General Equilibrium (without Production) 
or 
Exchange 
(Chapter 31)
General Equilibrium

- Events in one market have effects on other markets (spillovers)
- Demand for $x$ depends upon prices of complements, substitutes; income
- Supply of $x$ depends upon factor prices
- Previously, we’ve taken these as given—doing partial equilibrium analysis
- But it’s important to understand interdependence of markets—general equilibrium analysis

Partial equilibrium analysis says that competitive markets yield efficient outcomes—is this still true in general equilibrium?
Our approach:

- Simple environment—the *entire* economy
  - 2 kinds of goods
  - 2 people
- Focus on exchange
  - Abstract away from production of new goods
  - Give people endowments
  - Specify preferences
  - Allow them to trade
- Make predictions about behavior of utility-maximizers
- Evaluate welfare
Endowment Economy

- Consumers $A$ and $B$; goods 1 and 2
- Endowments: $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$
- Example: $\omega^A = (6, 4)$ and $\omega^B = (2, 2)$
- This means total endowment of good 1 is $\omega_1^A + \omega_1^B = 6 + 2 = 8$ and of good 2 is $\omega_2^A + \omega_2^B = 4 + 2 = 6$
Allocations

- Endowment represents where people start, but through trade, their allocations may change.
- General allocation or consumption: \( x^A = (x_1^A, x_2^A) \) and \( x^B = (x_1^B, x_2^B) \).
- \((x^A, x^B)\) is \textit{feasible} if it uses at most the aggregate endowment:
  \[
  x_1^A + x_1^B \leq \omega_1^A + \omega_1^B \quad \text{and} \quad x_2^A + x_2^B \leq \omega_2^A + \omega_2^B
  \]
- Helpful graphical tool: Edgeworth Box
- Allows us to simply depict all feasible allocations.
Edgeworth Box

The Endowment Allocation

\[ \omega_A = (6, 4) \]

\[ \omega_B = (2, 2) \]
Edgeworth Box

Feasible Reallocations
How do we think about equilibrium?

- In partial equilibrium analysis:
  - Treat each good separately
  - Find $p$ and $q$ that equate supply and demand
- But this is general equilibrium analysis: where do supply and demand come from?
- $A$ and $B$ can trade with each other
- For everything to be balanced, the amount that $A$ gives up has to equal amount that $B$ receives (for each good, and vice versa)
- In other words $Supply = Demand$ for each good
- This will determine prices for each good
- How do we find supply and demand curves?
- Go back to utility maximization problem
- Need to specify preferences to do this
Utility maximization

- Preferences are given
- Given prices for each good, endowment bundle serves as income
- Can write down budget constraint

\[ p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2 \]

- Solve utility maximization problem
- Gives you optimal allocation, as a function of price ratio
- \( x_1^* - \omega_1 > 0 \) means person demands more of good 1
- \( x_1^* - \omega_1 < 0 \) means person is willing to supply good 1
- Key question: what prices will make it so that A demand exactly as much of each good as B supplies?
Consumer A’s Preferences

Adding Preferences to the Box
For consumer A.

More preferred for consumer A.
Consumer B’s Preferences

Adding Preferences to the Box

For consumer B.

More preferred
Consumer B’s Preferences

Adding Preferences to the Box

For consumer B.

More preferred
Putting Them Both Together

Edgeworth’s Box

\[ x_2^A \]

\[ x_2^B \]

\[ x_1^A \]

\[ x_1^B \]

\[ \omega_1^A \]

\[ \omega_1^B \]

\[ \omega_2^A \]

\[ \omega_2^B \]

\[ O_A \]

\[ O_B \]
Pareto-improving allocations

- Given a particular allocation, a *Pareto-improving* allocation improves the welfare of at least one consumer *without reducing the welfare of another*.

- How do we depict Pareto-improving allocations in the Edgeworth box?
Pareto-Improving Allocations

Edgeworth’s Box
Pareto-Improving Allocations

The set of Pareto-improving allocations
Pareto-optimal allocations

- An allocation is *Pareto-optimal* if it is *feasible* and there is other feasible allocation that is a Pareto-improvement over it.
- In other words, there is no way to make anyone better off without making someone worse off.
- The set of all Pareto-optimal allocations is called the contract curve.
A Pareto-Optimal Allocation

Pareto-Optimality

A is strictly better off but B is strictly worse off

Both A and B are worse off

B is strictly better off but A is strictly worse off

Both A and B are worse off

A Pareto-Optimal Allocation

Pareto-Optimality

A is strictly better off but B is strictly worse off

Both A and B are worse off

B is strictly better off but A is strictly worse off

Both A and B are worse off
The Set of Pareto-Optimal Allocations

Pareto-Optimality

All the allocations marked by a black circle are Pareto-optimal.

The contract curve
Pareto-optimal Allocations

- From the figures, we can see that an allocation at which the indifference curves of the two consumers are tangent must be Pareto-optimal.
- Tangency implies they have the same slope.
- What is the slope of an indifference curve? The Marginal rate of substitution (MRS)!
- Condition for Pareto-optimality:

\[ MRS^A = \frac{\frac{\partial u^A(x_1^A, x_2^A)}{\partial x_1^A}}{\frac{\partial u^A(x_1^A, x_2^A)}{\partial x_2^A}} = \frac{\frac{\partial u^B(x_1^B, x_2^B)}{\partial x_1^B}}{\frac{\partial u^B(x_1^B, x_2^B)}{\partial x_2^B}} = MRS^B \]

- We also require feasibility:

\[ x_1^A + x_1^B = \omega_1^A + \omega_1^B \text{ and } x_2^A + x_2^B = \omega_2^A + \omega_2^B \]
Example

