Courtship as a Waiting Game

by

Ted Bergstrom and Mark Bagnoli
University of Michigan and University of Indiana

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In most times and places, men, on average, are older than their wives. A recent United Nations study reports the average age of marriage for each sex for more than 90 countries over the time interval between 1950 and 1985. [1] In every country and in every time period reported, the mean age at marriage of males exceeded that of females. The smallest difference in mean ages was 1 year (Ireland) and the largest difference was 10.9 years (Mali). In 1985 in the United States, the difference was 1.9 years, in Western Europe about 2.5 years, and in Southern and Eastern Europe about 3.5 years. In Japan the difference was 3.7 years, in India, nearly 5 years and in the Middle East, about 4 years. In the Caribbean the age gap is about 5 years, in Central America, about 4 years and in South America, between 2 and 3 years In African countries, this gap ranges between 5 and 10 years. In most countries, the age difference between the sexes at marriage has diminished substantially between 1950 and 1985, but nowhere has it disappeared altogether.[2]

This paper proposes a partial explanation for the difference in age-at-marriage of males and females, for why this difference is diminishing over time, and for why it tends to be greater in traditional societies than in modern societies. We suggest that this difference is a result of the different economic roles of males and females and a corresponding difference between the sexes in the rate at which evidence accumulates about one's "quality" as a possible marriage partner.

In societies where male roles as economic providers are relatively varied and specialized, information about an individual male's economic capabilities may be revealed only gradually after he has spent time in the work force. In contrast, for a female whose anticipated tasks will be childbearing, child care and traditional household labor, it may be that once she has reached physical maturity, the passage of time adds little information about her capabilities for these tasks.

We propose a model in which males who expect to prosper will delay marriage until the evidence of their success allows them to attract more desirable females. The most desirable females, on the other hand, have little to gain by postponing marriage, since the relevant

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[1] *Patterns of First Marriage: Timing and Prevalence* (1990). The average computed is the "singulate mean age at marriage". This statistic estimates the average number of years spent in the single state by those who marry before age 50, and is computed from census statistics on the proportion of the population who have never married in each age group. See Hajnal (1953) for details.

[2] In 72 of the 91 countries listed in Tables 1-4, the age gap decreased and in 14 countries the gap increased. Exceptions to this pattern are Japan, Germany, and several countries in Southern and Eastern Europe.

information about their quality is available at an earlier age. In the long-run stationary equilibrium of this model, males with poor prospects marry at an early age, while those who expect success will marry later in life. All females marry relatively early in life. The more desirable females marry successful older males and the less desirable females marry the young males who do not expect to prosper.

This model predicts that males who marry young will tend to have lower earnings in later life. There is evidence, at least for the United States, that this is the case. According to the 1980 U.S. census,[3] 35% of males aged 45-54 who married before age 20 had annual incomes below $10,000. For those who first married between ages 21 and 29, only 17.5% had incomes below $10,000. Median income of persons who married before 18 was $14,500, median income of those who married between 18 and 20 was $16,800, and median income of those who married between 22 and 29 was $19,000. [4]

The formal model presented in this paper is starkly oversimplified. We confine the analysis to two possible ages of marriage for each sex. We assume much more dramatic differences between the sexes than is justified by reality. Furthermore, the model lacks a realistic treatment of search costs. While our model lacks sophistication in many directions, it is unusual in its explicit treatment of the dynamics of an assignment equilibrium taking place in real time. Though make no claim that this model is detailed enough to "explain" the observed distributions of age-at-marriage and income after marriage, we believe that we have identified an important influence on marriage patterns and have taken a useful first step in untangling the logic of a dynamic marital "lemons" model. We hope that this model will useful as a building block for more realistic and detailed theories.

1. Preferences, Information, and the Distribution of Quality

We consider a population of constant size, in which people are born, marry, and die. In every year, equal numbers of males and females reach maturity. People can choose to marry either in the first or the second year of maturity. Those who marry in the first year are said to marry at Age 1 and those who marry in the second year of maturity are said to marry at Age 2. Marriages are monogamous and there is no divorce or remarriage.

