Econ 171 (Grossman) — Fall 2012
Exam 1
October 22

You have 75 minutes to take this exam. Please answer all 5 questions, each of which is worth 3 points, for a total of 15 points. Point subtotals are indicated. Show your work to obtain full credit. This exam is open paper, meaning you can use books, notes, written or printed material. No electronic devices (phones, calculators, translators, computers, tablets, etc.) are allowed.

1. Find the set of profiles of pure-strategies that are rationalizable. (1 point) For each strategy that you eliminate, name the strategy that dominates it. (2 points)

\[
\begin{array}{ccc}
X & Y & Z \\
A & 3,4 & 3,6 & 4,5 \\
B & 5,1 & 6,1 & 3,2 \\
C & 1,2 & 1,2 & 5,3 \\
\end{array}
\]
2. Find all Nash Equilibria of this matrix game. Clearly showing your steps toward your answer may help you get partial credit, even if the answer is not correct. (3 points)

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4,3</td>
<td>7,5</td>
<td>4,8</td>
</tr>
<tr>
<td>B</td>
<td>0,4</td>
<td>4,1</td>
<td>4,0</td>
</tr>
<tr>
<td>C</td>
<td>0,5</td>
<td>6,7</td>
<td>3,1</td>
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3. Thag and Zog live in neighboring valleys during the late Pleistocene Epoch. They eat berries and occasionally the meat of a mastodon. On a given day, each can individually choose to whether to gather berries or hunt, without knowing the choice of the other. Berries are nourishing, but protein-rich mastodon meat is much more so. However, while berries can be collected individually, in order to successfully take down a mastodon, both must participate in the hunt. If one tries to hunt by himself, he’ll go home hungry, without even any berries. Finally, the berries in Thag’s valley are more nourishing than those that can be found in Zog’s, though Thag’s berries are still not as nourishing as mastodon flesh.

(a) Write this game in normal form using a matrix. You are free to label the strategies and insert payoffs however you wish, but, given the description of the situation above, the labels and payoffs must be clear and sensible. Try to capture the essence of the game as simply as possible, incorporating what is described, but nothing more. (1 point)

(b) Find all Nash equilibria of this game. (2 points)
4. Thag and Zog and Ötzi live in three different valleys connected by a high mountain pass. Herds of docile mastodons graze on the alpine grass at the top of the pass. Each morning, Thag, Zog and Ötzi must independently decide whether or not to go hunt at the top of the pass. If there is a successful hunt, all three cavemen get exactly 1 day’s worth of food, regardless of who showed up to hunt, but a successful hunt requires at least two hunters to show up. Anyone who shows up to hunt uses up 1/2 of a day’s worth of food, regardless of whether or not the hunt was successful. (You can treat this as three people facing a public-good-provision problem, where the good requires at least two people to contribute, is valued at 1 by all participants, and where the non-refundable cost of contributing is 1/2.)

(a) Find all of the pure-strategy Nash equilibria of this game. (1 point)

(b) Find a mixed-strategy Nash equilibrium that is symmetric, meaning that all three players choose the same mixed strategy of hunting with probability \( p \). (1 point)

(c) What is the probability that the trio will successfully hunt the mastodon? (1 point—express in terms of \( p \) if you did not get the previous part)
5. Consider any $2 \times 2$ matrix game in which neither player has a strategy dominated by a *pure* strategy. In such a game, is it possible for there to be a strategy dominated by a *mixed* strategy? Briefly explain why or why not. (correct answer (including guesses) = 1 point; given correct answer, up to 2 more points for the explanation)