Choice Under Uncertainty (Chapter 12)
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Typical 100B problem

Breaking it down

- You’re asked to analyze economic behavior/outcomes/policy
  - Individual choice
  - Market behavior and welfare
  - Effectiveness/consequences of policy

- You need to break it down into smaller pieces

- Apply specific skills/tools to deal with each part

- Put parts together to solve overall problem

- zoom back out, refocus on big picture
  - Not just solving math problem
  - What insight do we gain from this?
Typical 100B problem

Example: uncertainty

- Given setup
- Separately derive budget constraint, indifference curves (find MRS)
- Solve $U$ max problem, optimal bundle
- Learn something about demand for insurance
Typical 100B problem

Example: market demand, equilibrium

- Given individual demands, info about supply
- Derive total demand, supply
- Solve for equilibrium $p, q$
- Learn something about behavior in the market
Typical 100B problem

Example: Changes to equilibrium (comparative statics)

- Given info about demand, supply
- Find equilibrium $p, q$
- Introduce demand/supply shift, tax, price floor, ceiling, quota, etc., calculate new $p, q$
- Observe something about effect on behavior, welfare
Example: Comparison of market structures

- Given market demand, costs/supply
- Find eq. $p, q$ for various market structures
- Compare behavior and welfare
Types of exam questions

One categorization: difficulty

- Easy, just about everyone should get
- Moderate, many, but not all should get
- Challenging, only a handful of the very best students will get
Types of exam questions

Another way to classify:

- Small, deals only with subpart of overall problem

- Large, deals with more parts or entire problem

- Pushes you to focus out on big picture, draw conclusions, push understanding further, deal with new complications— not necessarily more complicated math
What will the quizzes look like?

- Two multiple-choice questions
- Both type 1
- Diagnostic, small grade impact
- Checks for minimum necessary comprehension

Don’t think: I did well on the quiz, so I’m prepared for the exam

Do think: I did well on the quiz, so I can focus on the larger parts of the problem, big picture for the exam

Do think: I had trouble on the quiz— I really need to do something about this before the exam
States of Nature and Contingent Plans

- **States of Nature:**
  - “accident” (a) vs. “no accident” (na)
  - Probability of: accident = $\pi_a$, no accident = $\pi_{na}$; $\pi_a + \pi_{na} = 1$
  - Accident causes loss of $L$

- “Bundle” = state-contingent consumption plan: Specifies consumption level for each scenario (state)

- Option to buy some insurance: contracts are be state-contingent (e.g. insurer pays only if you have an accident)

- How much should you buy?
Deriving the Budget Constraint

Q: Where to start?
A: The bundle with which you are endowed.

- Without insurance, consumption is:
  - \( c_{na} = m \) if no accident
  - \( c_a = m - L \) if accident
- The endowment bundle displayed graphically:
Deriving Budget Constraint

Insurance contract: Buy $K of fire insurance at price $p$, claim $K$ from company if accident

- If no accident: $c_{na} = m - pK$
- If accident: $c_a = m - pK - L + K = m - L + (1 - p)K$
- Given $K$, it must be true that...(solve for $K$, substitute):

$$c_{na} = \frac{m - pL}{1 - p} - \frac{p}{1 - p} c_a$$

The endowment bundle.
Q: Why do people buy insurance when they face risk?

To answer this, we have to consider preferences

- $U(c_a, c_{na})$ captures attitude towards uncertainty/risk
- Person might be risk averse or risk neutral (or risk loving)
- Consider our three favorite examples:
  A. Perfect Substitutes
  B. Cobb-Douglas
  C. Perfect Complements
  D. Not sure
  E. Don’t have clicker yet

- CLICKER VOTE: which of these reflect some degree of risk aversion?
Insurance is a way of mitigating risk. If you are risk averse, you are happiest buying some positive amount of insurance.

Comparative statics:
- risk aversion $\uparrow \implies K$
- $p \uparrow \implies K \downarrow$
- $L \uparrow \implies K$

What about an algebraic solution? First, a detour...
Example: a lottery

- Win $90 or $0 equally likely
- $U(90) = 12$ and $U(0) = 2$
- Expected Utility is

\[ EU = 0.5 \times U(90) + 0.5 \times U(0) = 0.5 \times 12 + 0.5 \times 2 = 7. \]

- Expected Money is

\[ EM = 0.5 \times 90 + 0.5 \times 0 = $45. \]
Risk Attitudes

How do we characterize attitude towards risk?