Some people are more desirable marriage partners than others, but it is assumed that members of each sex agree in their rankings of the opposite sex. It is further assumed that all members of each sex have identical von Neumann-Morgenstern utility functions over lotteries in which their marriage partners are randomly selected from the opposite sex. Let x_i be the von Neumann-Morgenstern utility that males assign to the prospect of marrying person female i and let y_i be the von Neumann-Morgenstern utility that females assign to the prospect of marrying male i. We will call x_i or y_i, the "quality" of individual i. The

[3] *Subject Report 4C, Marital Characteristics*

[4] Ted Bergstrom and Robert Schoeni (1992) examined U.S. census data in detail and show that average male incomes are an increasing function of age-at-first-marriage until approximately age 30. But, interestingly, males marry for the first time after age 30 earn less than those who marry in their mid 20's.

quality of females is distributed over an interval $[L_g, U_g]$ with a cumulative distribution function, $F_g(x)$, and the quality of males is distributed over an interval $[L_b, U_b]$ with a c. d. f. , $F_b(y)$. Other things being equal, everyone would prefer marrying at Age 1 to marrying at Age 2. The utility cost of delaying marriage from Age 1 to Age 2 is c_b for males and c_g for females. Marrying even the least desirable member of the opposite sex is preferred to the prospect of remaining single.[5]

The quality of each female is known to all persons when she reaches Age 1. The quality of a male does not become public information until he reaches Age 2. At Age 1, each male knows what his own quality will be at Age 2.[6] To the females, his prospects are indistinguishable from those of his contemporaries, except insofar as his choice of when to marry acts as a signal.

2. Marriage Market Equilibrium

We model the marriage market as a game of incomplete information. Players have only two available strategies–to marry at Age 1 or to marry at Age 2. The quality of males of Age 2 and of females of all ages is common knowledge. The quality of a male of Age 1 is known only to himself. Members of each generation make simultaneous choices about when to marry, without observing the choices made by their contemporaries. Thus each individual believes that his or her choice of strategy will not alter the choices made by contemporaries. In equilibrium, although knowledge of the quality of specific Age 1 males is private information, the *distribution* of quality among young males who choose to marry and among young males who choose to wait will be common knowledge.

The payoffs to each strategy are determined by a matching rule applied to the set of people who choose to marry in any time period. Females who marry in any period are matched to males of corresponding *expected* quality who choose to marry in the same period. If the quality of all persons in the marriage market were public information, then this matching would be entirely straightforward. The most desirable male would be matched to the most desirable female, the second most desirable male to the second most desirable female, and so on until the supply of persons of at least one sex is exhausted. If the number of available persons of one sex exceeds that of the other, then some people from the lower tail of the quality distribution will be left unmatched. Unmatched persons of Age 1 may reappear in the marriage market in the next period.

The actual matching rule is complicated by the fact that males of Age 1 are indistinguishable to females and hence are of equal expected quality. Applying the principle of matching by corresponding rank leads to the following assignment. At time period, t, the best unmarried male of Age 2 will be matched to the best female who chooses to marry at

5 This assumption involves no loss of generality, since the population referred to consist only of those persons to whom being married is better than being single.

6 This model extends without formal alteration to the case where young males are not certain about how well they will turn out, but have some private information about their prospects. Then y_i is interpreted as i's *expectation* of his quality in period 2.

time t, the second best unmarried male will marry the second best unmarried female and so on until the supply of males whose quality exceeds the average of available Age 1 males is exhausted.

The assignment of partners for the remainder of the population follows directly from the principal of matching by corresponding rank and from the fact that females cannot distinguish between males who choose to marry at Age 1.

Let $N_m(t)$ be the number of Age 1 males who choose to marry at time t. Let $N_f(t)$ be the numbers of females who choose to marry at time t and who are not matched to a male of who is better than the a random draw from the available Age 1 males. There are three possible cases.

(1) If $N_m(t) = N_f(t)$, then each of the males who chose to marry at Age 1 will be randomly assigned a partner from the set of $N_f(t)$ females who want to marry in this period and are not already taken by an Age 2 male.

