Econ 171  Fall 2012  
Problem Set 2  
Solutions to the two-star problems.

In some cases I’ve provided more explanation than was asked of you.

** Problem 4  
Consider the game shown in the figure, with the payoffs \((a, b, c, d, e, f, g, h, i, j, k, l) = (2, 2, 1, 3, 3, 1, 4, 6, 2, 7)\).

(a) What is the backwards-induction strategy for each player?

Solution: The backwards-induction strategy profile is \((L_1L_2, r_1l_2r_3)\).

(b) What is the backwards-induction outcome?

Solution: The backwards-induction outcome is \((L_1, r_1)\) with payoffs \((2, 2)\).

(c) Two strategies can arguably be considered “equivalent” if they designate the same behavior at all decision nodes that the player will reach with positive probability given her own previous choices as specified by both of the two strategies. For each player, state which (if any) strategies are equivalent to others.

Solution: For Player 1, \(R_1L_2\) and \(R_1R_2\) are equivalent. For Player 2, \(r_1r_2l_3\) and \(r_1r_2r_3\) are equivalent and \(r_1l_2r_3\) and \(r_1l_2l_3\) are equivalent.

** Problem 5  
Watson 15.2

Solution:

(a) The Nash equilibria are \(\{(WY, AC), (ZX, BC), (WY, AD), (ZY, BC), (WX, BD)\}\).

Of these, only \((WY, AC)\) and \((ZX, BC)\) are subgame perfect.

(b) The Nash equilibria are \(\{(UE, BD), (DE, BC), (UF, BD), (DE, AC)\}\), but only \((UE, BD)\) and \((DE, BC)\) are subgame perfect.

** Problem 6  
Armies 1 and 2 are fighting over an island initially held by a battalion of army 2. Both armies have \(K\) battalions. Whenever the island is occupied by one army the opposing army can launch an attack. The outcome of the attack is that the occupying battalion and one of the attacking battalions are destroyed; the attacking army wins and, so long as it has battalions left, occupies the island with one battalion. Note, that after \(K\) battles neither army occupies the island because no army has any battalions left. The commander of each army is interested in maximizing the number of surviving battalions but also regards the occupation of the island as worth more than one battalion but less than two.

Predict the winner using SPNE for all \(K \geq 2\) (no formal proof required). Explain the intuition for the result. (Hint: try this for \(K = 2, 3, \) and 4 first.)
Solution: When $K$ is even, army 1 will attack and win the island. When $K$ is odd, army 1 will not attack and army 2 will retain possession of the island. To see why this result makes sense, consider what happens for specific small values of $K$.

- $K = 1$ Army 1 does not attack because that would leave it with zero battalions. Army 2 retains the island.
- $K = 2$ Army 1 will attack and occupy the island. Will army 2 try to retake the island? Now army 2 finds itself in the same position as army 1 was when $K = 1$, i.e. the previous subgame, so it will not attack. Thus army 1 wins the island.
- $K = 3$ If army 1 attacks, it will occupy the island and army 2 will be in a subgame identical to the one described above. This means army 2 would attack, retake the island and win. Knowing this and not wanting to needlessly lose battalions, army 1 does not attack and army 2 wins.
- $K = 4$ If army 1 attacks, it will occupy the island and army 2 will be in a subgame identical to the previous one, and therefore not attack. So army 1 attacks, 2 does not attack back, and army 1 wins.

Both armies know that whomever has possession when $K = 1$ will win the island because the other army will not be able to re-occupy the island after attacking. Backwards inducting, a non-occupying army will only attack if it has an even number of armies.

** Problem 7
Watson 19.6

Solution: For simplicity, let the offer always be stated in terms of the amount player 2 is to receive. Let $x$ be the offer in period 1, $y$ be the offer in period 2, and $z$ be the offer in period 3.

If period 3 is reached, player 2 will offer $z = 1$ and player 1 will accept, yielding a payoff of 1 for player 2. Thus, in period 2, player 2 will accept any offer that gives her at least $\delta$.

Knowing this, in period 2 (if it is reached) player 1 will offer just enough to make player 2 indifferent between accepting and rejecting to receive 1 in the next period. Thus, $y = \delta$.

Finally, in period 1, player 2 will accept any offer that gives her at least $x = \delta^2$ because she knows that she will only be offered $y = \delta$ in period 2. So player 1 will offer $x$ in the first period that is just high enough to make player 2 indifferent between accepting and rejecting to receive $\delta$ in the second period. Thus, player 1 offers $x = \delta^2$ and it is accepted.

** Problem 8
Watson 22.2

Note: You did not have to hand in this problem.
Solution:

(a) We need \( \frac{2}{1-\delta} \geq 4 + \frac{\delta}{1-\delta} \) in order to support cooperation. Solving for \( \delta \) yields \( \delta \geq \frac{2}{3} \).

(b) To support cooperation by player 1, we need \( \delta \geq \frac{1}{2} \) and to support cooperation by player 2, we need \( \delta \geq \frac{3}{5} \). So it must be that \( \delta \geq \frac{3}{5} \).

(c) Cooperation by player 1 requires \( \delta \geq \frac{4}{5} \) and player 2 has no incentive to deviate in the short run. Thus, for cooperation we need \( \delta \geq \frac{4}{5} \).