INCENTIVES AND PUBLIC INPUTS*

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Considered in this paper is a mechanism to coordinate the decision to provide a public input to a group of firms designed to overcome the 'free rider' problem. The coordinating agent relies on information communicated by the firms and it is shown that the mechanism provides an incentive for each firm to send truthful information so that an optimal quantity of the public input will be provided.

1. Introduction

1.1. The problem of incentives has arisen in a number of economic contexts, including most recently as a general issue in the design and analysis of resource allocation mechanisms [c.f. Hurwicz (1973)]. Most simply, the incentive problem concerns the rationale there is (or can be provided) for economic agents to follow prescribed rules of behavior designed to achieve some objective, usually an efficient or optimal allocation of resources. Typically the prescribed behavioral rules are some type of competitive market behavior such as, the communication of one's (net) demand for goods at prices that are announced by a central agent, e.g., a central planning bureau, market custodian, or auctioneer. The incentive problem concerns whether or not it is in an agent's interest to behave according to these rules and truthfully reveal his demand. Although it has been shown by Hurwicz (1972) that even in the most classical, no externality, private goods context it is typically in an agent's interest to violate the rules, the incentive problem is most familiar to economists as it arises with reference to public goods. Furthermore, it appears widely believed that no mechanism can be devised to optimally allocate public goods that avoids the incentive problem. The reason for this belief is easy to see.

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In the private goods, no externality case, if equilibrium prices are known and announced, an efficient allocation will result, under general convexity conditions, when agents maximize preferences or profits at these prices and demand or supply the maximizing quantities, i.e., behave competitively. However, in the public goods case, or more generally when externalities exist, no price system with a single price for every good will lead to an efficient allocation. In order to achieve efficiency through competitive behavior, different agents must face different prices for those commodities for which an externality in use exists [c.f. Arrow (1969)]. Furthermore, these individualized prices will depend directly on the individual characteristics, primarily the preferences, of each agent. Thus, in the absence of general knowledge of the agents' characteristics, any procedure for determining these prices must rely on the agents to truthfully reveal their characteristics. But as Samuelson (1954, p. 389) states, although 'one could imagine every person...indoctrinated to...reveal his preferences,...by departing from his indoctrinated rules, any one person can hope to snatch some selfish benefit...'. Thus, Kamien and Schwartz (1970, p. 19) argue that '...there is no market mechanism to determine either appropriate price or quantity' of a public good. And Buchanan (1968, p. 87) goes further, in asserting that 'it may prove almost impossible, however, to secure agreement among a large number of persons, and to enforce such agreements as are made. The reason for this lies in the 'free rider' (problem)... Even if an individual should enter into... (an) agreement, he will have a strong incentive to break his own contract, to chisel on the agreed terms.'

1.2.

In this paper we consider a group of firms, all of which use in their production processes a good that is nonexclusionary in use—a public input. Examples of such goods might be weather forecasts available to a group of commercial farmers, general research available to all firms, or advertising by an industry wide trade association.

It is well known that if each firm purchases the quantity of the public input that maximizes its own profits, given the quantities purchased by the other firms, the resulting (noncooperative) equilibrium will not be a joint profit maximum under general conditions. In fact, typically every firm would be better off if each purchased a larger quantity of the public input.2 Traditional

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1Although Samuelson was referring to the public goods case here, the statement is equally true for the private goods case as shown by Hurwicz (1972). The difference between the two cases is that although the efficient price for a private good depends on all agents' demands, if there are large numbers of agents this price will not be as sensitive to any one agent's demand as that agent's individualized price for the public good will be in the public good case.

2This is by no means universally true; in the case when then the public good is perfectly complementary in use to other inputs, the noncooperative equilibrium (if it exists) will be joint-profit-maximizing.
economic analysis suggests that centralizing the decision over the quantity of the public input to provide may lead to an improved solution. However, any scheme for centrally determining the quantity of the public input creates an incentive problem of inducing the firms to reveal their true preferences or demands for the input.

It has been argued by Kaizuka (1965, p. 118) that 'the benefiting firms have every incentive not to reveal their exact benefit from (the public input) ... An attempt to use decentralized pricing will here, too, invite game-theoretic dissembling on the part of producers. Hence in the absence of detailed social planning and fiats, any system of voting and taxing can be expected not to achieve social or even Pareto optimality.'

In section 2, we formulate the model and develop a general procedure for solving the incentive problem, Kaizuka's comment notwithstanding. The procedure is defined by rules that specify the quantity of the public input to be provided and the share of the costs to be borne by each firm given all the firm's messages. It is shown that the procedure provides the firms an incentive to send the 'correct' messages.

2. The public input model

2.1.