(2) If $N_m(t) < N_f(t)$, then the best $N_m(t)$ of the $N_f(t)$ available females will be randomly matched to the males of Age 1. The remaining $N_f(t) - N_m(t)$ females will be matched in order of corresponding quality with any remaining males of Age 2 who are of lower quality than the average available male of Age 1. Females left over at the end of this process will be left unmatched. Those who are of Age 1 may reenter the marriage market in the next period at Age 2.

(3) If $N_m(t) > N_f(t)$, then a random draw of $N_f(t)$ males from the set of available males of Age 1 will be paired with the $N_f(t)$ females who are available and have not been matched with a male of higher quality. Males who chose to marry at Age 1 but did not receive partners in the random assignment will be able to reenter marriage market in the next period at Age 2.

With these matching rules, the assignment of partners within the set of people who choose to marry in any given year has the *core* or *stable marriage assignment* property. Gale and Shapley (1962), Shapley and Shubik (1972). That is to say, no two people of opposite sexes *who marry in the same year* would both get higher expected utility from marrying each other than they do from their actual choices. [7]

Equilibrium must determine when each person chooses to marry as well as how the people who choose to marry at a given time are matched up. In equilibrium it must be that each person's choice of whether to marry at Age 1 or Age 2 maximizes his or her expected utility, given the choices of all other individuals.[8] Optimal strategies for

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[7] Because we have assumed that persons of each sex agree in their rankings of the opposite sex, the only assignment in the *core* from an *ex ante* standpoint is the assignment that matches persons in order of expected quality. For further references and a masterful survey of the general problem of stable marriage assignments, see Roth and Sotomayor (1990).

[8] This is a Bayesian equilibrium of an agreeably simple nature. In this model, we do not have to wonder what inferences are to be drawn about a player's type if he or she deviates from equilibrium behavior. By the time that a deviation is observed by another player, the deviator will have reached Age 2. The type of every Age 2 person is common knowledge, so there is no mystery in how to regard someone who has in the past deviated from equilibrium strategies.

individuals depend non-trivially on the actions of others because these choices determine the quality distribution in the marriage pool in each year and thus determine the payoffs from marrying at Age 1 or Age 2. In equilibrium, no person who marries at Age 1 would have a higher expected payoff from waiting to marry at Age 2 and no person who marries at Age 2 would have a higher expected payoff by marrying at Age 1.

By restricting strategic choices to a decision of whether to marry at Age 1 or Age 2, we have arbitrarily excluded strategies which may be preferred by some males. Since males who choose to marry young are matched randomly to females who are willing to marry young males, and since the quality of young females is common knowledge, one might expect some males to try a strategy of the form: *Go into the marriage market while young. If you are lucky enough to draw one of the better females who is willing to marry a young man, marry her. If you draw a female from the lower end of the distribution, don't marry but wait until you are older.* Indeed there is nothing in this model to prevent this strategy from being preferred by some males to accepting a random draw. A thorough treatment of strategies of this type must await a model with a more detailed search theory and with more than two possible ages of marriage.

3. Long-Run Stationary Equilibrium

Since we have assumed that the number of persons born in each year is constant and that quality distributions and preferences are the same in each generation, we can expect to find a long-run stationary equilibrium, in which each generation behaves in exactly the same way as all preceding generations.

Analysis of equilibrium is much simplified by the fact that in *any* equilibrium, whether it is stationary or not, the following is true:

Proposition 1. *At any time t, the set of males who choose to wait until Age 2 to marry will be an "upper tail" of the quality distribution. That is, a set of the form $\{y|y \geq y_t\}$ for some $y_t \in [L_b, U_b]$.*

Proof: Consider two males, born at the same time, of quality y' and y where $y' > y$. If these males both marry at Age 1, then they face the same lottery and their expected payoff will be the same. If they wait until Age 2 to marry, then the male of quality y' will be matched to a female whose quality is at least as great as the quality of the female matched to the male of quality y. From this it follows that if it is worthwhile for a male of quality y to wait until Age 2 to marry, any male of higher quality than y will find it worthwhile to wait. ∎

