Chapter 10

Our Town—An Exposé

This lecture was first written for an audience of students at the University of Canterbury in Christchurch, New Zealand. Hence the curious references to then current events.

Life in Our Town is simple. Folks here are interested in only three things. In Our Town, we still consider it indelicate to discuss one of them. So please assume that we are interested in only two things. These are hot dogs and the circus. There are only two kinds of people in town – the toads and the dudes. Toads don’t care at all about the circus but always prefer more hot dogs to less. Dudes like both hot dogs and circus. As it happens, preferences of dudes can be represented by the utility function $U_D(X_i, Y) = X_i + 2\sqrt{Y}$ while toads’ utility functions are simply $U_T(X_i, Y) = X_i$, where $X_i$ is the amount of hot dogs that person $i$ consumes and $Y$ denotes the size of the circus. Each citizen, $i$, of Our Town has an initial endowment of wealth $W_i$ which can be used either to buy hot dogs or to pay taxes. Tax revenue is used to pay for the circus. The bigger the circus, the more it costs. In fact, let us choose units of measurement for the size of the circus so that the cost of a circus of size $Y$ is just $\$Y$. Let us also suppose that hot dogs cost $\$1 each. There are $N$ people in Our Town. Let us define an allocation to be a vector $(X_1, \cdots, X_N, Y)$ where $X_i$ is the number of hot dogs consumed by person $i$ and $Y$ is the size of the circus. An allocation is feasible for the town if the total cost of hot dogs consumed plus the cost of the circus just equals total wealth of its citizens. The set of feasible allocations can therefore be denoted by

$$S = \{(X_1, \cdots, X_N, Y) | \sum_{i=1}^{N} X_i + Y = \sum_{i=1}^{N} W_i \}.$$ (10.1)
A feasible allocation is said to be *Pareto optimal* if there is no other feasible allocation that is as good for everyone and better for someone. A classic result of Samuelson is that a necessary condition for Pareto optimality in a place like *Our Town* is that the sum of everyone’s marginal rate of substitution of public for private goods must equal the marginal cost of public goods in terms of private goods. In our town the marginal cost of public goods is always one. Therefore the Samuelson condition takes the special form:

\[
\sum_{i=1}^{N} \frac{\partial U_i}{\partial Y} \div \frac{\partial U_i}{\partial X_i} = 1.
\] (10.2)

Recalling the special form of utility functions assumed we see that for a toad \( \frac{\partial U_i}{\partial Y} \div \frac{\partial U_i}{\partial X_i} \) is always zero. For a dude, we calculate \( \frac{\partial U_i}{\partial Y} \div \frac{\partial U_i}{\partial X_i} = \frac{1}{\sqrt{Y}} \). Therefore in Our Town equation 10.2 takes the special form

\[
N_D \frac{1}{\sqrt{Y}} = 1.
\] (10.3)

From 10.3 we see that the Pareto optimal amount of public goods for Our Town is

\[
Y = N_D^2.
\] (10.4)

*Our Town* is a democracy. Everybody pays the same tax rate. We decide by majority vote how much circus to have. Of course toads always vote for no public goods, since they have to pay taxes but don’t enjoy the circus. As it turns out, toads are in the minority in *Our Town*. Therefore dudes always out-vote the toads and get a positive amount of circus. (You might want to know why the toads haven’t all moved to a town that has a majority of toads and no circus. The answer is that some of the necessary jobs in town can only be done by toads. For example, we need a banker, a mortician, some accountants, an estate agent, a lawyer, a school principal and some mothers-in-law.)

