Public Goods (cont.)
(Chapter 36)
Today

- Optimal level of a public-goods provision
- Evaluations
- Demand revelation: the VCG mechanism
What is the optimal level of public goods provision?

- Recall: a non-rival, non-excludable good is a public good
- Last time: individuals have incentive to free-ride so market does not provide the efficient level of public goods
- If market failure means govt. should provide the public good
  - How much should it provide (if the quantity is variable)?
  - Indiv. demand is private—how might govt. actually calculate optimal level?
- How do we know much to make each person pay?
- Potential solutions: create incentives for people to reveal demand truthfully. (mechanism design)
- Then govt. knows whether/how much to provide and how should pay how much.
A public good with variable quantity

- Suppose there are two goods, \( x \) (private, i.e. money) and \( G \) (public)

- Cost of producing the public good: \( c(G) \)

- Two individuals \( A \) and \( B \), with private consumption \( x_A \) and \( x_B \)

- Budget: \( x_A + x_B + c(G) = \omega_A + \omega_B \)
A public good with variable quantity

- For each person, what is marginal benefit to switching some consumption from private to public good?
- \( MRS_A, MRS_B \)
- Each person separate equates marginal benefit with marginal cost.
- But because \( G \) is non-rival, 1 extra unit is fully consumed by both \( A \) and \( B \)
- So condition for socially-optimal (Pareto efficient) provision is
  
  \[ MRS_A + MRS_B = MC(G) \]

- More generally:
  
  \[ \sum_i MRS_i = MC(G) \]
How can we determine efficient provision level?

- We use a *revelation mechanism*: a scheme that makes it rational for individuals to truthfully reveal their private valuations of a public good
- Example: Vickrey-Groves-Clarke (VCG) mechanism (or Groves-Clarke tax, Clarke tax)
- How does it work?
- Intuition:
  - Your preferences impose externality or others...
  - Assign tax based upon your *stated* valuation and others’
  - Proper tax internalizes externality
VCG example

- 3 people
- Build a bridge? Yes/no decision
- Private valuations for bridge: $40 for A; $50 for B; $110 for C
- Costs $180 to build bridge, split equally ($60 each)
- Given $60 cost, each person’s net valuation is $v_A = -20$, $v_B = -10$ and $v_C = 50$.
- Is bridge efficient to build?
- Yes: $40 + 50 + 110 = 200 > 180$ or $v_A + v_B + v_C = -20 - 10 + 50 = 20 > 0$
Key concept: pivotal

- A person is pivotal if the inclusion of his/her voice (i.e. statement of preferences) would change the group’s decision.

- Example:
  - 5 people make a yes/no decision based on majority voting.
  - If vote is 3-2 in favor, each ‘yes’ voter is pivotal—switching vote changes outcome, but each ‘no’ voter is not.
  - If vote is 4-1 in favor, no one is pivotal—switching from ‘yes’ to ‘no’ or ‘no’ to ‘yes’ does not affect outcome.

- Bridge example: For each person we ask: would bridge be worth building if we only listened to the other people?
Key concept: pivotal

Is each person pivotal?

- **A is not pivotal:** other two have total net value $-10 + 50 = 40 < 0$. Adding A’s preferences makes the bridge seem less worthwhile, but not enough so to change the group decision.

- **B is not pivotal:** other two have total net value $-20 + 50 = 30 > 0$. Adding B’s preferences makes the bridge seem a little less worthwhile, but not enough to change the group decision.

- **C is pivotal:** other two have total net value $-10 - 20 = -30 < 0$. Without C’s input into the decision, others would decide not to build. Adding C’s voice is what makes the bridge seem worth building.
Key concept: pivotal

- A person who is pivotal is one who influences the collective outcome (yes/no)
- Since everyone cares about the outcome, a pivotal person imposes an external cost or benefit on others
- Idea behind VCG: use tax to make people pay (or get rewarded) for this externality
VCG mechanism: how it works

1. Everyone announces own valuation (true value is private so people *could* lie)
2. If sum of announced values exceeds cost (net value $\geq 0$, project (bridge) gets completed
3. For each person, calculate whether or not pivotal (based on announced values)
4. Collect tax
   - If not pivotal: no tax
   - If pivotal: tax equals net externality imposed on others
VCG mechanism: applying it

- Incentives created by VCG make lying unprofitable (so assume everyone reveals true valuation)
- \[40 + 50 + 110 = 200 > 180\], so bridge is built
- Calculating taxes
  - \(A\) is not pivotal so no tax.
  - \(B\) is not pivotal so no tax.
  - \(C\) is pivotal. Other two don’t like bridge, but she insists. She imposes \(40 - 60 = -20\) on \(A\) and \(50 - 60 = -10\) on \(B\). Combined net ext. is \(-30\), so \(C\) pays tax of 30
- We get efficient decision on bridge thanks to truthful revelation, but \(A\) and \(B\) are not happy about this and would not willingly participate in the mechanism.
- What if we change \(C\)’s valuation from 110 to 80?
VCG mechanism: does it really work?

What are A’s incentives, given B and C tell the truth?

- If truthful ($v = 40$), bridge is built, A pays no tax, and experiences $-20$ net value.
- If A states $v > 40$, bridge is still built, still not pivotal, so payoff unchanged.
- If A states $20 < v < 40$, bridge is still built, still not pivotal, so payoff unchanged.
- If A states $v < 20$, then bridge isn’t built, but A is pivotal in making that happen. A gets 0 net value because there is no bridge (an improvement over $-20$), but now she imposes a net externality on the others (helps B by 10, but deprives C of 50 net value), so she would have to pay a tax of $40$, yielding a payoff of $0 - 40 = -40$, which is worse what she gets from telling the truth.

So A cannot increase her payoff by lying.
VCG mechanism: does it really work?

Let’s do the same for $B$. Given $A$ and $C$ tell the truth:

- If truthful $(v = 50)$, bridge is built, $B$ pays no tax, and experiences $-10$ net value.
- If $B$ states $v > 50$, bridge is still built, still not pivotal, so payoff unchanged.
- If $B$ states $30 < v < 50$, bridge is still built, still not pivotal, so payoff unchanged.
- If $B$ states $v < 30$, then bridge isn’t built, but $B$ is pivotal in making that happen. $B$ gets 0 net value because there is no bridge (an improvement over $-10$), but now she imposes a net externality on the others (helps $A$ by 20, but deprives $C$ of 50 net value), so she would have to pay a tax of $30, yielding a payoff of $0 - 30 = -30$, which is worse what she gets from telling the truth.

So $B$ cannot increase her payoff by lying.
Finally, we repeat for $C$. Given $A$ and $B$ tell the truth:

- If truthful ($v = 110$), bridge is built, $C$ pays tax of $30$, which is subtracted from $50$ net value from bridge, so payoff is $20$.
- If $C$ states $v > 110$, bridge is still built, still pivotal, same tax, so payoff unchanged
- If $C$ states $90 < v < 110$, bridge is still built, still pivotal, same tax, so payoff unchanged
- If $C$ states $v < 90$, then bridge isn’t built, so $C$ doesn’t get net benefit of $50$, but saves $30$ tax because no longer pivotal. Ends up with zero, which is worse than $20$ from telling the truth.

So $C$ cannot increase her payoff by lying.