It turns out that in long-run equilibrium, all females marry at Age 1. There is a threshold level of quality, y^*, such that in each time period, males of higher quality than y^* marry at Age 2 and males of lower quality marry at Age 1. The highest quality male from a generation will marry at Age 2 to the highest quality female from the next younger generation. The second highest quality male will marry at Age 2 to the second highest quality female of Age 1, and so on until the threshold quality y^* is reached. Males of quality lower than y^* will choose to marry at Age 1 and will receive a random assignment from the set of Age 1 females who were not of sufficiently high quality to be matched with the available males of Age 2.

Let us define a function g such that $x = g(y)$ means that a female of quality x has the same ordinal rank among females as a male of quality y has among males. Thus $g(z)$ is the (unique) solution to the equation $F_b(z) = F_g(g(z))$. We further define $\mu_b(y)$ to be the quality of the "average male who is no better than a male of quality y" and define a similar notation $\mu_g(y)$ for females. Formally, $\mu_b(y) = \int_{L_b}^{y} z dF_b(z)/F_b(y)$ and $\mu_g(y) = \int_{L_g}^{y} z dF_g(z)/F_g(y)$.

In long run equilibrium, a male of threshold quality y^* will be just indifferent between marrying at Age 1 or marrying at Age 2. If he marries at Age 2, he will be matched to a female of quality $g(y^*)$ and his utility will be $g(y^*) - c_b$ where c_b is the utility cost of waiting. If he marries at Age 1, he will be indistinguishable from the other males who marry at Age 1 and will be randomly assigned to one of the females who can not marry an older male of quality $y > y^*$. The average quality of females in this pool is $\mu_g(g(y^*))$. Therefore his expected utility if he marries at Age 1 is $\mu_g(g(y^*))$ and he will be indifferent between marrying at Age 1 or Age 2 if

$$\mu_g(g(y^*)) = g(y^*) - c_b. \tag{1}$$

Long-run stationary equilibrium is fully characterized by the following result.

Proposition 2. *If $y^* \in [L_b, U_b]$ satisfies Equation 1, then there is a long-run stationary equilibrium such that in every generation, each male of quality $y \geq y^*$ marries at Age 2 to a female of Age 1 whose quality is $g(y)$ and each male of quality $y < y^*$ marries at Age 1 to a female randomly selected from the set of females in his own generation of quality $x < g(y^*)$. Conversely every long-run stationary equilibrium is of this type.*

Proof: The assertion that the proposed arrangement is an equilibrium will be demonstrated if we can show that no individual can gain by deviating from the proposed equilibrium strategy. Consider a male of quality $y > y^*$. If he chooses to marry at Age 2, he will be matched to a female of quality $g(y)$ and his payoff will be $g(y) - c_b$. If he chooses to marry at Age 1, he will have an expected payoff of $\mu_g(g(y^*))$.[9] Since g is an increasing function of y, it follows from Equation 1 that he cannot gain by marrying at Age 1 rather than at Age 2.

Consider a male of quality $y < y^*$. If he marries at Age 1, he will have a random draw from the set of females of quality less than $g(y^*)$ and his expected payoff will be $\mu_g(g(y^*))$.

[9] Since we have assumed that people in the same generation choose their age of marriage simultaneously, his choice to marry at Age 1 will not change the set of females who choose to marry at Age 1, nor will it change the set of unmarried Age 2 males. Therefore the pool of females who are available to marry Age 1 males does not change in response to his decision to marry at Age 1. It follows that the expected payoff from marrying at age 1 remains $\mu_g(g(y^*))$ whether or not he chooses to marry at Age 1.

If he waits until Age 2 to marry, then his quality will be common knowledge. All of the males from his own generation of quality $y \geq y^*$ will be in the marriage pool at this time and will be matched to all of the Age 1 females of quality $x \geq g(y^*)$. Therefore his payoff from marrying at Age 2 will be smaller than $g(y^*) - c_b$. From Equation 1, it follows that he cannot gain by marrying at Age 2 rather than at Age 1.

