1. Consider an exchange economy consisting of two people, A and B, endowed with two goods, 1 and 2. Each person is initially endowed with 5 of each good. Their preferences are given by \( U^A(x_1, x_2) = x_2^1 x_2 \) and \( U^B(x_1, x_2) = x_1 x_2^2 \). This question asks you to assume that \( p_2 = 1 \), then find the competitive equilibrium allocations and \( p_1 \). (Points split equally across parts)

(a) Find the MRS for each consumer

\[ MRS^A = \frac{2x_2}{x_1} \quad \text{and} \quad MRS^B = \frac{x_1}{2x_2} \]

(b) Write the equation of the contract curve (express \( x_2^A \) as a function of \( x_1^A \))

\[ x_2^A = \frac{10x_1^4}{40 - 3x_1^4} \]

(c) Write the equation of the budget constraint (in person A’s coordinates) as a function of \( p_1 \)

\[ \text{Answer: Some form of } x_2^A - 5 = -p_1(x_1^A - 5) \]

(d) What expression involving \( x_1^A \) and \( x_2^A \) can we substitute for \( p_1 \) in your answer to the previous part?

\[ \text{Answer: The MRS of } A, \text{ which is } \frac{2x_2}{x_1} \]

(e) Find the competitive equilibrium price, \( p_1 \) and allocations, \( x^A = (x_1^A, x_2^A) \) and \( x^B = (x_1^B, x_2^B) \)

\[ \text{Answer: This amounts to finding the point that is on both the contract curve and budget constraint. The solution is } x^A = (20/3, 10/3) \text{ and } x^B = (10/3, 20/3), \text{ from which you can determine } p_1 = 1. \]

2. A bakery and coffee shop operate next to each other downtown. The bakery sells bags of donuts (\( d \)) for $30 each and the coffee shop sells bags of coffee (\( k \)) for $15. Making donuts and coffee is costless, but each shop needs to advertise to attract customers. Some of the customers lured by the advertising from one shop will also make a purchase at the other shop. Specifically, the bakery needs to spend \( c(d) = d^2 - \frac{dk}{30} \) on advertising in order to sell \( d \) donuts and the coffee shop must spend \( c(k) = k^2 - \frac{dk}{15} \) on advertising in order to sell \( k \) cups of coffee.

(a) (2 points) What is the bakery’s marginal private benefit of selling a bag of donuts and the marginal private cost? What is the coffee shop’s marginal private benefit of selling a bag of coffee and the marginal private cost?

\[ \text{Answer: } MPB_d = p_d = 30, \quad MPC_d = \frac{2d}{15} - \frac{k}{30}, \quad MPB_k = p_k = 15, \quad MPC_k = \frac{2k}{15} - \frac{d}{30} \]

(b) (2 points) How many bags of donuts and coffee will be sold in the competitive marketplace?

\[ \text{Answer: } d = 270, \quad k = 180 \]

(c) (2 points) The city looks out for its business owners and hires a consultant to determine how much the two shops should advertise in order to maximize total profits. What level of \( d \) and \( k \) does the consultant recommend?

\[ \text{Answer: } d = 375, \quad k = 300 \]
(d) (4 points) The city decides to subsidize $d$ and $k$ so as to induce the owners to choose the amounts recommended by the consultant. What should be the subsidy per bag of donuts, $s_d$, and the subsidy per bag of coffee, $s_k$?

Answer: $s_d = 10$ and $s_k = 12.5$