Test 2, Version A (9:30 lecture)
Econ 134A, Winter 2012
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(a) (4 points) If the return on a security is 10%, the beta of the security is 2, and the return on the market is 7%, what is the risk free rate? Assume the linear CAPM equation from class.

**Answer:**

\[
E(R_i) = R_f + \beta_i (E(R_m) - R_F)
\]

\[
10\% = R_f + 2(7\% - R_f)
\]

\[
10\% = R_f + 14\% - 2R_f
\]

\[
-4\% = -R_f
\]

\[
R_f = 4\%
\]
(b) (3 points) What is the stated annual interest, compounded weekly, if the effective annual interest rate is 28.8%? Assume 52 weeks per year in your calculation.

- **Answer:**

\[
1 + EAIR = \left(1 + \frac{SAIR}{52}\right)^{52}
\]

\[
\left(\sqrt[52]{1.288} - 1\right) \cdot 52 = SAIR
\]

25.3708% = SAIR
(c) (4 points) All risks in this problem are measured as standard deviations of a portfolio’s return: Systematic risk for investment markets is currently assumed to be at 8% in Massifornia. Unsystemic risk is assumed to be \((50/N)\)%, with \(N\) denoting the number of stocks in one’s portfolio. What is the minimum standard deviation possible in Massifornia? Justify your answer to receive full credit.

**Answer:** Minimum standard deviation is 8%.

\[
\lim_{N \to \infty} \left( 8 + \frac{50}{N} \right) \% = (8 + 0) \% = 8\%
\]
(d) (4 points) In 1954, US Treasury bills had an effective annual return of 0.93%. The Consumer Price Index, which is used as a measure of inflation, was -0.74%. What was the real effective annual return of US Treasury bills in 1954?

- **Answer:**

\[
1 + \text{nominal} = (1 + \text{real})(1 + \text{inflation})
\]

\[
1 + .0093 = (1 + \text{real})(1 - .0074)
\]

\[
1.0093 = (1 + \text{real})(.9926)
\]

\[
1.0168245 = 1 + \text{real}
\]

\[
1.68245\% = \text{real}
\]

- **Common Wrong Answer (-3 points):**

\[.93\% + .74\% = 1.67\%\]
(e)(4 points) If the return of a sample of three stocks is 4%, 2%, and 8%, what is the standard deviation of this sample?

**Answer:**

\[
\text{Average} = \frac{4\% - 2\% + 8\%}{3} = 3 \frac{1}{3}\%
\]

\[
\text{Variance} = \frac{1}{2} \left[ (4\% - 3 \frac{1}{3}\%)^2 + (-2\% - 3 \frac{1}{3}\%)^2 + (8\% - 3 \frac{1}{3}\%)^2 \right]
= .002533\bar{3}
\]

Standard Deviation = \sqrt{.002533\bar{3}} = 5.03322\%
(f) If the effective annual interest rate is 5.1%, how many years will it take for a bank deposit to double?

Answer:

\[(1 + 0.51)^T = 2\]

\[T = \frac{\log 2}{\log 1.051} = 13.9348 \text{ years}\]
(8 points) Aubrey Avery Automotive is set to pay a $1 dividend per share later today, followed by 30% annual growth for each of the next 5 years. After that, the growth rate will be -6% per year forever. If the effective annual discount rate for this company is 5%, what is the present value of each share of stock? (Note that dividends are paid yearly by Aubrey Avery Automotive.)

**Answer:**

\[ PV = 1 + \frac{1.30}{1.05} + \frac{1.30^2}{1.05^2} + \frac{1.30^3}{1.05^3} + \frac{1.30^4}{1.05^4} + \frac{1.30^5}{1.05^4} \cdot \frac{1}{.05 - (-.06)} \cdot \frac{1}{1.05^4} \]

\[ PV = 1 + 1.2381 + 1.5328 + 1.8978 + 2.3497 + 27.76942 \]

\[ PV = 35.79 \]
(8 points) Today is March 3, 2013. Simple Sipple Drinks, Inc. is offering to sell some bonds today. The face value of each bond is $350, which gets paid 3 years from today. The bond offers to pay 8% coupons 1 year, 2 years, and 3 years from today. At the beginning of today, the effective annual discount rate is 8%. At the end of the day, the effective annual discount rate is 7%. How much does the value of the bond change today? (Justify your answer completely to get full credit.)

- **Answer:**

  - Beginning of the day:

    \[ PV = \frac{28}{1.08} + \frac{28}{1.08^2} + \frac{28 + 350}{1.08^3} = 350 \]

  - End of day:

    \[ PV = \frac{28}{1.07} + \frac{28}{1.07^2} + \frac{28 + 350}{1.07^3} = 359.19 \]

  Price went up by $359.19 − $350 = $9.19
(10 points) Abi Loney is considering to invest in a new deep sea diver. If she purchases the diver, she must make a down payment of $40,000 today, and another payment of $80,000 two years from today. The diver gives a $120,000 benefit one year from today. For what effective annual discount rates should Abi buy the diver?

- **Answer:** Abi should buy the diver for discount rates that make NPV of investment greater than zero.

\[-40,000 + \frac{120,000}{1 + r} \frac{80,000}{(1 + r)^2} > 0\]

\[-1(1 + r)^2 + 3(1 + r) - 2 > 0\]

Let \(X = 1 + r\)

\[-X^2 + 3X - 2 > 0\]

\[-(X^2 - 3X + 2) > 0\]

\[-(X - 2)(X - 1) > 0\]
So

\[
1 + r - 2 = 0 \rightarrow r = 1
\]
\[
1 + r - 1 = 0 \rightarrow r = 0
\]

Check Regions

\[
r = 0.5 \quad \rightarrow \quad -40,000 + \frac{120,000}{1 + 0.5} - \frac{80,000}{(1 + 0.5)^2} = $4,444.44
\]
\[
r = 1.5 \quad \rightarrow \quad -40,000 + \frac{120,000}{1 + 1.5} - \frac{80,000}{(1 + 1.5)^2} = -$4,800
\]

Answer:

\[
0 \leq r \leq 1
\]
(11 points) Assume that there are two known states of the world: Alpha and Beta, each occurring with 50% probability. Delta Delta Gums, Inc. has a 6% return in the Alpha state and 19% return in the Beta state. Chick-a-Cola, Inc. has an 8% return in the Alpha state and 10% return in the Beta state. What is the variance of a portfolio that contains 60% of Delta Delta and 40% of Chick-a-Cola?

**Answer:**

\[
\text{Delta average return} = \frac{6\% + 19\%}{2} = 12.5\%
\]

\[
\text{Chick average return} = \frac{8\% + 10\%}{2} = 9\%
\]

\[
\text{Cov}(D, C) = .5(.06 - .125)(.08 - .09) + .5(.19 - .125)(.10 - .09) = 0.00065
\]
\[
Var(D) = .5(0.06 - 0.125)^2 + .5(0.19 - 0.125)^2 \\
= 0.004225 \\
s.d.(D) = \sqrt{0.004225} = 0.065
\]

\[
Var(C) = .5(0.08 - 0.09)^2 + .5(0.1 - 0.09)^2 \\
= 0.0001 \\
s.d.(C) = \sqrt{0.0001} = .01
\]

Portfolio Variance:
\[
(0.6)^2(0.004225) + (0.4)^2(0.0001) + 2(0.6)(0.4)(0.00065) = .001849
\]