Consider any female. If she deviates from the strategy of marrying at Age 1, the expected quality of her partner will be no higher than the expected quality she can get at Age 1.[10] Since waiting is costly, she would not gain from choosing to marry at Age 2 rather than at Age 1.

We have shown that if y^* satisfies Equation 1, no person can gain by deviating from the proposed equilibrium strategies. All that remains is to show that every long-run stationary equilibrium is of the type described in this proposition. From Proposition 1, it follows that in any equilibrium, the set of males divides into an upper quality interval who marry at Age 2 and a lower quality interval who marry at Age 1. If equilibrium is to be stationary, then the threshold quality at which these groups divide must be some constant y^*. If the pool of available males is the same in every period, then (since waiting is costly) it can never be worthwhile for females to choose to marry at Age 2 rather than at Age 1. Therefore in equilibrium all females must marry at Age 1. Finally it is straightforward to verify that males better than y^* will choose marriage at Age 2 and males worse than y^* will choose marriage at Age 1 only if Equation 1 is satisfied. ∎

4. Existence and Uniqueness of Long-run Equilibrium

The questions of existence and uniqueness of long-run equilibrium reduce to the question of whether Equation 1 has a solution and whether that solution is unique. Let us define the difference between a male or female's own quality, z, and the quality of the average male or female who is no better than z. Let $\delta_b(z) = z - \mu_b(z)$ and $\delta_g(z) = z - \mu_g(z)$. Then Equation 1 is equivalent to

$$\delta_g(g(y^*)) = c_b. \tag{2}$$

The following two assumptions will be sufficient for existence and uniqueness, respectively, of a solution to Equations 1 and 2.

Assumption 1. *The distribution function for quality of each sex is continuous and the difference between the quality of the most desirable female and the average quality of females exceeds the cost, c_b, to a male of waiting to marry at Age 2.*

Assumption 2. *The function, $\delta_g(x)$, (which is the difference between x and the average quality of females worse than x) is a monotone increasing function of x.*

[10] There is a slight complication. If she decided to delay marriage, then when her age is 1, the number of males in the marriage market would exceed the number of females by 1. Therefore a randomly selected male who chose to marry at Age 1 would not find a mate. He would reappear in the marriage market in the next year. But the addition of a randomly selected male from the set of males of quality $y < y^*$ will not improve the expected quality assignment for any female who waits until the next period to marry.

Proposition 3. *If Assumption 1 holds, then there exists at least one long-run stationary state equilibrium where y^* solves Equation 1.*

Proof: By Assumption 1, $\delta(U_g) > c_b$. From the definition of the function $\delta()$, it follows that $\delta(L_g) = 0 < c_b$. The function $\delta(x)$ inherits continuity from the distribution function for x. Therefore from the intermediate value theorem, there must be at least one solution, x^*, to the equation $\delta(x^*) = c_b$. Let $y^* = g^{-1}(x^*)$. Then $\delta(g(y^*)) = c_b$. Therefore there exists a solution to Equations 1 and 2. From Proposition 2, it follows that there exists a long-run stationary equilibrium. ∎

Proposition 4. *If Assumption 2 holds, then any long-run stationary equilibrium is unique.*

Proof:

From Assumption 2 and the monotonicity of g, it must be that $\delta_g(g(y)) - c_b$ is a monotonic increasing function of y and hence there can be only one y^* for which $\delta(g(y^*)) = c_b$. From Proposition 2 it follows that every long-run stationary equilibrium must satisfy this equation. ∎

