}
\]

\[
(1 + X\%)^2 = 1.104202
\]

\[
(1 + X\%) = 1.0508104
\]

\[
X = 5.08104\%
\]
(b) (5 points) There are two states of the world, Good and Bad, each occurring with 50% probability. Gobble Turkey’s returns will be 5% in the Bad state and 45% in the Good state. Chick Steak’s returns will be 10% in the Bad state and 20% in the Good state. Find the correlation coefficient of these two stocks.

**Answer:**

Gobble average returns: \[ \frac{5\% + 45\%}{2} = 25\% \]

Chuck average returns: \[ \frac{10\% + 20\%}{2} = 15\% \]

\[
\text{Cov}(G, C) = \frac{1}{2} \left[ (0.05 - 0.25)(0.1 - 0.15) + (0.45 - 0.25)(0.2 - 0.15) \right]
\]

\[= \frac{1}{2} (0.01 + 0.01) = 0.01 \]

\[
\text{Var}(G) = \frac{1}{2} \left[ (0.05 - 0.25)^2 + (0.45 - 0.25)^2 \right]
\]

\[= \frac{1}{2} [0.04 + 0.04] = 0.04 \rightarrow \text{s.d.}(G) = 0.2 \]

\[
\text{Var}(C) = \frac{1}{2} \left[ (0.1 - 0.15)^2 + (0.2 - 0.15)^2 \right]
\]

\[= \frac{1}{2} [0.0025 + 0.0025] = 0.0025 \rightarrow \text{s.d.}(C) = 0.05 \]

\[
\rho = \frac{0.01}{0.2 \cdot 0.05} = 1
\]
(c) (5 points) Use the [table on the exam] to calculate the average equity risk premium for a fictional country for the years 1987-1991. Use an arithmetic mean here.

**Answer:**

\[
\begin{align*}
1987 &: \quad 10.5\% - 1.2\% = 9.3\% \\
1988 &: \quad -5.6\% - 1.5\% = -7.1\% \\
1989 &: \quad 21.1\% - 4.6\% = 16.5\% \\
1990 &: \quad 0.1\% - 2.5\% = -2.4\% \\
1991 &: \quad 4.5\% - 0.3\% = 4.2\%
\end{align*}
\]

\[
\frac{9.3\% - 7.1\% + 16.5\% - 2.4\% + 4.2\%}{5} = 4.1\%
\]
(d) (4 points) Jayden has invested $1,000 in a company today. In return, he expects to receive $600 one year from today and $500 two years from today. If the effective annual discount rate is 5%, what is the profitability index of this investment?

**Answer:**

$$ PV_{benefits} = \frac{600}{1.05} + \frac{500}{1.05^2} = 1024.94 $$

$$ P.I. = \frac{1024.94}{1000} = 1.02494 $$
(8 points) Wexaco, Inc., stock is expected to pay dividends every 6 months, starting 4 months from today. The last dividend was paid 2 months ago. The dividend payment 2 months ago was $0.50 per share. The executives at Wexaco have announced that each dividend paid in the future will be 7% higher than the previous dividend. If we expect this increase to go on forever, how much is each share worth today if the effective annual discount rate is 15%?

**Answer:**

- **Interest rate every 6 months:**
  \[ \sqrt{1.15} - 1 = 7.23805\% \]

- **Perpetuity that pays every 6 months starting 6 months from now:**
  \[
  \frac{0.50(1.07)}{0.0723805 - 0.07} = $224.74
  \]

- **Since 1st payment will actually be in 4 months, we need to add 2 month-interest.**

- **Rate every 2 months:**
  \[ \sqrt[6]{1.15} - 1 = 2.35671\% \]

\[
PV = $224.74(1.0235671) = $230.04
\]
(10 points) Mason Ree is a builder who needs a loan for his small business. He will receive $100,000 today from Building Tomorrow National Bank, and will pay back $10,000 next year. Each subsequent year, he will pay back 20% more until the balance gets paid off completely N years from today. Find N if the effective annual discount rate is 20%. (Hint: The growing annuity formula may not be helpful for you, so you may need to find another way to figure this problem out.)

