Choice Under Uncertainty
(Chapter 12)

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# Teaching Assistants

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Expected Utility of a Risky Prospect
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  Risk Attitudes

Constrained Utility Maximization: Part II
Typical 100B problem

Breaking it down

• You’re asked to analyze economic behavior/outcomes/policy
  • Individual choice
  • Market behavior and welfare
  • Effectiveness/consequences of policy

• You need to break it down into smaller pieces

• Apply specific skills/tools to deal with each part

• Put parts together to solve overall problem
Typical 100B problem

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• zoom back out, refocus on big picture
  • Not just solving math problem
  • What insight do we gain from this?
Typical 100B problem

Example: uncertainty

- Given setup
- Separately derive budget constraint, indifference curves (find MRS)
- Solve $U$ max problem, optimal bundle
- Learn something about demand for insurance
Typical 100B problem

Example: market demand, equilibrium

• Given individual demands, info about supply

• Derive total demand, supply

• Solve for equilibrium $p, q$

• Learn something about behavior in the market
Typical 100B problem

Example: Changes to equilibrium (comparative statics)

- Given info about demand, supply
- Find equilibrium $p, q$
- Introduce demand/supply shift, tax, price floor, ceiling, quota, etc., calculate new $p, q$
- Observe something about effect on behavior, welfare
Typical 100B problem

Example: Comparison of market structures

• Given market demand, costs/supply

• Find eq. \( p, q \) for various market structures

• Compare behavior and welfare
Types of exam questions

One categorization: difficulty

- Easy, just about everyone should get
- Moderate, many, but not all should get
- Challenging, only a handful of the very best students will get
Types of exam questions

Another way to classify:

- Small, deals only with subpart of overall problem
- Large, deals with more parts or entire problem
- Pushes you to focus out on big picture, draw conclusions, push understanding further, deal with new complications— not necessarily more complicated math
What will the quizzes look like?

- Two multiple-choice questions
- Both type 1
- Diagnostic, small grade impact
- Checks for minimum necessary comprehension

Don’t think: I did well on the quiz, so I’m prepared for the exam

Do think: I did well on the quiz, so I can focus on the larger parts of the problem, big picture for the exam

Do think: I had trouble on the quiz— I really need to do something about this before the exam
States of Nature and Contingent Plans

- States of Nature:
  - “accident” (a) vs. “no accident” (na)
  - Probability of: accident = $\pi_a$, no accident = $\pi_{na}$; $\pi_a + \pi_{na} = 1$
  - Accident causes loss of $L$

- “Bundle” = state-contingent consumption plan: Specifies consumption level for each scenario (state)

- Option to buy some insurance: contracts are state-contingent (e.g. insurer pays only if you have an accident)

- How much should you buy?
**Deriving the budget constraint**

Q: Where to start?

Without insurance, consumption is:

\[ c_{na} = m \] if no accident

\[ c_a = m - L \] if accident

The endowment bundle displayed graphically:

\[ C_{na} \quad C_a \quad m \quad m - L \]
Deriving the budget constraint

Q: Where to start?
A: The bundle with which you are endowed.
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Insurance contract: Buy $K$ of accident insurance at price $p$, claim $K$ from company if accident

- If no accident: $c_{na} = m - pK$
- If accident: $c_a = m - pK - L + K = m - L + (1 - p)K$
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- Given $K$, it must be true that... (solve for $K$, substitute):

\[
c_{na} = \frac{m - pL}{1 - p} - \frac{p}{1 - p} c_a
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![Diagram showing the endowment bundle with coordinates labeled as $m$, $m - L$, and $m - pL$ on the axes.]
Deriving the budget constraint: Now you try it!

Being forgetful, you have a 10% chance of losing your $100 Nokia phone and be left with nothing. Nokia offers “flake” insurance for your phone at the price of 20 cents ($0.20) for each dollar of protection and you can buy as much or as little as you want. Let $c_l$ and $c_{nl}$ represent your wealth in the cases that you lose and do not lose it, respectively. Which equation represents the your budget constraint?

Clicker Vote

- A) $c_{nl} = 80 - \frac{c_l}{3}$
- B) $c_{nl} = 60 - \frac{c_l}{5}$
- C) $c_{nl} = 100 - \frac{c_l}{4}$
- D) $c_{nl} = 75 - \frac{c_l}{3}$
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Preferences

Q: Why do people buy insurance when they face risk?

