Blue-book – 21 out of 50 pts. (2 qns., 10 pts. each + 1)

Answer these questions in your blue-book. Show your work and intermediate steps for partial credit. Points are split equally across parts. Your score will only be based on the marks in your blue-book. You will not receive any credit for anything written on your exam paper. You will receive 1 extra point for correctly writing your name, perm number, version (A,B,C, or D), and TA’s name on your blue-book.

1. Consider an exchange economy consisting of two people, A and B, endowed with two goods, 1 and 2. Person A is initially endowed with $\omega^A = (4, 8)$ and person B is initially endowed with $\omega^B = (4, 0)$. Their preferences are given by $U^A(x_1, x_2) = x_1 x_2$ and $U^B(x_1, x_2) = x_1^2 x_2^2$.

(a) Write the equation of the contract curve (express $x^A_2$ as a function of $x^A_1$).

Answer: $x^A_2 = x^A_1$

(b) Let $p_2 = 1$. Find the competitive equilibrium price, $p_1$, and allocations, $x^A = (x^A_1, x^A_2)$ and $x^B = (x^B_1, x^B_2)$.

Answer: $p_1 = 1$, $x^A = (6, 6)$, and $x^B = (2, 2)$

(c) Now suppose that person B’s preferences are instead given by $U^B(x_1, x_2) = \min\{x_1, x_2\}$. Given a price of $p_1 = \frac{1}{3}$ (and assuming $p_2 = 1$), what Pareto-optimal allocation will result from trade between the two people?

Answer: $x^A = (7, 7)$ and $x^B = (1, 1)$.

(d) Briefly explain why this price and allocation combination are not a competitive equilibrium.

Answer: At these prices, this allocation maximized utility for B, but A is not. Because good 1 is cheaper than in equilibrium, A would still prefer to buy more good 1 and sell more good 2 and the market does not clear.

(e) Let $p_2 = 1$. Find the actual competitive equilibrium price, $p_1$, and allocations, $x^A = (x^A_1, x^A_2)$ and $x^B = (x^B_1, x^B_2)$.

Answer: They are the same as before: $p_1 = 1$, $x^A = (6, 6)$, and $x^B = (2, 2)$. 


2. Ten (10) fishermen live on the shores of Lake George, a freshwater lake with a surface area of 10 hectares. Each fisherman can decide how many fishing trips to take per year and over how large an area in the lake to cast his nets. Each fishing trip costs $1 in gas and supplies, regardless of the area fished, and the total dollar value of fish caught in a patch of the lake with area $A$ is given by $f(A, X) = \frac{A^{\frac{3}{5}} X}{5}$, where $X$ is the total number of fishing trips taken by all fishermen. (You can assume that all fishermen fishing the same area get the same value of fish per boat.)

(a) Each fisherman takes the behavior of others as given. If a fisherman decided to take a trip, how large an area, $A^*$, would he choose to fish? Since everyone chooses to fish $A^*$, plug in this value to find the total dollar value of fish caught, as a function only of the number of trips, i.e. $f(A^*, X)$.

Answer: Because the value of fish caught is increasing in the area fished, he would choose as large an area as possible, or all 10 hectares of the lake. Because everyone fishes all 10 hectares each trip, the total dollar value as a function of trips will be $f(10, X) = \frac{10^{\frac{3}{5}} \sqrt{X}}{5} = 20\sqrt{X}$.

(b) When each fisherman independently decides how many trips to take, how many trips will be taken overall and what will the total profits for all 10 fishermen be?

Answer: They will fish until there are zero profits. The total cost of $X$ trips is $X$, so profits are zero when $20\sqrt{X} = X$, or $X = 400$.

(c) The fishermen hold a meeting to figure out a way increase all their profits. One suggestion is to form a collective which will act as a single decision-making body overseeing the fishing rights to the lake. It will allow only the optimal number of fishing trips and divide all profits evenly among all fishermen. Under this proposal, how many trips will be taken and what will the total profits for all 10 fishermen be?

Answer: They will fish to the point at which the marginal benefit equals the marginal cost. The marginal benefit is $\frac{10}{\sqrt{X}}$ and the marginal cost is 1, so $X = 100$. This gives profits of $20\sqrt{100} - 100 = 100$.

(d) Another suggestion is that the fishermen divide up the lake into 10 single-hectare parcels and give each of the 10 fishermen the exclusive rights to fish one of those 1-hectare patches of water. Under this proposal, how many trips will be taken and what will the total profits for all 10 fishermen be? (Assume that fisherman are allowed to take fractional numbers of trips.)

Answer: Now each fisherman has only one hectare and will fish that hectare until the marginal benefit equals the marginal cost. The value of fish from one hectare is $f(1, X) = \frac{\sqrt{X}}{5}$, so the marginal benefit is $\frac{1}{10\sqrt{X}}$. The marginal cost is still 1, so the optimal number of trips for each fisherman is $\frac{1}{100}$ and the profit for each fisherman is $\frac{1}{60} - \frac{1}{100} = \frac{1}{100}$. So the total trips and profits for all fishermen are $\frac{1}{10}$ and $\frac{1}{10}$.

(e) Both proposals try to avoid the tragedy of the commons by assigning private fishing rights—in the first case the rights to the entire lake are assigned to the fisherman’s collective and in the second case the rights to each hectare are assigned to a different fisherman. Do both proposals avoid the full depletion of the lakes resources? Which is more successful at increasing the fisherman’s profits? Very briefly explain in intuitive terms why the other proposal is not as successful.

Answer: Both proposals yield positive profits and thus avoid the full depletion of the lakes resources, but the former (the collective) is much more profitable. When the lake is split up into small parcels the fishing rights are all private so there is no longer any free-riding or tragedy of the commons. However, fishing is much less efficient than when the fisherman can cast their nets over the entire lake. So, while assigning property rights may solve the tragedy of the commons, it is possible to go too far and assign too many property rights to too many decision-makers.