**Answer:**
- PV of 1st payment:
  \[
  \frac{10,000}{1.2} = 8,333.33
  \]
- PV of 2nd payment
  \[
  \frac{10,000(1.2)}{1.2^2} = 8,333.33
  \]
- PV of nth payment
  \[
  \frac{10,000(1.2)^{n-1}}{1.2^n} = 8,333.33
  \]
Since the PV of each payment is $8,333.33, 12 payments are needed: \( \frac{100,000}{8,333.33} \)
(12 points) Olivia is set to open Olivia’s Originals, a business specializing in one-of-a-kind items. She has just taken out a bank loan of $100,000, which charges 12% stated annual interest, compounded monthly. This loan is amortized such that equal payments must be made monthly over the next 15 years. However, she can pay off the loan at any time she wants, as long as she pays an additional 1% of the remaining balance when she does so. 
Today, after signing the loan documents and receiving her money, she finds out that she received a French Express credit card, which offers a 9% stated annual interest rate, compounded monthly. This credit card charges a 2% fee for any balances that are transferred.
Olivia’s effective annual discount rate is 15%.
Should Olivia pay off the bank loan, or would she be better off transferring her balance to the credit card and make monthly payments of $1,500 until the credit card is paid off? (Assume that Olivia can only follow through on these two options.)
You need to completely justify your answer to get full credit.
Effective monthly discount rate

$$\sqrt[12]{1.15} - 1 = 1.17149\%$$

Choice 1: Pay off loan

$$100,000 = \frac{C}{.01} \left[ 1 - \frac{1}{1.01^{180}} \right]$$

$$C = \frac{1200.17}{.0117149} \left[ 1 - \frac{1}{1.0117149^{180}} \right] = 89,857.72$$

PV of these payments:

Choice 2: Transfer today

Need $100,000(1.01) to payoff loan $101,000 with transfer fee: $101,000(1.02) = $103,020

$$\frac{100,000}{.0075} \left[ 1 - \frac{1}{1.0075^T} \right]$$

$$0.5151 = 1 - \frac{1}{1.0075^T}$$

$$\frac{1}{1.0075^T} = 0.4849$$

$$1.0075^T = 2.0623$$

$$T = \frac{\log 2.0623}{\log 1.0075} = 96.87$$
- Final payment is $1305.34
- PV of these payments

\[
\frac{1500}{.0117149} \left[ 1 - \frac{1}{1.0117149^{96}} \right] + \frac{1305.34}{1.0117149^{97}} = \$86,606.60
\]

- Choose the transfer: $86,606.60 < $89,857.72
(9 points) Shark Fin Loans is quoting customers 20% annual interest as follows: If you borrow $20,000 today, you will be charged $4,000 in interest. The $24,000 you pay back will be divided into 2 payments of $12,000 each. The first payment will be made 6 months from today and the second payment will be made 12 months from today. What is the true effective annual interest rate of this loan?
\textbf{Answer:}

\[ 20,000 = \frac{12,000}{(1 + r)^{\frac{1}{2}}} + \frac{12,000}{1 + r} \]

Let \( X = (1 + r)^{\frac{1}{2}} \)

\[ 20,000 = \frac{12,000}{X} + \frac{12,000}{X^2} \]
\[ 20,000X^2 = 12,000X + 12,000 \]
\[ 5X^2 - 3X - 3 = 0 \]
\[ X = \frac{3 \pm \sqrt{9 - 4(5)(-3)}}{10} \]
\[ X = \frac{3 \pm \sqrt{69}}{10} = 1.1306624 \]
\[ X = (1 + r)^{\frac{1}{2}} = 1.1306624 \]
\[ r = 27.8397\% \]