Consider a collection of $n$ firms, indexed $i = 1, \ldots, n$, which use a particular commodity $K$ in their production processes. The commodity $K$ is a public input to the $n$ firms since the total quantity is available to all firms and the use of it by one firm does not diminish its availability to the other firms. The net revenue $r_i$ of each firm $i$ (gross revenues less costs of inputs other than $K$) is given by a function $R_i$ of the total available quantity $K$, however provided, and some local decisions $L_i$, e.g., a vector of other inputs, variables specifying the choice of technique, etc.,

$$r_i = R_i(K, L_i), \quad i = 1, \ldots, n. \quad (1)$$

Assuming (a) the total available quantity of $K$ is known by all firms at the time they make their decisions $L_i$, (b) for every quantity $K$ there exists a value of $L_i$ that maximizes net revenues, (c) the firms are profit-maximizing, and (d) the difference between profits and net revenues is independent of the firm's choice of $L_i$; we may restrict attention to the (maximum net) revenue functions $\pi_i$ defined by

$$\pi_i(K) = \max_{L_i} R_i(K, L_i), \quad i = 1, \ldots, n. \quad (2)$$
The revenue functions $\pi_i$ are assumed to satisfy the regularity condition:

For every $i$, $\pi_i(K)$ is monotonic increasing, strictly concave, and everywhere differentiable for $K \geq 0$ with $\lim_{K \to \infty} \pi_i'(K) = 0$.

Let $\Pi$ denote the set of all such functions. (3)

2.2.

Suppose the public input $K$ is available to the $n$ firms in any quantity at a (positive) market price $p$. If the firms must independently purchase any $K$, the familiar free rider problem exists. Since firm $i$ will benefit from the total purchases made by the firms, it will add to this total only if the marginal benefit of doing so (the marginal revenue $\pi_i'$ evaluated at the level of $K$ provided by the other firms) exceeds the marginal cost to the firm (the market price $p$). Thus, at any aggregate quantity $K$ where each firm's marginal benefit is less than the price, none of the firms will purchase more of the input. This holds even if, in addition, the aggregate marginal benefit (the sum of the $n$ firm's marginal benefits) exceeds the price indicating that the firms could all profit by purchasing more of $K$ and sharing the cost.

Since independent purchases will typically lead to such a suboptimal purchase of the public input, there is the recognized advantage in coordinating decisions over the purchase of the public input. One method for accomplishing this coordination is to centralize the decision. We therefore consider an agent, hereafter called the center, whose task it is to determine the quantity of the public input to be provided the firms and the share of its costs to be borne by each firm. The center may be thought of either as an agent hired by the firms to perform this task in accordance with some agreed upon rules, or alternatively as a central authority (a government or the administration of a large conglomerate organization composed of the $n$ firms as divisions) with the power to purchase the public input and to levy charges against the firms to finance its costs.

The center's objective in determining the quantity of $K$ to provide is to maximize joint profits:

Choose $K^* \geq 0$ to maximize $\sum_i \pi_i(K) - p \cdot K$. (4)

If the center knows the revenue functions $\pi_i$, this problem is trivial. If, however, the center does not know these functions, as we assume to be the case, in order to calculate the optimal or joint profit-maximizing quantity of $K$, it must acquire information from the firms regarding the functions $\pi_i$.

3 The assumption of a constant market-price could be relaxed at the expense of additional notational complexity but without the benefit of additional insight.

4 Under the regularity condition (3), this will occur for some $K > 0$ as long as the price $p$ is less than the sum of the firms' marginal revenue evaluated at zero, i.e., $p < \sum_i \pi_i'(0)$. 
We suppose, therefore, that each firm $i$ communicates information to the center in the form of a function $M_i$ from the set $\Pi$ of all revenue functions satisfying the regularity condition (3) which the center interprets as the firm’s revenue function $\pi_i$. Thus, given any $n$-tuple of messages $M = (M_1, \ldots, M_n)$, the center then maximizes ‘reported’ joint profits,

$$\text{Choose } \hat{K}(M) \geq 0 \text{ to maximize } \sum_i M_i(K) - p \cdot K.$$

(5)

Since each message $M_i$ must satisfy the regularity condition (3), $\hat{K}(M)$ exists and is unique if $p > 0$. Furthermore, comparing (4) and (5), if the firms send the center their true revenue functions, i.e., $M_i^* = \pi_i$, the center’s rule will select the optimal quantity, i.e., $\hat{K}(M^*) = K^*$.

2.3.

However, depending on the method by which the purchase of the public input is financed, it may not be in the interest of the individual firms to send their true revenue functions. The incentive problem asks if it is possible to induce the firms to send their true revenue functions by levying charges in an appropriate manner.

Formally, the center assesses the firms by choosing functions $C_i$ of the messages $M = (M_1, \ldots, M_n)$ it receives from the firms -- $C_i(M)$ is the $i$th firm’s cost share for the public input $\hat{K}(M)$ purchased by the center. Although it would be natural to do so, the choice of cost share functions is not constrained to satisfy the budget constraint,

$$\sum_i C_i(M) - p \cdot \hat{K}(M).$$

(6)

In other words, the center is permitted to run a surplus or a deficit (see below, sections 2.6–2.8).

