Consider the moving-average process

\[ Y_t = U_t + \theta U_{t-1}, \quad t = 1, \ldots, n \]

in which \( \{U_t\}_{t=1}^n \) is a sequence of uncorrelated random variables with mean 0 and variance \( \sigma^2 \).

a) Derive the log-likelihood function for the joint density.

b) Use the Kalman filter to derive the log-likelihood function.