Recall: $EU = 7$ and $EM = $45

- $U(45) > 7 \implies$ risk-averse
- $U(45) < 7 \implies$ risk-loving
- $U(45) = 7 \implies$ risk-neutral
Risk Attitudes

We typically assume diminishing marginal utility (DMU) of wealth.

\[ EU < U(EM) \ldots \]

this implies risk aversion!
Risk Attitudes

Example: Risk-loving preferences

\[ EU > U(EM) \]
Risk Attitudes

Example: Risk-neutral preferences

\[ EU = U(EM) \]
Optimal Choice (Algebra)

Calculating the MRS

- \( EU = \pi_a U(c_a) + \pi_{na} U(c_{na}) \)
- Indifference curve \( \implies \) constant EU
- Differentiate:
  - \( dEU = 0 = \pi_a MU(c_a)dc_a + \pi_{na} MU(c_{na})dc_{na} \)
  - \( MRS = \frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})} \)
- Solution satisfies
  \[
  \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})} = \frac{p}{1 - p}.
  \]
How optimal insurance purchase \((K)\) and consumption levels \(c_a, c_{na}\) depend upon probabilities (given) and price.

Q: What determines the price of insurance?

A: Market conditions

Consider a competitive insurance market:

- Free entry \(\implies\) zero expected economic profit
- \(pK - \pi_a K - (1 - \pi_a)0 = (p - \pi_a)K = 0.\)
- \(\implies p = \pi_a\)

Insurance is fair
With fair insurance, rational choice satisfies

\[
\frac{\pi_a}{\pi_{na}} = \frac{\pi_a}{1 - \pi_a} = \frac{p}{1 - p} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}.
\]

In other words, \( MU(c_a) = MU(c_{na}) \)

Risk-aversion \( \Rightarrow \) \( c_a = c_{na} \)

Full insurance!
Suppose the insurance market is not competitive

- Insurers can expect positive profits

- \( pk - \pi_a K - (1 - \pi_a)0 = (p - \pi_a)K > 0 \)

- Then \( p > \pi_a \) and \( \frac{p}{1-p} > \frac{\pi_a}{1-\pi_a} \)

- \( \implies MU(c_a) > MU(c_{na}) \)

- Risk-averse \( \implies c_f < c_{na} \): less than full (not-fair) insurance
I flip a fair coin. Heads: I pay you $120; tails: you pay me $100. *Any takers?*

**CLICKER VOTE:**

A  Accept  
B  No thank you!
Proposed Gamble: II

What if I offered this same gamble at the beginning of every lecture (and you had to tell me today what you would choose each time)?

CLICKER VOTE:

A  Accept every time
B  Reject every time
C  Some combination
Why is the same gamble more attractive when it is repeated?

- Each gamble has positive expected value
- Each coin toss is independent
- Law of Large Numbers: expected money from compound gamble = $N$ times the $EM = \text{a big positive number}$
- Portfolio of gambles is diverse, so very little chance of net loss
Example:

- Two firms, A and B. Shares cost $10
- With prob = .5, \( \Pi_A = 100 \) and \( \Pi_B = 20 \)
- With prob = .5, \( \Pi_A = 20 \) and \( \Pi_B = 100 \)
- You have $100 to invest. How?
Diversification

Example:

- Buy only firm A’s stock?
- $100/10 = 10 shares
- Earn $1000 w/ prob .5 and $200 w/ prob .5
- Expected earning: $500 + $100 = $600
- Same for buying only B
Diversification

Example:

- Buy 5 shares of each firm?
- Earn $600 for sure
- Diversification has maintained expected earnings while lowering risk
- Typically there’s a tradeoff between earnings and risk
Recap

What are rational responses to risk?

- Buying insurance
- A diverse portfolio of contingent consumption goods (assets)
How do insurance companies operate?

- You buy insurance in response to risk
- Insurance company gets your premium, but now faces risk of having to pay claim
- To the extent that claims are independent, this is ok for them because they have a diverse portfolio of risks
- Same w/ home lenders: they get your mortgage payments, but lose if you default
- To diversify risk, lenders wad contracts/mortgages together into bundles, then sell them (in pieces) as relatively safe (diversified) securities
- Thus, our risk and insurance courses through the veins of the financial system
How do insurance companies operate?

So what can and does go wrong?

- Diversification works if risks are independent, but not if correlated
- My proposed gamble: imagine if I decided outcome w/ one coin-toss at the end of the quarter. Takers?
- Risk of house burning down: Seattle vs. SoCal
- Insurance companies are exposed to systemic risks
- Wildfires, earthquakes, hurricanes can wipe out entire cities/regions at once
- Natural disasters are disasters for insurers
- Insurers know this: there is an enormous re-insurance industry