Given the center’s purchasing rule $\hat{K}(\cdot)$ and cost share functions $C_i(\cdot)$, the firms’ profits are defined by

$$\omega_i(M; C_i) = \pi_i[\hat{K}(M)] - C_i(M), \quad i = 1, \ldots, n.$$

(7)

Since the center’s rule $\hat{K}(\cdot)$ gives the optimal quantity $K^*$ when the firms send their true revenue functions, the incentive problem asks if there exist cost share functions $C_i^*$ such that each firm will maximize its own profits $\omega_i(M; C_i^*)$ by sending its true revenue function.

\textbf{Incentive Problem.} Find cost share functions $C_i^*$, $i = 1, \ldots, n$, such that, for every $i$, $M_i^* \equiv \pi_i$ maximizes $\omega_i(M, C_i^*)$ over all $M_i$ in $\Pi$ for any $M \setminus M_i \equiv$
(\(M_1, \ldots, M_{i-1}, M_{i+1}, \ldots, M_n\)), that is,

\[
\omega_i(M_i|M_1^*, C_1^*) \geq \omega_i(M_i, C_i^*),
\]

(8)

for all \(M_i\) in \(\Pi\), where \(M_i|M_1^* \equiv (M_1, \ldots, M_i^*, \ldots, M_n)\).

Any \(n\)-tuple \(C^* = (C_1^*, \ldots, C_n^*)\) of cost share functions solving the incentive problem is called an optimal incentive structure.

Several important properties of an optimal incentive structure deserve emphasis. First, although both the quantity \(\hat{K}(M)\) and the cost share \(C_i^*(M)\) depend, in general, on all the messages received by the center, under an optimal incentive structure the true revenue function maximizes the firm’s profits independent of the messages sent by the other firms.\(^5\)

Second, since the cost share rules \(C_i^*\) are functions only of the messages received by the center, the center need not know either the true revenue functions or the actual revenue received by the firms in order to determine the cost shares. An optimal incentive structure provides a justification for the center to assume the firms are sending their true revenue functions.

Third, under an optimal incentive structure, the cost share of a firm is independent of how well or poorly the other firms behave in implementing their best local decisions \(L_j\). Although the \(i\)th firm’s profit depends on the others’ messages \(M_j\), it is independent of their decisions \(L_j\) and their actual revenues \(R_j[\hat{K}(M), L_j], j \neq i\) [see (2)].

Fourth, under an optimal incentive structure, each firm’s actual profits depend on all the ‘local’ decisions \(L_i\), since its actual profits are given by

\[
R_i[\hat{K}(M), L_i] - C_i^*(M).
\]

Thus, each firm has an incentive to be as efficient as possible in selecting its own local decisions.

2.4.

It is actually quite easy to solve the incentive problem stated in (8). Consider the cost share functions \(C_i^*\) defined by

\[
C_i^*(M) = -\sum_{j \neq i} M_j[\hat{K}(M)] + p \cdot \hat{K}(M) + A_i(M|M_j),
\]

\[
i = 1, \ldots, n,
\]

(9)

where \(A_i(M|M_j)\) is a number that may depend on the \((n-1)\)-tuple of messages.

\(^5\)This property is stronger than the Nash equilibrium property which requires only that \(M_i^*\) maximize \(\omega_i(M_i|M_i, C_i^*)\) for every \(i\), or that sending the true message is best when the other firms are also sending their true messages.
Note that although the messages $M_j$ are functions of $K$, the quantity $A_i(M_j)$ is independent of $K$ as well as, of course, the message $M_i$. 6

To show that $C^*$ is an optimal incentive structure it is only necessary to recall that $\tilde{R}(M_i|M_i^*)$ maximizes the joint-profits

$$M_i^*(K) + \sum_{j \neq i} M_j(K) - p \cdot K.$$  

Thus, from (7),

$$\omega_i(M_i^*, C_i^*) + A_i(M_i) = M_i^*[\tilde{R}(M_i|M_i^*)] + \sum_{j \neq i} M_j[\tilde{R}(M_i|M_i^*)]$$

$$-p \cdot \tilde{K}(M_i|M_i^*)$$

$$\geq M_i^*(K) + \sum_{j \neq i} M_j(K) - p \cdot K,$$

for all $K$. (10)

In particular, the inequality holds for $\tilde{R}(M_i|M_i)$, for all $M_i$. Thus,

$$\omega_i(M_i^*, C_i^*) + A_i(M_i) \geq \omega_i(M_i^*, C_i^*) + A_i(M_i),$$

or, since $A_i(M_i)$ is independent of $M_i$,

$$\omega_i(M_i^*, C_i^*) \geq \omega_i(M_i^*, C_i^*),$$

for all $M_i$, (12)

which is the requirement for an optimal incentive structure [see (8)].

