Instructions: Write your name, perm #, and TA in the blanks on this page and write your perm # and version # on your Scantron sheet. There are two parts to this exam. Part I consists of 15 multiple-choice questions (2 pts. each) and Part II consists of 2 longer problems (10 pts. each). No calculators are allowed. You have three hours (180min.) to complete this exam. If you get stuck on something, I suggest moving on and coming back later if/when you have time. Good luck!

Part I – Multiple Choice (30 out of 50 points)
Indicate your answers on the Scantron sheet. Make sure you put your response on the correct line.

1) The inverse demand function for diamonds is \( P = 1500 - 2Q \). The market for diamonds consists of two firms, Shiny and Dull. Shiny’s cost function is \( c(q) = 300q + 5,000 \) and Dull’s cost function is \( c(q) = 300q + 10,000 \). Which of the following statements is true?
   a. Cournot equilibrium total output is 400. Stackelberg eq. total output is 450.
   b. Cournot equilibrium total output is 400. Stackelberg eq. total output is 300.
   c. Cournot equilibrium total output is 200. Stackelberg eq. total output is 450.
   d. Cournot equilibrium total output is 200. Stackelberg eq. total output is 150.
   e. Cournot equilibrium total output is 200. Stackelberg eq. total output is 300.

2) Cournot duopolists Firm 1 and Firm 2 face inverse demand \( P = 24 - Q \) and each has a marginal cost of 6. Firm 2 revamps it’s production process, allowing to produce at marginal cost of 3 (while Firm 1 still has MC of 6). How does this affect Firm 1?
   a. Its output is unchanged, but it’s profits fall by 6.
   b. Its output and profits are unchanged.
   c. Its output drops by 1 and it’s profits fall by 11.
   d. Its output is unchanged, but its profits fall by 12.
   e. Its output increases by 1 and it’s profits fall by 1.

3) In the small town of Broadlands, there are currently only two coffee shops, The Java House and The Drip. The two shops compete as Cournot duopolists. The inverse demand function for coffee in Broadlands is given by \( P = 150 - Q \) and each firm has the same cost function of \( c(q) = q^2 \). What is total equilibrium output?
   a. \( 296/3 \)
b. 60
c. 75
d. 148/3
e. 37.5

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Which of the following statements about the above matrix game is true?

a. If \( a > 3 \) and \( b > 1 \) then \((U,R)\) is a Nash equilibrium.
b. If \( a < 7 \) and \( b > 4 \) then there is no Nash equilibrium.
c. If \( a > 7 \) and \( b > 1 \) then \((U,R)\) is a Nash equilibrium.
d. If \( a < 3 \) and \( b > 1 \) then there are two Nash equilibria.
e. If \( a > 3 \) and \( b < 4 \) then \((U,R)\) is a Nash equilibrium.

5) In an exchange economy, there are two people (A and B), and two goods (X and Y). The utility functions of A and B are given by \( U_A = X_A Y_A \) and \( U_B = X_B Y_B \).

Person A starts with 30 units of X and zero units of Y.

Person B starts with zero units of X and 10 units of Y.

Which expression below gives an equation for the contract curve?

a. \( Y_A = 30X_A \)
b. \( X_A = 10Y_A \)
c. \( Y_A = 3X_A \)
d. \( X_A = 3Y_A \)
e. \( Y_A = X_A \)

6) (Continued from #5 above) If the price of good Y is 1, then what is the price of good X in competitive equilibrium?

a. 1/3
b. 4
c. 3
d. 1/2
e. 2

7) In an exchange economy, there are two people (A and B), and two goods (X and Y). The utility functions of A and B are given by \( U_A = X_A^{2/3} Y_A^{1/3} \) and \( U_B = X_B^{3/4} Y_B^{1/4} \).

There are 10 units of X and 10 units of Y in total.

