Sequential Games & The Extensive Form
Sequential Games

How can we incorporate timing into our analysis of strategic behavior?

- Need a way to formalize games that incorporates timing
- Extensive form
- Need solution concepts that take timing into account
- Subgame-Perfect Nash Equilibrium (SPNE)
Introduction to the extensive form

Figure: Sequential BoS
**Extensive form basics**

- **Node:** where a decision occurs; Branches, represent individual actions. Initial, vs. decision vs. terminal node vs. chance. Player actions and labels: don’t use same labels in different places.

- **Information sets.** Players know structure of tree, but not necessarily where they are in the tree. Info set is a place where decision is made.

- **Outcomes represented by terminal nodes, payoffs represent preferences, just like in normal form.**
BoS in extensive form

Figure: BoS in extensive form
Strategies in extensive form games

- A strategy is a complete contingent plan for how to act at every decision node.

- List strategies for each player in BoS game.

- List strategies for each player in Sequential BoS game.

- How are they different?

- Can you write each game in *normal form*?

Sequential game players can condition choices on observed behavior, so the normal form representation is different from that of the same game played simultaneously.
Game tree rules

- Every node is the successor of the initial node, and the initial node is the only one with this property.
- Every node except initial node has exactly 1 immediate predecessor. Initial node has none.
- Multiple branches extending from the same node have different action labels.
- Each information set contains decision nodes for only one player.
- All nodes in a given information set must have the same number of immediate successors and they must have the same set of action labels on the branches leading to these successors.

Can you draw a game tree representing the extensive form of the grab-the-quarter game?
A game similar to grab-the-quarter

Figure: Centipede Game
The Entry Game

Figure: The entry game
Entry game in normal form

<table>
<thead>
<tr>
<th></th>
<th>$F$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In</strong></td>
<td>$-1, -1$</td>
<td>$2, 1$</td>
</tr>
<tr>
<td><strong>Out</strong></td>
<td>$1, 0$</td>
<td>$1, 0$</td>
</tr>
</tbody>
</table>

**Figure:** The entry game in normal form