2.5.

The optimal incentive structure $C^*$ is interpreted most easily by defining the cost shares independent of the particular rule used to determine the quantity of $K$. Specifically, for every quantity $K$, define functions $\hat{C}_i$ of $K$ and the messages $M$ by

$$\hat{C}_i(K, M) = p \cdot K - \sum_{j \neq i} M_j(K) + A_i(M_i), \quad i = 1, \ldots, n. \quad (13)$$

When the center uses the rule $\bar{R}(\cdot)$ to determine the quantity of $K$ to purchase, the cost shares defined by $\hat{C}_i$ are the same as those given by the optimal incentive structure $C^*$,

$$\hat{C}_i(\bar{R}(M), M) = C_i^*(M), \quad \text{for all } M, \quad i = 1, \ldots, n. \quad (14)$$

6For example, $A_i(M_i|M_i)$ might be defined by $A_i(M_i|M_i) = \sum_{j \neq i} M_j(\bar{K}) - p \cdot \bar{K}$ where $\bar{K}$ is some fixed, predetermined, quantity. In section 2.8 another example is given.
To interpret the cost share functions $C_i$, recall that the center interprets each firm’s message $M_i$ as the firm’s revenue function. Thus $C_i$ assesses firm $i$ for the full cost of the public input, but offsets this by the full amount of ‘reported’ revenues.\footnote{Of course, if firm $j$ communicates a false revenue function; i.e., $M_j \neq \pi_j$, the cost share function $C_i$ will calculate $i$’s cost using the reported revenue schedule instead of the true one since the center would not know the true one in this case.} Additionally, the firm is assessed an amount $A_i(M_i)$ which to this point is given by an arbitrary function of $M_i$. Since this later amount is independent of $M_i$, it has no effect in influencing firm $i$’s choice of its best message. It’s role is examined in the next section.

Now writing each firm’s profits in terms of the functions $C_i$, they are functions $\omega_i$ of the quantity $K$, the messages $M$, and the cost share function $C_i$,

$$\omega_i(K, M, C_i) = \pi_i(K) - \hat{C}_i(K, M), \quad i = 1, \ldots, n.$$  

(15)

The marginal profitability of the public input for the $i$th firm and hence the value to firm $i$ of the marginal unit of $K$ is

$$\frac{\partial \omega_i}{\partial K} = \pi_i'(K) + \sum_{j \neq i} M_j'(K) - p.$$  

(16)

However, when the center receives the messages $M_i$ from the firms and endeavors to maximize joint profits, the marginal joint profitability for the public input as perceived by the center is

$$\sum_{j=1}^{n} M_j'(K) - p.$$  

(17)

Thus, if firm $i$ reports truthfully, i.e., sends $M_i^* = \pi_i$; the center will value the marginal unit at every level of $K$ the same as firm $i$. Also, when all firms are communicating truthfully, each firm’s profit $\omega_i(K, M^*; C_i)$ is maximized at the same quantity $\hat{K}(M^*) = K^*$ – the true joint profit-maximizing quantity – although there is no reason for all the firms’ profits to be equal at this quantity since the quantities $A_i(M^*; M_i^*)$ need not be identical for all $i$.

By this discussion, the optimal incentive structure may be interpreted as a scheme to induce each firm to evaluate the public input in terms of its true marginal social benefit, $\sum_j \pi_j(K)$, and its full marginal social cost, $p$.

2.6.

Since the center’s choice of cost share functions $C_i$ is not constrained by the budget constraint (6),

$$\sum_i C_i(M) = p \cdot \hat{K}(M),$$

218  

T. Groves and M. Loeb, Incentives and public inputs
under an optimal incentive structure \( C^* \) the center may run a surplus or a deficit. The magnitude of the surplus or deficit depends on the functions \( A_i(\cdot) \) chosen; under \( C^* \) the net surplus may be defined as

\[
\text{Net surplus} \equiv \sum_i C_i^*(M) - p \cdot \hat{K}(M) = \sum_i A_i(M|M_i) - (n-1) \left\{ \sum_i M_i[\hat{K}(M)] - p \cdot \hat{K}(M) \right\}. \quad (18)
\]

Although in special cases it is possible to find functions \( A_i(\cdot) \) that will ensure a zero net surplus, in general such functions do not exist.\(^8\)

An example where it is possible to balance the center's budget is the quadratic model in which each firm's gross profit function \( \pi_i(K) \) is given by

\[
\pi_i(K) = \alpha_i K - \frac{1}{2} K^2, \quad i = 1, \ldots, n. \quad (19)
\]