Which of the following gives a condition for Pareto optimality?
8) (Continued from #7 above) Suppose person A is originally endowed with all 10 units of good X and person B is originally endowed with all 10 units of good Y. Let \( p \) be the price of good X and let the price of good Y be 1. What is the competitive equilibrium value of \( p \)?

a. \( p = \frac{4}{9} \)

b. \( p = \frac{9}{4} \)

c. \( p = 2 \)

d. \( p = \frac{1}{2} \)

e. \( p = 3 \)

9) Firm 1 produces output X with a cost function \( C_1(X) = \frac{X^2}{200} \). Firm 2 produces output Y with a cost function \( C_2(X, Y) = \frac{Y^2}{100} - 2X \). Both firms face competitive product markets. The competitive price of X is $4 and the competitive price of Y is $5. There is no entry or exit into this market. Which of the following is true?

a. In equilibrium, the competitive amount of Y produced is greater than the socially optimal amount of Y.

b. The socially optimal amount of X produced is 200 less than the amount produced in the competitive equilibrium.

c. The social optimum is that 200 units of X and 250 units of Y by produced.

d. The socially optimal amount of X produced is 200 more than the amount produced in the competitive equilibrium.

e. The socially optimal amount of X is the same as the amount of X produced in a competitive equilibrium.

10) An airport is located next to a banana farm. The airport can fly X planes per day, but the burning fuel will cause air pollution in the amount of Y per day. The total profits for the airport per day are given by \( 50X - X^2 + (9 - 2Y)^2 \). The banana farm needs clean air to produce bananas, B, so the air pollution caused by the airport makes growing bananas more costly. The banana farm’s profit function is \( 20B - (B + Y)^2 \). The banana farm asks the government to make the airport emit less pollution per day. As a result, the government decides to tax the airport per plane to get the airport to emit the socially optimal level of pollution, which they found to be Y=7. What should be the per unit tax?
11) A village owns a common pasture where villagers graze their sheep. The cost to a sheep owner of owning and caring for the sheep is $2. The total revenue from all sheep on the common pasture is \( TR(s) = (20 - 3s)s \), where \( s \) is the number of sheep on the pasture. The local government notices that total profits from the pasture are not maximized if the villagers are allowed to pasture their sheep for free. The government decides to allow a sheep to use the common pasture only if its owner buys a sheep license for a price of \( St \). To maximize total profits (of villagers and the government) how many licenses should the government sell and how much should each license cost?

- a. \( s = 3, t = 9 \)
- b. \( s = 6, t = 0 \)
- c. \( s = 6, t = 12 \)
- d. \( s = 3, t = 6 \)
- e. \( s = 3, t = 2 \)

12) A quiet town in Kansas has 10 people, all of whom have the same preferences. There is one private good and one public good. Each person \( i \) in town has utility 

\[ U(x_i, y) = x_i + y^{1/2} \]

where \( x_i \) is private good for person \( i \) and \( y \) is the amount of public good that the town provides. If the town already provides 1 unit of the public good, what is the marginal social utility of additional units of the public good?

- a. 20
- b. 2
- c. 10
- d. 5
- e. \( \frac{1}{2} \)

13) Bob and Larry have preferences defined over artichokes, \( a \), and trampolines, \( t \). They have identical utility functions, 

\[ U(a, t) = a + 2000t^{1/2} \]

Each artichoke costs $1 and each trampoline costs $1,000. Bob and Larry like to share, and indeed trampolines are a public good for them. Artichokes, however, are a private good. We don’t know their exact incomes, but we do know that each of them earns at least $10,000. The Pareto efficient number of trampolines for them is

- a. 4
- b. 1
c. impossible to determine without knowing how the costs will be shared
d. 2
e. 6

14) Barry and Ray are thinking of buying a sofa. Barry’s utility function is $U_B(S, M_B) = (1 + S)M_B$ and Ray’s utility function is $U_R(S, M_R) = (2 + S)M_R$, where $S = 0$ if they don’t get the sofa and $S = 1$ if they do and where $M_B$ and $M_R$ are the amounts of money they have respectively to spend on their private consumptions. Bob has a total of $2,000 to spend on the sofa and other stuff. Ray has a total of $300 to spend on the sofa and other stuff. The maximum amount that they could pay for the sofa and still arrange to both be better off than without it is

a. $1,100
b. $800
c. $1,400
d. $850
e. $3200

15) Arne, Barack, and Christy are deciding whether or not to pool their money to buy a $1200 HDTV to put in the oval office. They agree that if they do buy it, they’ll split the cost evenly. Arne values the HDTV at $300, Barack at $600, and Christy at $100. Which best describes the equilibrium outcome if they use a Groves-Clarke tax mechanism to make their decision?