An Example: Suppose that the quality of females is uniformly distributed on an interval $[0, a]$ and the quality of males is uniformly distributed on the interval $[0, b]$. Then the function that maps males to females of corresponding quality rank is $g(y) = \frac{a}{b}y$. For the uniform distribution, the average quality of females worse than x is just $x/2$. Thus we have $\mu_g(x) = x/2$ and $\delta_g(x) = x - \mu_g(x) = x/2$. We see that $\delta_g(x)$ is an increasing function of x, so that Assumption 3 is satisfied. In fact we can readily solve for the unique equilibrium. The equilibrium condition, $\delta_g(g(y^*)) = c_b$ will be satisfied if $\frac{a}{2b}y^* = c_b$ or equivalently if $y^* = \frac{2bc_b}{a}$. Therefore if $0 < 2c_b < a$, there will exist a unique solution for y^* in the interval $(0, b)$. In long-run equilibrium all males of quality $y < y^* = \frac{2bc_b}{a}$ will choose to marry at Age 1. Males who marry at Age 1 will get a random draw from the population of females of Age 1 whose quality is lower than $g(y^*) = 2c_b$. The expected payoff of a draw from this pool will then be c_b. If a male of quality y^* marries at Age 2, he will be paired with a female of quality $g(y^*) = 2c_b$, but he has to bear the cost of waiting until Age 2. His utility payoff from waiting is $2c_b - c_b = c_b$, which is the same as the payoff from marrying at Age 1. Females of quality $x > 2c_b$, will marry Age 2 males of quality $\frac{bx}{a}$. Females of quality $x < 2c_b$ will get a random draw from the population of males who choose to marry at Age 1.

We have shown that monotonicity of the function $\delta_g(z)$ is sufficient for the uniqueness of equilibrium and as our example shows, if the quality of females is uniformly distributed, then $\delta_g(x)$ is strictly monotone increasing. It would be useful to know more generally, what probability distributions have this property. As it happens, the class of distributions that have this property is quite large and many of its members can be identified by an easily checked sufficient condition.

It turns out that a necessary and sufficient condition for $\delta_g(x)$ to be an increasing function of x is that the log of the integral of the cumulative density function be a concave function.[11] This fact is not as useful as one might hope because it is rarely possible to find a closed form expression for the cumulative density function, let alone its integral. Therefore it is not easy to verify whether a random variable has this property. But we are rescued by the remarkable fact that "Log-concavity begets log-concavity" (under integration). This result seems to have been discovered by Prekova (1973) and has since appeared in several places in the literature on operations research, statistics and economics (See, for example, Pratt (1981), Goldberger (1983), Flinn and Heckman (1983), Caplin and Nalebuff (1988), and Dierker (1989).). This result, which is proved in the Appendix is.

Lemma 1. *If $f(x)$ is a differentiable, log-concave[12] function on the real interval $[a, b]$, then the function $F(x) = \int_a^x f(t)dt$ is also log concave on $[a, b]$ and so in turn will be the function $G(x) = \int_a^x F(t)dt$.*

In a recent study, Bagnoli and Bergstrom (1989) examine the log concavity of density functions, cumulative density functions, and their integrals for numerous common probability distributions. All of the following probability distributions have log concave densities and hence monotone increasing $\delta_g(x)$ functions. Uniform, normal, logistic, extreme value, chi-squared, chi, exponential, and Laplace. Therefore according to Theorem 2, if the distribution of female quality belongs to any of these families equilbrium will be unique. The following probability distributions have log concave density functions for some but not all parameter values. Weibull, power function, gamma, beta.

Log concavity of the density function is a sufficient, but not a necessary condition for $\delta_g(x)$ to be monotone increasing. Bagnoli and Bergstrom (1989) show that although the log normal distribution and the Pareto distribution do not have log concave density functions, they do have log concave cumulative density functions and monotone increasing $\delta(x)$.[13]

5. The Trajectory to Long-Run Equilibrium

If the population starts out in long run stationary equilibrium, it will remain there. But if initially the population is not in long run stationary equilibrium, it will *not* immediately jump to a stationary equilibrium.[14] Somehow the system must move gradually toward

[11] To see this, integrate the expression for $\delta_g(x)$ by parts.

[12] A function f is said to be log-concave if $\log f$ is a concave function.

[13] In fact, we are aware of no probability distributions famous enough to be "named", for which $\delta(x)$ is not monotone increasing. However, as Bagnoli and Bergstrom (1989) showed, the probability distributions defined as "mirror images" of the Pareto distribution and the log normal distribution have respectively monotone increasing and nonmonotonic $\delta(x)$.