In this simple example, the message \( M_i \) for each firm may be taken to be the linear coefficient \( \alpha_i \) and, thus, if the firms send the messages \( a_i, i = 1, \ldots, n \), the optimal rule for the center to use to determine the quantity of the public input is

\[
R(a) = \frac{\sum_i a_i - p}{n}, \quad \text{for } \sum a_i - p > 0. \quad (20)
\]

It is straightforward to verify that if the functions \( A_i(\cdot) \) are defined by

\[
A_i(a|a_i) = \frac{1}{2n} \left[ \left( \sum_{j \neq i} a_j - p \right)^2 + \frac{1}{2n(n-2)} \sum_{j \neq k} a_ia_k - \frac{1}{2n^2} p^2 \right], \quad (21)
\]

then \( C_i^*, i = 1, \ldots, n \), is an optimal incentive structure where

\[
C_i^*(a) \equiv p \cdot \hat{K}(a) - \sum_{j \neq i} M_j[\hat{K}(a)] + A_i(a|a_i)
\]

\[
= \frac{1}{2n} \left[ a_i^2 - \frac{1}{n} \left( \sum_j a_j - p \right)^2 \right] + \frac{1}{2n(n-2)} \sum_{j \neq k} a_ia_k - \frac{1}{2n^2} p^2,
\]

and \( M_j(K) = a_j K - \frac{1}{2} K^2 \). With a little algebra it may also be verified that this

\(^8\)If the set \( \Pi \) of allowable revenue functions is parameterized by some real variable so that each message \( M_i \) is just the parameter defining the particular revenue function, and if the net surplus is a polynomial function of the messages \( M \) of degree less than or equal \( n-1 \), then, the budget can be balanced always. This suggests that if the number of firms is sufficiently large, the net surplus can be made small in an approximation sense.
optimal incentive structure has the budget balance property,

$$\sum_i C'_i(a) - \lambda \hat{K}(a) = 0.$$  

Thus, for this special example, if the center uses the rules $\hat{K}(\cdot)$ and $C'_i(\cdot)$, every firm will have an incentive to send its 'true' message, i.e., $a^*_t = x_t$, regardless of the messages sent by the other firms and furthermore the center is assured of having a zero net surplus regardless of the messages sent by the firms.

2.7. The impossibility of balancing the center's budget in general with an optimal incentive structure of the form $C^*$ given in (9) raises two problems. First, if the center's surplus is negative (a budget deficit), a subsidy is provided the $n$ firms which must be raised by the center somehow and which also suggests that new firms that could not survive in the absence of the subsidy might enter and seek to be included among the original $n$ firms. Second, if the center's surplus is positive, in aggregate the group of firms is being taxed which raises the possibility that some firms which in the absence of the tax would enter or remain in the group might stay out of or exit from the group.

Since our model is a static, partial equilibrium model, we are unable to analyze the effect of the surplus on entry. However, even if the surplus were zero, since aggregate profits would be increased by the center's coordination in providing the public input, presumably entrants would be attracted to the group. But analysis of entrants is outside our model since we consider a group of $n$ firms, not necessarily all in the same industry, and do not specify what potential entrants there might be in the complete economy. Furthermore, although the center's surplus, whether positive, negative, or zero, may have an effect on entry, it is not a priori obvious that the total allocation of resources – both private and public – resulting from the implementation of an optimal incentive structure will be less efficient than the initial allocation of resources or even non-Pareto Optimal.

An evaluation of the total resource allocation depends on many issues outside the scope of the partial equilibrium model presented here. In particular, one would need to know the structure of the industries to which the $n$ firms belong – whether they are competitive, oligopolistic, or monopolistic. For example, the $n$ firms might be a group of electric power companies, each a natural monopoly in its own region and perhaps publicly regulated. The public input in question might be, say, research and development of nuclear power technology. In this example, entry effects would be nonexistent.

Additionally, the effect of the surplus on entry depends on the relative importance of the public input to the firms. The loss in joint profits from not
coordinating the provision of the public input may be large absolutely yet small relative to the total profits of the $n$ firms.

More generally, the entry issue depends on the full dynamic general equilibrium context of the problem. If it is the government that is serving as the center and is raising a surplus from the firms or running a deficit, one would need to know how the surplus is being disposed of or how the deficit is raised and then how this affects the intertemporal behavior of other economic agents. It is possible, however, to construct a static general equilibrium model—an Arrow-Debreu economy with public inputs—and prove that a competitive equilibrium with central coordination of the public input decision using an optimal incentive structure gives a Pareto Optimal allocation of resources. This, of course, would not answer entirely the entry question, but no static, partial or general equilibrium model can.

2.8.

