1. Consider the following extensive-form game:

\[
\begin{array}{cccc}
A & 1 & B & 2 \\
C & 2 & D & 0,0 \\
& 1,1 & 0,0 & 2,2 \\
& 0,0 & 2,2 & 0,2 \\
\end{array}
\]

(a) [2] List all the Nash equilibria and indicate which are subgame-perfect.

\textit{Answer:} \((B, CE)\) and \((A, CF)\) are SPNE; \((B, DE)\) and \((B, DF)\) are NE, but not SPNE.
(b) [2] Now consider the following game: First Player 1 decides (Y or N) whether or not to play. If she chooses N, the game ends. If she chooses Y, then Player 2 decides (Y or N) whether or not to play. If he chooses N the game ends. If he chooses Y, then they go ahead and play BoS with the payoffs shown below. A player who opts out by choosing N gets 2 and the other player gets 0. List the two subgame-perfect Nash equilibria for this game.

\[
\begin{array}{cc}
O & B \\
O & 3,1 & 0,0 \\
B & 0,0 & 1,3 \\
\end{array}
\]

Answer: \((NO, NO)\) and \((NB, YB)\)

(c) [1] Briefly explain how the SPNE arrive at the same outcome in two different ways.

Answer: In both equilibria, Player 1 opts out immediately. Any SPNE must specify that either \((O, O)\) or \((B, B)\) would be played in the BoS subgame. In each of these equilibria, the player with the lower payoff would have preferred the outside option of 2. Either player 1 opts out because the BoS equilibrium doesn’t favor her or she opts out because she knows that Player 2 will otherwise, because the BoS equilibrium doesn’t favor him.
2. Two people play a standard, four-period alternating-offer bargaining game over a 1 unit surplus with discount factor $\delta \in (0, 1)$ for every period of delay. There is one twist, though—Player 2 has a doctor’s appointment in period 2 and can’t make it to the bargaining table. This means that if Player 1’s period 1 offer is rejected, the players meet again in period 3 (after a two-period delay), in which Player 2 makes an offer. If that is rejected, Player 1 makes the offer in the fourth and final period.

(a) [2] In the unique SPNE, what is the offer made in each period? Express your answer as an ordered pair, where the first number represents Player 1’s share of the surplus and the second represents Player 2’s share.

Answer: Player 1 offers $(1, 0)$ in period 4, Player 2 offers $(\delta, 1 - \delta)$ in period 3, and Player 1 offers $(1 - \delta^2(1 - \delta), \delta^2(1 - \delta))$ in period 1.

(b) [2] Suppose Player 2 has the option of postponing her doctor’s appointment until period 3. This would mean that she would make the offer in period 2, they would skip period 3 and would return for Player 1’s offer in period 4, after a two-period delay. Should she postpone the appointment? Answer yes or no, and show how you arrive at this answer. You can assume a specific value of $\delta$ in your answer (e.g., $\delta = 1/2$) and still get full credit, but the actual answer does not depend on $\delta$ and I will smile more if your argument does not depend on a specific value of $\delta$.

Answer: In any SPNE, the period 1 offer is accepted, so Player 2’s payoff in the original version of the game is $\delta^2(1 - \delta)$, or $\delta^2 - \delta^3$. If Player 2 postpones, her SPNE payoff is $\delta(1 - \delta^2)$, or $\delta - \delta^3$. This is because 1 would offer $(1, 0)$ in period 4 and 2 would offer $(\delta^2, 1 - \delta^2)$ in period 2. Because $\delta > \delta^2$, her payoff is higher if she postpones her appointment.

(c) [1] Briefly explain intuitively why your answer to (b) makes sense.

Answer: Delay is costly, so you have more bargaining power if a rejection of your offer would lead to a longer delay. This means that 2 is better off having the extra long delay be after her offer, rather than before.
3. Two people play an infinitely repeated prisoners’ dilemma with payoffs shown below.

\[
\begin{array}{cc}
C & D \\
C & 3.3 & 0.4 \\
D & 6.0 & 1.2 \\
\end{array}
\]

(a) [1] For what values of the discount factor, \( \delta \), can we observe a subgame-perfect Nash equilibrium in which both players always choose \( D \), no matter what happens?

**Answer:** Because \((D, D)\) is a NE in the subgame, both players choosing a strategy of always playing \( D \) will always be a SPNE, regardless of \( \delta \).

(b) [2] If Player 2 uses a grim trigger strategy, how high must the discount factor be to sustain cooperation from Player 1?

**Answer:** Player 1’s payoff from cooperating would be \( \frac{3}{1-\delta} \) and from defecting would be \( 6 + \delta \frac{1}{1-\delta} \). For cooperation to be a best response we need \( 3 \geq 6 - 6\delta + \delta \), or \( \delta \geq 3/5 \).

(c) [2] Consider a game in which the stage game shown below is repeated for two periods and there is no discounting. Fully describe a subgame perfect Nash equilibrium in which the players select \((U, L)\) in the first period.

\[
\begin{array}{ccc}
& L & M \\
U & 8.8 & 0.9 & 0.0 \\
C & 9.0 & 0.0 & 3.1 \\
D & 0.0 & 1.3 & 3.3 \\
\end{array}
\]

**Answer:** In this game \((U, L)\) is treated as cooperating, \((U, M)\) amounts to defection by Player 2 and \((C, L)\) is defection by Player 1. There are three NE: \((D, M)\), \((C, R)\), and \((D, R)\). The first two can be used as punishment for Players 1 and 2 respectively, and the third one can be used as a reward for good behavior by both (or for anything else).

The SPNE is as follows: if Player 2 defects (\((U, M)\) is played) in the first period, then the players coordinate on \((C, R)\) in the second period. If Player 1 defects (\((C, L)\) is played) in the first period, then the players play \((D, M)\) in the second period. Otherwise, the players play \((D, R)\) in the second period.