[14] This problem is echoed by occasional informal arguments to the effect that "Maybe the reason that women marry older men that somehow people got started doing things this way and now it can't be stopped because there are so many unmarried older men around who compete the women away from younger men."

equilibrium. During the process of adjustment to long run equilibrium, some people are going to have to be left without partners.

When the system does not start out in long run equilibrium, the dynamics are complicated by the fact that in some time periods, females will choose to delay their date of marriage because the supplies of available males and females males may be more favorable to them in the second period of their lives than in the first. A complete general characterization of the behavior of the system outside of long-run equilibrium appears to be very difficult. Here we settle for a pair of general results, one for each sex, and an example.

Proposition 1, which we proved earlier, that along any equilibrium path, the set of males who choose to wait until Age 2 to marry is an upper tail of the distribution of males. The behavior of females is much more complicated and not fully described here, but we do have one rather interesting general result.

Proposition 7. *In equilibrium, no female will ever marry a male of Age 2 whose quality rank is higher than her own.*

Proof: If in period t, some female marries a male of higher quality rank than her own, then there will have to be some young female of higher quality who does not marry a male of quality rank as high as her own in period t. This second female must, therefore, have voluntarily postponed marriage. But if she is willing to postpone her marriage, she must get a male whose quality exceeds that of her quality-match by at least c_b. This means that a third female of yet higher quality must be displaced one generation later. The process would have to continue, with females of ever higher quality in later generations being displaced. Eventually there would be no male sufficiently good to compensate the best displaced female for waiting until Age 2. ∎

We conclude with an example which is simple enough so that we can work out an exact solution for the pattern of marriage that starts out from a position off of the long run equilibrium path and moves gradually toward long run equilibrium.

An Example: For each sex, "quality" is uniformly distributed on the interval $[0, 1]$. Equal numbers of males and females are born in each period. In the initial period, there are no unmarried persons of Age 2 available from either sex. The utility cost of marrying at Age 2, rather than at Age 1 is $c < 1/2$ for members of either sex. A person's desirability to members of the opposite sex neither increases nor decreases between Ages 1 and 2.

In long run equilibrium, males of quality lower than $2c$ marry at Age 1, males of quality higher than $2c$ marry at Age 2, and all females marry at Age 1. But this population will not go all the way to long run equilibrium in a single step. If it did so, then no Age 1 males of quality $y > 2c$ would marry and since there are no males of Age 2 available, any female who marries in the first period would have to accept a random young male, whose expected quality would be c. But by waiting until the next period when some high quality Age 2 males become available, females of the highest quality could get spouses of nearly quality 1. Since by assumption, $1 - 2c > 0$, it must be that $1 - c > c$, so some of the best females will be better off waiting to marry at Age 2.

For this example, the pattern of ages at marriage converges to long run equilibrium in a simple, but rather surprising way. The proportion of males who choose to marry at Age 2 goes immediately to the equilibrium level and stays there. But females divide into four groups. In each period after the first, an interval of females at the top of the quality distribution marries at Age 1 to males of Age 2. An intermediate quality interval waits until Age 2 to marry at which time they marry males of Age 2. An interval of females just below these marries at Age 1 to males of Age 1. Finally at the bottom of the quality distribution of females is an interval of females who are left without partners.

Let X_t^1 denote the set of females born in year t who marry at Age 1 to males of Age 2. Let X_t^2 be the set of females born in year t who marry at Age 2 to males of Age 2, let X_t^3 be the set of females born in year t who marry at Age 2 to males of Age 1, let X_t^4 be the set of females born in year t who marry at Age 1 to males of Age 1 and let X_t^5 be the set of females born in year t who are left without mates. If initially there are no unmarried persons of Age 2, each of these sets is an interval. These intervals take the following form: $X_t^1 = (x_t^1, 1)$, $X_t^2 = (x_t^2, x_t^1)$, X_t^3 is empty, $X_t^4 = (x_t^4, x_t^2)$ and $X_t^5 = (0, x_t^4)$. Specifically, it turns out that