Concerning the possible effects of the surplus on the exit of some of the $n$ original firms, the question is deeper than just whether in aggregate the firms are being taxed or not. Even if the surplus is negative, indicating that in aggregate the $n$ firms are receiving a subsidy, whether any one firm receives a subsidy or is taxed to such an extent that it leaves the group is yet to be examined. To answer this question we will exhibit a family of optimal incentive structures [a subset of all those defined by (9)] that (a) guarantee the center a positive surplus (thus avoiding the question of how a deficit is to be raised), and (b) leave each firm better off than it would be in the absence of the center’s coordination or in a reference initial situation. Also, we suggest a plausible method by which the $n$ firms could agree to adopt one member of this family of optimal incentive structures.

To begin let $(\hat{K}_j) \equiv (\hat{K}_1, \ldots, \hat{K}_n)$ denote an $n$-tuple of nonnegative quantities of the public input that would be provided by the $n$ firms in the absence of the center’s coordination. For example, $(\hat{K}_j)$ might be the quantities the $n$ firms will individually purchase if there is no centralized coordination of the public input decision—formally, a Nash or noncooperative equilibrium of the $n$-person game with payoff functions $\pi_i(\sum_j K_j) - pK_i$. Or, $(\hat{K}_j)$ may be the quantities the $n$ firms jointly agree each firm will be responsible for providing. Or, $(\hat{K}_j)$ may be the quantities the firms will purchase if some of the firms agree among themselves and the others do not cooperate and remain free riders. In any case, the $n$-tuple $(\hat{K}_j)$ will be called the initial situation in which the total amount of the public input provided, the initial quantity is, of course,

*See Groves and Ledyard (1974) for an indication of how such a model might be constructed and such results proved. Their model, however, concerns the more difficult case of public consumption goods and not public inputs, but the issues are similar.
\[ \tilde{K} = \sum_j \tilde{K}_j \] which may be greater than, less than, or equal to the true joint-profit maximizing quantity \( K^* \).

Now, consider an optimal incentive structure \( C^* \) of the form [repeating (9)]

\[ C_i^*(M) = p \cdot \tilde{K}(M) - \sum_{j \neq i} M_j [\tilde{K}(M)] + A_i(M \setminus M_i), \]

in which the functions \( A_i(M \setminus M_i) \) are defined in terms of the initial situation \( \langle \tilde{K} \rangle \) and nonnegative weights \( \theta_i \), summing to unity by

\[ A_i(M \setminus M_i; \langle \tilde{K} \rangle, \theta_i) = p \cdot \tilde{K}_i - \theta_i p \cdot \tilde{K} \]

\[ + \max_k \left\{ \sum_{j \neq i} M_j(K) - (1 - \theta_i)p \cdot K \right\}. \] (24)

The cost functions \( C_i^*(\cdot) \) may then be written as

\[ C_i^*(M; \langle \tilde{K} \rangle, \theta_i) = \theta_i p \tilde{K}(M) - \tilde{K}_i + p \cdot \tilde{K}_i \]

\[ - \sum_{j \neq i} [M_j(\tilde{K}(M)) - \theta_j p \cdot \tilde{K}(M)] \]

\[ + \max_k \sum_{j \neq i} [M_j(K) - \theta_j p \cdot K]. \] (25)

Since \( C_i^* \geq \theta_i p \tilde{K}(M) + p \cdot \tilde{K}_i - \theta_i p \cdot \tilde{K}, \) \( \sum_j \tilde{K}_j = \tilde{K}, \) and \( \sum \theta_j = 1, \) the center’s net surplus is always nonnegative,

\[ \sum_j C_j^* - p \tilde{K}(M) \geq \sum_j \theta_j p \tilde{K}(M) - p \tilde{K}(M) + p \sum_j \tilde{K}_j \sum \theta_j p \tilde{K} = 0. \] (26)

This incentive structure may be given the following description: Let the weights \( \langle \theta_j \rangle = (\theta_1, \ldots, \theta_n) \) be called the normal cost share distribution and \( M_j(K) - \theta_j p \cdot K \) the (reported) net normal profit of firm \( j. \) The incentive structure \( C^*_i \) of (25) assesses each firm (a) its cost in the initial situation, \( p \cdot \tilde{K}_i; \) (b) plus (minus) its normal cost share of the increased (decreased) quantity of public input provided, \( \theta_i p \tilde{K}(M) - \tilde{K}_i; \) (c) plus the total difference in the (reported) net normal profits caused by the center’s choice of the joint (reported) profit-maximizing quantity \( \tilde{K}(M) \) rather than the quantity maximizing the other firms’ (reported) net normal profits. In other words, this incentive structure assesses each firm, in addition to (a) and (b) the full (reported) impact that it has on all the other firms.

