All-Pay Auctions

• In an all-pay auction, every bidder pays what they bid regardless of whether or not they win.

• Examples:
  – Elections
  – Almost any kind of contest or sports event
  – Research and Development
  – Wars
  – Lobbying

• Since bids are wasted if you don’t win, bidders have a strong incentive to bid aggressively if they bid at all.
All-Pay Auctions

• A simple all-pay auction:
  – The object for sale is worth 100 to all of $n$ identical bidders and all of them know this valuation exactly.
  – Clearly no one will bid more than 100.
  – Each bidder’s bid is an amount $x$ in the interval $[0,100]$.
  – The winner is the highest bidder and gets a payoff of $100 - x$
  – The payoff of all other bidders is $-x$

• There is no pure strategy equilibrium of this game.

• A mixed strategy of a player is a probability distribution over the possible bids between 0 and 100.

• We represent a mixed strategy by a probability distribution function $P$, where $P(x)$ which gives the probability that a player bids less than some amount $x$ between 0 and 100.
All-Pay Auctions

• We look for a symmetric equilibrium.
  – Suppose bidders 2, ..., n used the mixed strategy \( P \) and consider bidder 1.
  – Suppose bidder 1 bids \( x \).
  – Her expected payoff is given by
    \[
    P(x)^{n-1}(100-x) + (1- P(x)^{n-1})(-x)
    = 100P(x)^{n-1} - x
    \]
  – In order for bidder 1 to be willing to randomize we require \( 100P(x)^{n-1} - x = c \).
All-Pay Auctions

- Moreover, since P is a probability distribution function we require \( P(0) = 0 \)
- Hence, \( c = 0 \)
- Hence,

\[
P(x) = \left( \frac{x}{100} \right)^{\frac{1}{n-1}}
\]
All-Pay Auctions

- Equilibrium mixed strategies in this example:
Cummulative Distribution Function of Bids for All Pay Auction
Spring 2013

- 2 bidders
- 5 bidders
- 10 bidders
Cummulative Distribution Function of Bids for All Pay Auction
Winter 2016

[Graph showing the cumulative distribution function of bids for different numbers of bidders: 2 bidders, 5 bidders, and 10 bidders.]
# All Pay Auction

<table>
<thead>
<tr>
<th></th>
<th>2-bidder</th>
<th>5-bidder</th>
<th>10-bidder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Winner Profit</td>
<td>29.23</td>
<td>8.00</td>
<td>0.34</td>
</tr>
<tr>
<td>Average Loss of Losers</td>
<td>32.31</td>
<td>24.17</td>
<td>29.63</td>
</tr>
<tr>
<td>Average Overall Profit</td>
<td>-1.54</td>
<td>-17.74</td>
<td>-26.63</td>
</tr>
</tbody>
</table>