a. They don’t buy the HDTV and no one pays anything
b. They don’t buy the HDTV; Arne and Christy pay Barack $300
c. They buy the HDTV and each pays $400 to the store.
d. They buy the HDTV, each paying $400 to the store; Barack pays Arne $100 and Christy $300
e. They don’t buy the HDTV; Christy pays $100 to the tax administrator
Part II – Long Problems (20 out of 50 points)

Answer both questions in the space provided below. Indicate your answer by circling it or putting a box around it. Show your work/reasoning to obtain partial credit for an incorrect answer. Each sub-part is worth 1 point, with partial-credit given in half-point increments.

1) Myra and Kathleen are roommates who spend 8 hours studying together each day. One thing they have in common is their love of eating fresh fruit. However, they differ in their study habits—Myra likes to listen to loud music as she studies, but Kathleen, who hears every song that Myra plays, finds it distracting. On a given day, Myra’s utility is given by $U_m(F_m, T) = F_m + T$, where $F_m$ is the number of pieces of fruit Myra eats that day and $T$ is the number hours she spends listening to her tunes while studying. Kathleen’s utility is given by $U_k(F_k, T) = F_k - T^2/8$, where $F_k$ is the number of pieces of fruit Kathleen eats that day. Each roommate has a daily allowance of $4 (none of which can be saved for another day). A piece of fruit costs $1, but it doesn’t cost Myra anything to play her music.

a. How much fruit does each roommate eat and how much music does Myra play if they each choose individually?

b. What is the resulting level of utility for each roommate? What is the total utility?

c. What is the socially optimal (total-utility-maximizing) level of fruit consumption for each person and music listening for Myra? What is the resulting level of utility for each roommate? What is the total utility?

d. Suppose the dorm elders wanted were to tax loud music so as to maximize total utility. What would be the optimal tax to charge for an hour of loud music to give them the best chance of obtaining the optimal level of music?

e. Supposing that the elders pour all tax revenue into fresh fruit, to be distributed evenly among Myra and Kathleen, assuming that Myra chooses
the socially optimal T, what are their resulting utility levels? What is the total utility?

f. Instead of a tax, the elders rule that you need your roommate’s permission to play music. How much would Kathleen demand that Myra pay her to play an hour of music?

g. Assuming that Myra chooses the socially optimal T, what are the resulting utility levels? What is the total utility?

h. An alternative policy is that you don’t need your roommates’ permission to play loud music. In this case, how much would Myra demand that Kathleen pay her to refrain from playing music for an hour?

i. What level of T will Myra choose? What are the resulting utility levels and total utility?

j. In this specific example, which policy (music tax, right-to-quiet, right-to-noise) is the only one that would actually guarantee the socially-optimal level of music? Explain.
2) Duopolists 1 and 2 face inverse demand $P = 2 - Q$ for their product, where $Q$ is their total output, and each has a constant marginal cost of 1, with no fixed cost.

   a. Find the Cournot equilibrium total market quantity and price.

   b. Find the consumer surplus, industry profit, and total welfare.

   c. What will the price and total quantity be if they form a cartel?

   d. Find the consumer surplus, industry profit, and total welfare in this case.

   e. What price and total quantity would result from perfect competition? Find the consumer surplus, industry profit, and total welfare in this case.

   f. Briefly summarize the pattern you see in market behavior and welfare across these three cases.

   g. Suppose the government decides to subsidize the good by 1 dollar per unit. What is the effect on the Cournot equilibrium behavior and welfare?

   h. What is the effect of the subsidy on behavior and welfare if the firms are acting as a cartel?
i. In these two cases, does the subsidy raise or lower total welfare? Without doing the math, would this also be true for the perfectly competitive case? Explain in one short sentence.

j. In which case (Cournot or cartel) is the welfare effect of the subsidy greater? Very briefly explain why.