Eq. (25) defines an optimal incentive structure for any normal cost share distribution \( \langle \theta_j \rangle. \) Furthermore, each incentive structure in this family guarantees
the center a nonnegative surplus. However, given any particular member of the family, i.e., a specific cost share distribution, any one firm might be better off in the initial situation \( \langle \vec{R}_j \rangle \) than with the optimal quantity \( \hat{K}(\pi) = K^* \) when it is charged \( C^*_i(\pi; \langle \vec{R}_j \rangle, \theta_i) \). This depends on the particular normal cost share \( \theta_i \) chosen. If \( \theta_i = 0 \), then firm \( i \) is better off with the optimal quantity \( K^* \) than in the initial situation \( \langle \vec{R}_j \rangle \), but if \( \theta_i \) is close to unity, it is better off in the initial situation. The following proposition establishes, however, that there are many possible choices of the cost share distribution such that every firm is better off with \( K^* \) when charged \( C^*_i(\pi; \langle \vec{R}_j \rangle, \theta_i) \) than it is in the initial situation.

**Proposition.** Under the regularity condition (3), if the optimal quantity \( K^* \) is not equal to the initial quantity \( \hat{K} \), then there exists a nonempty open convex set depending only on \( \vec{R}, \Theta(\vec{K}) \), in the unit simplex such that for every cost share distribution \( \langle \theta_j \rangle \) in \( \Theta(\vec{K}) \), each firm is better off with \( K^* = \hat{K}(\pi) \) than in the initial situation; i.e.,

\[
\pi_i(K^*) - C^*_i(\pi; \langle \vec{R}_j \rangle, \theta_i) > \pi_i(R) - p \cdot \vec{R}_i, \quad \text{for all } i,
\]

for any \( \langle \theta_j \rangle \in \Theta(\vec{K}) \).

**Proof.** Consider the cost shares

\[
\hat{\theta}_i \equiv \frac{\pi'_i(K^*)}{\sum_j \pi'_j(K^*)}, \quad i = 1, \ldots, n.
\]

Under the regularity condition (3), \( \hat{\theta}_i > 0 \) and clearly \( \hat{\theta}_j = 1 \). Furthermore, it is easily verified that

\[
C^*_i(\pi; \langle \vec{R}_j \rangle, \theta_i) = \frac{p}{\sum_j \pi'_j(K^*)} \cdot \pi'_i(K^*)(K^* - \hat{K}) + p \cdot \hat{R}_i.
\]

Thus,

\[
\pi_i(K^*) - C^*_i(\pi; \langle \vec{R}_j \rangle, \theta_i) + p \cdot \hat{R}_i = \left[ \pi'_i(K^*) - \frac{p}{\sum_j \pi'_j(K^*)} \pi'_j(K^*) \right] (K^* - \hat{K}),
\]

for some \( K^i \) between \( K^* \) and \( \hat{K} \).

If \( K^* - \hat{K} > 0 \), then \( K^* > 0, \sum_j \pi'_j(K^*) = p \), and since \( \pi_i(\cdot) \) is strictly concave, \( \pi'_i(K^i) > \pi'_i(K^*) \). Thus, if \( \theta_i = \hat{\theta}_i \),

\[
\pi_i(K^*) - C^*_i(\pi) - p \cdot \hat{R}_i, \quad \text{if } K^* > \hat{K}.
\]
If $K^* < \bar{K}$, then $\pi'_i(K^*) < \pi'_i(K^*)$, and

$$\pi'_i(K') - \frac{p}{\sum_j \pi'_j(K^*)} \pi'_i(K^*) < \pi'_i(K^*) \left(1 - \frac{p}{\sum_j \pi'_j(K^*)}\right) \leq 0,$$

since $\pi'_i(K^*) > 0$ and $\sum_j \pi'_j(K^*) \leq p$. Thus, if $\theta_i = \bar{\theta}_i$,

$$\pi'_i(K^*) - C^*_i > \pi'_i(K^*) - p \cdot \bar{K}_i,$$

if $K^* < \bar{K}$ also,

and we have established that $\langle \bar{\theta}_i \rangle$ is in $\Theta(\bar{K})$.

The proposition follows by verifying that the cost $C^*_i(\pi; \langle \bar{K}_i \rangle, \theta_i)$ as a function of $\theta_i$ is continuous and monotonic increasing in $\theta_i$.

This proposition only establishes that, given any initial situation $\langle \bar{K}_j \rangle$ where $\bar{K} \neq K^*$, there exist many normal cost share distributions $\langle \theta_j \rangle$ such that each firm is better off with the center’s coordination than it is in the initial situation. How any particular distribution with this property may be chosen has not been specified. A simple possibility would be for the firms to bargain among themselves until they agree on the distribution and then announce it to the center.

