Econ 171  Fall 2012
Problem Set 3 - Solutions
Due Wednesday, December 5

Important: hand in only the two-star problems. There are no one-star problems on this problem set. The notation a.b denotes problem number b from Chapter a in Watson.

** Problem 1
Consider a two-player Bayesian game where both players are not sure whether they are playing the game X or game Y, and they both think that the two games are equally likely. This game has a unique Bayesian Nash equilibrium, which involves only pure strategies. What is it? (Hint: start by looking for Player 2’s best response to each of Player 1’s actions.)

\[
\begin{array}{c|ccc}
 & L & M & R \\
\hline
T & 1.2 & 1.0 & 1.3 \\
B & 2.2 & 0.0 & 0.3 \\
\end{array}
\]

** X **

\[
\begin{array}{c|ccc}
 & L & M & R \\
\hline
T & 1.2 & 1.3 & 1.0 \\
B & 2.2 & 0.3 & 0.0 \\
\end{array}
\]

** Y **

Solution: The unique BNE is \((B, L)\), yielding each player a payoff of 2. Player 1’s payoffs do not depend upon which version of the game is actually being played. Her best response to \(L\) is to play \(B\) and \(T\) is a best response to \(M\) or \(R\). If 1 plays \(T\), then both \(M\) and \(R\) give Player 2 an expected utility of .15, so her best response is \(L\). Similarly, Player 2’s best response to \(B\) is \(L\). So in expected utility, \(L\) is a dominant strategy for 2, and 1 best responds with \(B\).

** Problem 2
Now consider a variant of this game (from Problem 1) in which Player 2 knows which game is being played (but Player 1 still does not). This game also has a unique Bayesian Nash equilibrium. What is it? (Hint: Player 2’s strategy must specify what she chooses in the case that the game is \(X\) and in the case that it is \(Y\).) Compare Player 2’s payoff in the games from Problems 1 and 2. What seems strange about this?

Solution: The unique BNE is \((T, (R, M))\). Player 2 now knows the game that is being played, and each type of Player 2 has a dominant strategy \((R\) for the type that knows the game is \(X\) and \(M\) for the type that knows that the game is \(Y\)). Since there is no chance that 2 will play \(L\), Player 1’s unique best response is to play \(T\).

In the first part, each player earned a payoff of 2. In the second part, Player 2 actually has more information about what game is actually being played and ends up only earning 0.3 (in either case). At first it may seem a bit strange that 2 is worse off
knowing the game than she is not knowing it. This happens because the uninformed Player 2 uses $L$ as a compromise. When she knows the game, she will choose either $M$ or $R$, tailoring her action for fit the game. What hurts her is the fact that 1 knows that she knows this information.

** Problem 3  

Watson 26.6

*Solution:* $(LL', U)$

** Problem 4  

Firm 1 is considering taking over Firm 2. It does not know Firm 2’s current value, but believes that is equally likely to be any dollar amount from 0 to 100. If Firm 1 takes over firm 2, it will be worth 50% more than its current value, which Firm 2 knows to be $x$. Firm 1 can bid any amount $y$ to take over Firm 2 and Firm 2 can accept or reject this offer. If 2 accepts 1’s offer, 1’s payoff is $\frac{3}{2}x - y$ and 2’s payoff is $y$. If 2 rejects 1’s offer, 1’s payoff is 0 and 2’s payoff is $x$. Find a Nash equilibrium of this game. What does this situation have to do with dating and shopping for used cars?

*Solution:* Firm 1 will bid zero and Firm 2 will accept any offer greater than or equal to $x$. Firm 2’s simply accepts offers that are higher than the firm’s own value. Firm 1 knows that the value of a firm that accepts an offer of $y$ is anywhere from 0 to $y$. Thus, the expected value of a firm that accepts is $y/2$, which means that Firm 1’s expected payoff as a function of it’s bid is $\frac{3}{2}(y/2) - y = -\frac{1}{4}y$. In other words, it expects to lose money on any positive bid it makes. It’s best response, then is to bid zero. Just like in dating and the used-car market, this market is plagued by adverse selection, which in this case leads the market to unravel completely.

** Problem 5  

Find all perfect-Bayesian equilibria of the game in Figure 1.

![Figure 1: A variant of the entry game.](image)

*Solution:* We use the two NE, $(R, A)$ and $(O, F)$, as the starting point for finding the PBE. For $(R, A)$, the only belief 2 can have that is consistent with 1’s strategy is the $Pr(R) = 1$, and $A$ is a best-response to this belief. This gives us the following PBE:
[(R, A), Pr(R) = 1]. For (O, F), 1’s strategy places no restriction on 2’s beliefs, so any belief Pr(R) = p is consistent. However, F is only a best response for 2 if p ≤ 1/2, so another set of PBE are: [(O, F), Pr(R) ≤ 1/2].