Identifying Pareto-optimal allocations

• Recall total endoments: $\omega^A_1 + \omega^B_1 = 6 + 2 = 8$ and $\omega^A_2 + \omega^B_2 = 4 + 2 = 6$
• Let $u^A(x^A_1, x^A_2) = \ln(x^A_1) + 2 \ln(x^A_2)$ and $u^B(x^B_1, x^B_2) = \ln(x^B_1) + 2 \ln(x^B_2)$.
• MRS of consumer $A$:

$$MRS^A = \frac{\frac{1}{x^A_1}}{\frac{2}{x^A_2}} = \frac{x^A_2}{2x^A_1}$$

• Similarly,

$$MRS^B = \frac{\frac{1}{x^B_1}}{\frac{2}{x^B_2}} = \frac{x^B_2}{2x^B_1}$$

• So a Pareto-optimum is a feasible allocation for which

$$\frac{x^A_2}{2x^A_1} = \frac{x^B_2}{2x^B_1}$$
Clicker Vote

Which of these allocations is Pareto Optimal?

A) \((x^A_1, x^A_2, x^B_1, x^B_2) = (4, 2, 6, 3)\)
B) \((x^A_1, x^A_2, x^B_1, x^B_2) = (4, 1, 4, 5)\)
C) \((x^A_1, x^A_2, x^B_1, x^B_2) = (4, 4, 5, 1)\)
D) \((x^A_1, x^A_2, x^B_1, x^B_2) = (4, 3, 4, 3)\)
E) \((x^A_1, x^A_2, x^B_1, x^B_2) = (6, 4.5, 2, 1.5)\)
Example

Identifying Pareto-optimal allocations

• Recall that a Pareto-optimum is a feasible allocation for which

$$\frac{x_2^A}{2x_1^A} = \frac{x_2^B}{2x_1^B}$$

• In other words, A and B need to have the same $x_1 : x_2$ ratio, which makes sense, because they have identical preferences.

• Clicker option A meets the tangency condition, but is not feasible.

• B is feasible, but does not meet the tangency condition.

• C satisfies neither condition.

• However, both D and E satisfy both conditions, so they both are Pareto-optimal.
Example

Finding all Pareto-optimal allocations (deriving the contract curve)

- We can simplify tangency condition to:

\[
\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}
\]

- Recall endowment/feasibility requirement:

\[x_1^A + x_1^B = 8 \text{ and } x_2^A + x_2^B = 6\]

- Re-write tangency condition, substituting \(x_1^B = 8 - x_1^A\) and \(x_2^B = 6 - x_2^A\):

\[
\frac{x_2^A}{x_1^A} = \frac{6 - x_2^A}{8 - x_1^A}
\]

or

\[
\frac{x_2^A}{x_1^A} = \frac{3}{4}
\]

- This is the equation of the contract curve.
- In this case, it’s just the diagonal of the rectangle.
Core Allocations

How can we narrow down our prediction about the outcome resulting from trade?

- We’ve derived the contract curve: \( x_2^A = \frac{3}{4} x_1^A \)
- Note that clicker options D and E are both on this curve.
- However, Pareto Optimal, but given her endowment, \((\omega^B = (2, 2))\), Person B would never agree to trade to allocation \(E = (6, 4.5, 2, 1.5)\).
- We need to restrict attention to Pareto-optimal allocations that are Pareto-improvements over the initial endowment.
- These allocations are called *core allocations, or the core*—the set of all PO allocations that are welfare improving for both consumers relative to their own endowments
- An allocation will be in the core if it is feasible and it is not *blocked* by any consumer
Core Allocations

The Core

Pareto-optimal trades blocked by B

Pareto-optimal trades blocked by A
The Core

Pareto-optimal trades not blocked by A or B are the core.
A competitive general equilibrium is defined by prices \((p_1, p_2)\) and allocations \((x^A, x^B)\) such that

1. **Utility Maximization**: Given \((p_1, p_2)\), the allocation \((x^A, X^B)\) solves each consumer’s utility maximization problem. That is, \(x^A\) is the solution to

\[
\max_{(x_1^A, x_2^A)} U^A(x_1^A, x_2^A)
\]

such that

\[
p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A,
\]

and similarly for \(B\).

2. **Market Clearing**: the total demand of good 1 is equal to the total endowment (supply) of good 1 and similarly for good 2:

\[
x_1^A + x_1^B \leq \omega_1^A + \omega_1^B \quad \text{and} \quad x_2^A + x_2^B \leq \omega_2^A + \omega_2^B
\]
Trade in Competitive Markets

A’s utility maximization:

For consumer $A$.

\[ p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A \]
B’s utility maximization

Trade in Competitive Markets

For consumer B.

\[ p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B \]
A’s utility maximization:
Trade in Competitive Markets

B’s utility maximization
Trade in Competitive Markets

Market Clearing:

\[
\begin{align*}
&x_1^A + x_1^B = \omega_1^A + \omega_1^B \\
&x_2^A + x_2^B = \omega_2^A + \omega_2^B
\end{align*}
\]
What can we say about welfare?

Two important results:

1. First Welfare Theorem: Given that consumers’ preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy’s endowment. That is, \( CE \Rightarrow PO \).

2. Second Welfare Theorem: any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately arranged among the consumers. That is, \( PO \Rightarrow CE \).
What can we say about welfare?

First Welfare Theorem: CE must be on contract curve.

Implemented by competitive trading from the endowment \( \omega \).
What can we say about welfare?

Second Welfare Theorem: any PO allocation is a CE...
What can we say about welfare?

Second Welfare Theorem: any PO allocation is a CE... from an appropriate starting point!

But this allocation is implemented by competitive trading from $\theta$. 