In detail, the entire process of implementing the central coordination of the public input decision might be envisioned as follows:

A group of $n$ firms use in common some public input. Let us suppose that, whatever prior arrangements have or have not been made to coordinate their purchases of the input, their current purchases are $\langle \bar{K}_j \rangle$, the initial situation, and that the total amount provided $\bar{K}$ is recognized or suspected to be inefficient or not joint-profit-maximizing, although the precise optimal quantity $K^*$ is, of course, unknown. Now, suppose some agent, who might be the manager of one of the $n$ firms or an outside agent or the government, proposes the following arrangement to the firms:

'The agent, hereafter called the center, will provide the service of centrally purchasing and providing the public input to the $n$ firms. The center will use the rule $\bar{K}(\cdot)$ defined by (5) to calculate the quantity of the public input to provide and will charge each firm $C^*_i(M; \langle \bar{K}_i \rangle, \theta_i)$ where $\langle \theta_j \rangle$ is a normal cost share distribution agreed upon by the firms. Upon acceptance of this arrangement, the firms will jointly communicate their choice of $\langle \theta_j \rangle$ to the center. The center will then ask each firm for its message $M_i$ and execute its decisions, providing $\bar{K}(M)$ to the firms and billing each firm $C^*_i(M; \langle \bar{K}_i \rangle, \theta_i)$.'

The results established above imply:

(1) The $n$ firms have an incentive to accept this arrangement since there are many normal cost share distribution $\langle \theta_j \rangle$ that will leave them all better off than they are or would be in the initial situation (Proposition).
(2) The center is assured of not running a deficit [(26)].

(3) The center's surplus will be strictly positive when the firms report truthfully, i.e., send $M_i^* = \pi_i$ for any cost share distribution $\langle \theta_j \rangle$ except when $\theta_i = \bar{\theta}_i$ (defined in proof of proposition) for all $i$, in which case the surplus is zero [follows from summing (28) over all $i$].

(4) Given any cost share distribution, each firm has an incentive to report truthfully its revenue function $\pi_i$ to the center, regardless of what any of the other firms are communicating [by (12) since $C^*$ is optimal].

(5) Given any cost share distribution, if the firms respond to the incentives and communicate truthfully (as is in each firm's own interest), the center will provide the true joint-profit-maximizing quantity of the public input $K^*$ [by definition of the rule $\hat{K}(\cdot)$ since $K^* = \hat{K}(\pi_i)$].

Conclusion (4) has the further implication that any agreement a group of firms might reach while bargaining over $\langle \theta_j \rangle$ as to the subsequent messages $M_i$ that they will send is not stable. As Buchanan (1968, p. 87) has stated: 'Even if an individual should enter into ... (an) agreement, he will have a strong incentive to break his own contract, to chisel on the agreed terms.' However, Buchanan advanced this argument as a reason why agreements to provide optimal quantities of public goods are not stable. We use the same argument as a reason why the mechanism proposed here will lead to the optimal provision of the public input. Once the agreed upon $\langle \theta_j \rangle$ is announced to the center, each firm does best for itself by reporting its true revenue function $\pi_i$, regardless of the other firms messages and regardless of what agreements might have been concluded as to the messages they would send the center.

Conclusion (3) also shows that in aggregate the best agreement for the $n$ firms to conclude is for $\theta_i = \bar{\theta}_i$ for all $i$ since then, the aggregate net profits of the firms (revenues less costs paid to the center) equals the maximum joint-profits when they reveal their true revenue functions. However, even if the firms are unable to discover this particular cost share distribution and agree instead on some other $\langle \theta_j \rangle$, they each will have an incentive to report truthfully [Conclusion (4)]. Thus, the result that each firm will have an incentive to report truthfully, and consequently, that the true optimal or joint-profit-maximizing quantity of the public input will be provided does not depend on the firms' abilities to discover through bargaining the particular cost share distribution $\langle \theta_j \rangle$.

The bargaining is merely a device to insure every firm's willing participation in the centralized coordination arrangement. If the center is the government and the firms are not allowed to escape the arrangement, then the bargaining can be dispensed with and $\langle \theta_j \rangle$ selected arbitrarily. Since it would be reasonable to permit the government to know the actual realized revenues of the firms (but not their revenue functions, of course), it would be an easy matter to select the $\langle \theta_j \rangle$ such that no firm would be bankrupt by the cost rules.
2.9.

In conclusion we note some directions for extensions of the methods and models of this paper. First of all, although the commodity $K$ was interpreted as a public input, it could just as easily be interpreted as an input used by one (or many firms) which cause a negative externality (diseconomy) on the other firms [see Groves (1974)]. More abstractly, the methods are applicable to $n$-person games with freely transferable utility in which decisions affecting more than one player (i.e., externalities) are centralized.\(^\text{10}\)

In addition, a recent paper by Groves and Ledyard (1974) show that these methods may also be used for solving the incentive problem for the general public goods model where the public good is a consumption good entering each consumer's utility function at the same level. The cost or compensation functions $C_a(\cdot)$ for this model, enter the consumers' budget equations instead of defining direct transfers of utility.

\(^{10}\)This would include 'team' problems and for a discussion of incentive problems in team models, see Groves (1974).

References


