1. Consider the extensive-form game shown in Figure 1.

   ![Game Tree](image)

   **Figure 1:** The game from question 1.

(a) [5] List the strategies for each player.

   Answer: \( S_1 = \{ Ll, Lr, Rl, Rr \} \) and \( S_2 = \{ l_1l_2, l_1r_2, r_1l_2, r_1r_2 \} \)

(b) [5] Find the Nash Equilibria.

   Answer: \((Ll, l_1l_2)\) and \((Lr, l_1l_2)\)

(c) [5] Apply backwards induction and state the resulting SPNE.

   Answer:
2. Consider the game shown in Figure 2.

(a) [5] Find all the pure-strategy Nash equilibria of this game.

*Answer*: $(BE, C), (AE, D), \text{ and } (AF, D)$

(b) [5] Which of these are subgame perfect?

*Answer*: $(BE, C)$ and $(AF, D)$

(c) [5] **Challenge question**: There is a concept called forward induction that can be used to argue that one of the SPNE is unreasonable. We haven’t studied it in class, but the name kind of gives you a hint as to how to apply it. Look at the SPNE and try to come up with an argument for why one of them should be eliminated. *Briefly* explain your argument.

*Answer*: For player 1, note that $BF$ is dominated by $AF$ (or $AE$). Thus, if player 2 believes that 1 is rational, she would never expect 1 to play $F$ in the subgame, if it is reached. So when player 2 is at her decision node, she ought to believe that 1 chose $B$ and will go on to choose $E$. In that case, 2 would choose $C$. Therefore, the $(AF, D)$ equilibrium should be eliminated.
3. Players 1 and 2 play a three-period alternating-offer bargaining game over a pie of size 2. They have discount factor $\delta$, with $0 < \delta < 1$. In the first period, player 1 makes an offer, which 2 can either accept or reject. If 2 rejects, she gets to make an offer in the second period. If 1 rejects this offer, then in the third period the pie shrinks to size 1 and player 1 can make one final offer. If this offer is rejected, then both players get nothing.

(a) [5] In the unique SPNE of this game, what offer does player 1 make in period 3? Does player 2 accept or reject it? (Write the offer as $(x, y)$, where $x$ is the amount that player 1 would get and $y$ is the amount that player 2 would get.)

Answer: Player 1 offers $(1, 0)$ and it is accepted.

(b) [5] What offer does player 2 make in period 2? Does player 1 accept or reject it?

Answer: Player 2 offers $(\delta, 2 - \delta)$ and it is accepted.

(c) [5] What offer does player 1 make in period 1? Does player 2 accept or reject it?

Answer: Player 1 offers $(2 - \delta(2 - \delta), \delta(2 - \delta))$ and it is accepted.
4. Consider two players who own a firm and want to dissolve their partnership. Each owns half of the firm. The value of the firm for players A and B is \(v_a\) and \(v_B\), respectively, where \(v_A > v_B > 0\). They have agreed to proceed as follows.

Player A sets a price \(p\) for half of the firm. Player B then decides whether to sell his share or to buy A’s share at this price.

- If B decides to sell his share, then A will own the entire firm and pays \(p\) to B. This yields payoffs \(v_A - p\) and \(p\), for players A and B, respectively.
- If B decides to buy, then B owns the entire firm and pays \(p\) to A, yielding payoffs \(p\) and \(v_B - p\) for A and B, respectively.

(a) [5] Find B’s best response, as a function of the \(p\) chosen by A.

Answer: If \(p < \frac{v_B}{2}\), B’s best response is to buy. If \(p = \frac{v_B}{2}\), both buy and sell are best responses. If \(p > \frac{v_B}{2}\), B’s best response is to sell.

(b) [5] Is it the case that B strictly prefers one of the actions (buy or sell) in equilibrium? Why or why not? Which action must B actually choose in equilibrium? Why can’t B choose the other action?

Answer: In equilibrium, A’s offer must make B indifferent between buying and selling. If B strictly prefers to buy, then the price is too low and A could increase her own payoff by raising it. If B strictly prefers to sell, then the price is too high and A could increase her own payoff by lowering it.

However, in equilibrium, B must choose to sell when indifferent. If here were buying, then A’s payoff as a function of \(p\) would be \(p\) if \(p \leq \frac{v_B}{2}\) and \(v_A - p\) if \(p > \frac{v_B}{2}\). Then, no price would maximize the payoff of A, which is inconsistent with equilibrium.

(c) [5] Fully describe the subgame-perfect Nash equilibrium of this game.

Answer: A offers \(p = \frac{v_B}{2}\) and B sells if and only if \(p \geq \frac{v_B}{2}\).