To answer this, we have to consider preferences

- $U(c_a, c_{na})$ captures attitude towards uncertainty/risk
- Person might be risk averse or risk neutral (or risk loving)
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- Consider our three favorite examples:
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  - B Cobb-Douglas
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- CLICKER VOTE: which of these does not reflect any degree of risk aversion?
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Choosing corner solutions implies choosing very risky plan
Optimal Choice (Graph)

Risk $\implies$ your endowment is away from the 45-degree line. Insurance is a way of mitigating risk. Risk aversion $\implies$ you are happiest buying some positive amount of insurance $\implies$ you closer to the 45-degree line.

Need to understand preferences to get an algebraic solution.

Expected utility theory presents a way to think about how people evaluate risk.
Expected utility example: a lottery

- Win $90 or $0, equally likely

- *Expected Money* is

\[
EM = 0.5 \times 90 + 0.5 \times 0 = 45.
\]
Expected utility example: a lottery

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- \( U(90) = 12 \) and \( U(0) = 2 \)
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Expected Utility Theory: take the sum of utilities from each outcome, weighted by probability of that outcome
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\[ EM = .5 \times 90 + .5 \times 0 = $45. \]

- \( U(90) = 12 \) and \( U(0) = 2 \)

- *Expected Utility* is

\[ EU = .5 \times U(90) + .5 \times U(0) = .5 \times 12 + .5 \times 2 = 7. \]

Expected Utility Theory: take the sum of utilities from each outcome, weighted by probability of that outcome
Risk Attitudes

How do we characterize attitude towards risk?

Recall: $EU = 7$ and $EM = $45

- $U(45) > 7 \implies$ risk-averse
- $U(45) < 7 \implies$ risk-loving
- $U(45) = 7 \implies$ risk-neutral
Risk Attitudes

We typically assume diminishing marginal utility (DMU) of wealth.

So $EU < U(EM)\ldots$

this implies risk aversion!
Risk Attitudes

Example: Risk-loving preferences

EU > U(EM)
Risk Attitudes

Example: Risk-neutral preferences

\[ EU = U(EM) \]
Optimal Choice (Algebra)

Calculating the MRS

- \( EU = \pi_a U(c_a) + \pi_{na} U(c_{na}) \)
- Indifference curve \( \implies \) constant EU
Optimal Choice (Algebra)

Calculating the MRS

- \( EU = \pi_a U(c_a) + \pi_{na} U(c_{na}) \)
- Indifference curve \( \implies \) constant EU
- Differentiate:
  - \( dEU = 0 = \pi_a MU(c_a)dc_a + \pi_{na} MU(c_{na})dc_{na} \)
  - \( MRS = \frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})} \)
Optimal Choice (Algebra)

Calculating the MRS

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  - \( MRS = \frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})} \)
- Solution satisfies
  \[
  \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})} = \frac{p}{1 - p}.
  \]
Competitive Insurance

How optimal insurance purchase (\(K\)) and consumption levels \(c_a, c_{na}\) depend upon probabilities (given) and price.

Q: What determines the price of insurance?

A: Market conditions

Consider a competitive insurance market:

- Free entry \(\implies\) zero expected economic profit

- So \(pK - \pi_a K - (1 - \pi_a)0 = (p - \pi_a)K = 0\).

- \(\implies p = \pi_a\)

- Insurance is fair
Competitive Insurance

• With fair insurance, rational choice satisfies

\[
\frac{\pi_a}{\pi_{na}} = \frac{\pi_a}{1 - \pi_a} = \frac{p}{1 - p} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}.
\]

• In other words, \( MU(c_a) = MU(c_{na}) \)

• Risk-aversion \( \implies c_a = c_{na} \)

• Full insurance!
Not-Fair Insurance

Suppose the insurance market is not competitive

- Insurers can expect positive profits

- \( pK - \pi_a K - (1 - \pi_a)0 = (p - \pi_a)K > 0 \)

- Then \( p > \pi_a \) and \( \frac{p}{1-p} > \frac{\pi_a}{1-\pi_a} \)

- \( \Rightarrow MU(c_a) > MU(c_{na}) \)

- Risk-averse \( \Rightarrow c_a < c_{na} \): less than full (not-fair) insurance
What is a rational response to uncertainty?

- If you are risk averse, you will want to buy insurance.
- How much?
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  • Fair insurance (i.e. $p = \pi_a$) $\implies$ a person with any degree of risk aversion will fully insure, have the exact same consumption level in no matter what happens
  • Less-than-fair insurance (i.e. $p > \pi_a$) $\implies$ a risk averse person will buy some insurance, but will not fully insure, i.e. she will still have lower consumption if there is an accident. The lower the price and the greater the aversion to risk the closer she will be to full insurance.