$$x_t^1 = 2c + (1 - 2c)\left(\frac{1}{2}\right)^{t-1}, \qquad x_t^2 = 2c + (1 - 2c)\left(\frac{1}{2}\right)^{t},$$

$$\text{and} \quad x_t^4 = (1 - 2c)\left(\frac{1}{2}\right)^{t}.$$

This means that x_t^1 starts out at 1 in the first period. In the second period, x_t^1 moves half way from 1 to the equilibrium value $2c$ and in each subsequent period again moves half way from its previous location to $2c$. Notice also that for all t, $x_t^2 = x_{t-1}^1$ and that the length of the interval X_t^2 of females who marry at Age 2 is halved in every period and is being squeezed asymptotically to $2c$. The interval set X_t^5 of females who are left unmatched is being halved in every period. In the limit, the behavior of females approaches the long run equilibrium in which all females of quality $x > 2c$ belong to X_t^1 and all females of quality $x < 2c$ belong to X_t^4.

6. Remarks and Possible Extensions

Gary Becker (1974) suggests a reason to expect that high-wage males might marry earlier rather than later. Becker argues that high-wage males have more to gain from marriage than low-wage males because they will enjoy greater returns to specialization (by marrying low-wage females who will specialize in doing household work.) Since there is more to be gained from being married, they will spend less time searching and hence marry earlier. Michael Keeley (1977) investigates this relation empirically, using a sample of households from the 1967 Survey of Economic Opportunity. Although he finds a positive relation between age-at-first-marriage and income if one does not include years of schooling as an explanatory variable, he finds a *negative* relation between age at first marriage and income when years of schooling is included. Using 1982 census data, Bergstrom and Schoeni (1992) find that controlling for education does reduce the positive relation between age-at-first-marriage and income in later life, but even in this case, expected male income in later life increases with age-at-first marriage up to an age in the mid-twenties and then decreases for older ages.

As a test of our model, it seems inappropriate to "control for education" in exploring the relation between age-at-first-marriage and economic success. To do so seems to beg the question of why it is that people who get more years of education tend to marry later. It is hard to see why the benefits from marriage are likely to be smaller for those who are attending universities than for persons of the same age who are working for wages.[15] One of the most convincing ways that a young man can demonstrate to potential mates that he is able and diligent is to finish a college degree.

Of course we would not be so narrow-minded as to claim that our model is a full explanation of when people marry or that the considerations suggested by Becker and Keeley can be neglected. To confront the data more convincingly, one would like to have a much more elaborate model than we have presented. The model should be enriched to incorporate search costs, to allow more varied roles for females, to allow the gradual accretion of evidence about members of each sex as time passes, and to take into account the role of nonhuman wealth. There is much interesting work to be done.

[15] Those who have observed fraternities at large universities will find it hard to believe that this environment is as well suited to scholarship as married life.

Appendix

Proof of Lemma 1 (Log concavity begets Log concavity). [16]

By elementary calculus, $F(x)$ will be log concave if $0 \geq (F'(x)/F(x)) = f'(x)F(x) - f(x)^2$. If f is log concave, then also by elementary calculus, it must be that for $x \leq t$, $f'(x)/f(x) \geq f'(t)/f(t)$. Therefore, for all $x \in [a, b]$,

$$\frac{f'(x)}{f(x)}F(x) = \frac{f'(x)}{f(x)} \int_a^x f(t)dt \leq \int_a^x \frac{f'(t)}{f(t)} f(t)dt.$$

But

$$\int_a^x \frac{f'(t)}{f(t)} f(t)dt = \int_a^x f'(t)dt = f(x) - f(a).$$

Therefore

$$\frac{f'(x)}{f(x)}F(x) \leq f(x) - f(a) \leq f(x).$$

and hence $0 \geq f'(x)F(x) - f(x)^2$. Therefore $F(x)$ is log concave. Since $F(x)$ is log concave, the argument of the previous paragraph can be applied to show that $G(x)$ inherits log-concavity from $F(x)$.

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[16] The idea for this proof is borrowed from Dierker's proof of the same proposition.

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