Oligopoly (contd.)
Chapter 27
Midterm 2

- Bring pencil/pen, bluebook, pink scantron

- 8 MC questions, like last time

- Important skills:
  - Find monopoly $p, q$
  - Analyze effects of policy (e.g. tax, subsidy, price ceiling) on monopoly
  - Max profits using price discrimination (lecture and especially workouts 25.5-8)
  - Analyze duopoly (Cournot, Bertrand, Stackelberg, collusion)
Considerations:

- Do firms compete on price or quantity?
- Do firms act sequentially (leader/followers) or simultaneously (equilibrium)
- Stackelberg models: quantity leadership
- Cournot equilibrium models: simultaneous choice quantity competition
- Bertrand equilibrium models: simultaneous choice price competition
Today

- Price competition (Bertrand duopoly)
- More quantity competition
  - Stackelberg Duopoly
  - Cartels/collusion
Bertrand Duopoly: price competition

- Firms compete on price
- No clear leader, follower so firms effectively choose $p$ simultaneously
- Take the other firm’s price as given
- Market demand determines equilibrium output
- Both choose same price: divide demand evenly
- One sets lower price: that firm captures entire market
Suppose two firms have same $MC$

What price to charge?
Bertrand Duopoly: price competition

- Suppose two firms have same $MC$
- What price to charge?
- Apply Nash equilibrium
  - Each chooses optimal $p_i$ given $p_j$
  - No one has incentive to deviate
- If pricing above marginal cost, each has incentive to undercut competitor ($p_i > p_j > MC$ is not an equilibrium)
Bertrand Duopoly: price competition

- Suppose two firms have same $MC$
- What price to charge?
- Apply Nash equilibrium
  - Each chooses optimal $p_i$ given $p_j$
  - No one has incentive to deviate
- If pricing above marginal cost, each has incentive to undercut competitor ($p_i > p_j > MC$ is not an equilibrium)
- $p_i = p_j = MC$ is the only possible equilibrium
- Zero profits for both, but no incentive to deviate:
  - Higher price means no sales
  - Lower price means losses
Stackelberg Duopoly: leader/follower

1. Two firms compete in the same market
   - Firm 1 chooses $q_1$
   - Firm 2 observes $q_1$, chooses $q_2$
   - This determines total $Q$
   - ...which determines price
Stackelberg Duopoly: check your intuition

Clicker Vote

Who has the advantage?

A) First mover
B) Second mover
C) Neither
D) Depends
Stackelberg Duopoly: how to solve the model

Analyze using *backwards induction*

- Start at the end: what does Firm 2 do given $q_1$?
- Derive *reaction function* just like we did for Cournot
- Then find optimal $q_1$, given Firm 1 can deduce 2’s reaction
Stackelberg Duopoly: back to our example

Find 2’s reaction function (recall: \( p = a - Q, \ MC = c \))

- Firm 2’s profits, given \( q_1 \):
  \[
  \Pi_2(q_1, q_2) = (p - c)q_2 = (a - q_1 - q_2 - c)q_2
  \]
  implies \( q_2^*(q_1) = \frac{a-q_1-c}{2} \)

- Realizing this, 1 factors in 2’s response when computing profits:
  \[
  \Pi_1(q_1, q_2^*(q_1)) = (p - c)q_1
  = (a - q_1 - q_2^*(q_1) - c)q_1
  = (a - q_1 - \frac{a-q_1-c}{2} - c)q_1
  = \frac{a-q_1-c}{2} q_1
  \]
Stackelberg Duopoly: back to our example

\[ \Pi_1(q_1, q_2^*(q_1)) = \frac{a - q_1 - c}{2} q_1 \]

- Differentiate to find \( q_1^* \)
  \[ \frac{\partial \Pi_1}{\partial q_1} = \frac{a - 2q_1 - c}{2} = 0 \]

- Solving yields \( q_1^* = \frac{a-c}{2} \)
- Plug into reaction function:
  \[ q_2^*(q_1^*) = \frac{a - q_1^* - c}{2} = \frac{a - \frac{a-c}{2} - c}{2} = \frac{a - c}{4} \]

- Same per unit profit, so \( q_1 > q_2 \Rightarrow \pi_1 > \pi_2 \)
- First-mover advantage
Stackelberg Duopoly: back to our example

Fill in details:

- \( q_1^* = \frac{a-c}{2}, \; q_2^* = \frac{a-c}{4} \)

- \( Q = \frac{3}{4}(a - c) \) so \( P = \frac{a+3c}{4} \)

- \( p - c = \frac{a-c}{4} \), so \( \pi_1 = \frac{(a-c)^2}{8}, \; \pi_2 = \frac{(a-c)^2}{16} \), and 
  \( \Pi = \frac{2}{16}(a - c)^2 \)

- \( CS = \frac{9}{32}(a - c)^2 \) so \( W = \Pi + CS = \frac{15}{32}(a - c)^2 \)
Can firms make more profit by colluding and behaving as a monopoly?

- The cartel will behave as if it’s a monopoly
- \( \Rightarrow \) \( MR = MC \)
- \( Q = \frac{1}{2}(a - c) \) and \( q_1 = q_2 = \frac{1}{4}(a - c) \), \( P = \frac{a+c}{2} \)
- Profits: \( \Pi = \frac{1}{4}(a - c)^2 \) so \( \pi_1 = \pi_2 = \frac{1}{8}(a - c)^2 \)
- Compare to Cournot: lower \( Q, CS, W \); higher \( P, \Pi \)
### Summary Table

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<td>15/32</td>
<td>4/9</td>
<td>$\frac{n^2+2n}{2(n+1)^2}$</td>
<td>3/8</td>
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Note: quantities ($q_1$, $q_2$, $Q$) in units of $a - c$; welfare ($\Pi$, $CS$, $W$) in units of $(a - c)^2$. 
So what is stopping firms colluding and making more profits?

- Anti-trust regulation
- Ok, but even without regulation, e.g. what problem does OPEC face?
So what is stopping firms colluding and making more profits?

- Anti-trust regulation
- Ok, but even without regulation, e.g. what problem does OPEC face?
- Collusion is not individually rational: each firm has incentive to cheat and produce more
- E.g. increase $q$ and earn more profit (foreshadowing: how is this like tragedy of the commons?)
- E.g. slightly decrease $p$ and capture entire market
- Cartels can only succeed if they can effectively monitor and punish cheating, which is difficult, esp. if collusion is illegal
What are the incentives facing each firm?

Suppose each duopolist can choose to cooperate (C) and produce the collusive quantity \( q_i = \frac{a-c}{4} \), or to cheat/defect (D) and produce the Cournot quantity \( q_i = \frac{a-c}{3} \).

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<td><strong>D</strong></td>
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Duopolists’ Dilemma

Note: \( \frac{1}{8} = \frac{18}{144} \) and \( \frac{1}{9} = \frac{16}{144} \)