How much circus would a dude like to have? Where \( Y \) is the amount of circus, his tax bill will be just \( \frac{Y}{N} \). Therefore his after-tax wealth is just \( W_i - \frac{Y}{N} \). Therefore he will be able to consume \( X_i = W_i - \frac{Y}{N} \) hot dogs when the amount of circus is \( Y \). His utility would then be

\[
U_D(W_i - \frac{Y}{N}, Y) = W_i - \frac{Y}{N} + 2\sqrt{Y}
\] (10.5)
From 10.5 we see that

$$\frac{d}{dY} U_D(W_i - \frac{Y}{N}, Y) = \frac{1}{\sqrt{Y}} - \frac{1}{N},$$

(10.6)

Therefore a dude’s utility is an increasing function of $Y$ for $Y < N^2$, a decreasing function of $Y$ for $Y > N^2$ and is maximized at $Y = N^2$. Since there are more dudes than toads, it is clear that the only amount of circus that “wins” in majority voting is

$$Y = N^2.$$

(10.7)

For a long time the toads in Our Town have been grousing about high taxes and too much circus. Dudes never paid much attention. The other day an economist visited us. (Claimed he wasn’t a toad). He said the toads were right. He showed us equation 10.4 and pointed out that we have more than the Pareto efficient amount of public goods. He said he had just come from Their Town in the next county, where the problem was just the opposite. A majority of the people in Their Town (but not everyone) are toads. They have no circus at all.

This economist suggested that we try a different political system where we require unanimity instead of majority rule. But, since we have people with different tastes, we would have to set different tax rates for different people so as to get unanimity about quantities. He called this idea Lindahl equilibrium. In Our Town, the only way we could get the toads to agree to any positive amount of circus is if we don’t tax them for the circus. Then dudes would have to pay all the taxes. Suppose that all dudes are taxed at the same rate. Then each dude would have a tax bill of $\frac{Y}{N_D}$. He could therefore consume $X_i = W_i - \frac{Y}{N_D}$ hot dogs and would have a utility of

$$U_D(W_i - \frac{Y}{N_D}, Y) = W_i - \frac{Y}{N_D} + 2\sqrt{Y}$$

(10.8)

This is maximized when $Y = N^2_D$. Therefore all dudes would choose the amount $N^2_D$ as their most preferred quantity of circus. Since toads pay no taxes and have no interest in the circus, this amount is as good as any other amount for them. Therefore the amount, $N^2_D$, receives unanimous approval. Therefore the Lindahl equilibrium is the allocation in which $Y = N^2_D$, $X_i = W_i$ if $i$ is a toad and $X_i = W_i - \frac{Y}{N_D} = W_i - N_D$ if $i$ is a dude.

The economist said that Lindahl equilibrium was both more equitable and more efficient than our old ways. The toads said he was right. The
dudes were not so sure. A dude made the following calculations. Under the
current system a dude has the utility:

$$W_i - \frac{N^2}{N} + 2\sqrt{N^2} = W_i + N. \quad (10.9)$$

Under the Lindahl system a dude has the utility

$$W_i - \frac{N^2_D}{N_D} + 2\sqrt{N^2_D} = W_i + N_D. \quad (10.10)$$

Since $N > N_D$, moving to the Lindahl system is bad for dudes. The
economist said that the dude had a point (though he was being a bit piggish).
But the economist said that since we know that the current system is not
Pareto optimal, it should be possible for the toads to bribe the dudes to
move to Lindahl equilibrium. The economist pointed out that under the
current system each toad has a utility of

$$X_i = W_i - \frac{N^2}{N} = W_i - N \quad (10.11)$$

while under the Lindahl system he would have no taxes so his utility would be

$$X_i = W_i. \quad (10.12)$$

We can see from expressions (9) and (10) that a dude could be bribed to
accept the Lindahl system if he was given $N - N_D = N_T$ hot dogs. Since
there are $N_D$ dudes, it would take $N_D N_T$ hot dogs to bribe all of the dudes
to accept the Lindahl system. Therefore if each toad gave up $N_D$ hot dogs
to bribe the dudes, there would be just enough hot dogs to do so. If this is
done, each toad would have a utility of

$$X_i = W_i - N_D. \quad (10.13)$$

Equation expresses the utility of each dude in the Lindahl system without
bribes. With bribes of $N_T$ for each dude, the utility of each dude would be

$$W_i + N_D + N_T = W_i + N \quad (10.14)$$

which is the same as his utility under the current system. Since Expression
10.13 is greater than Expression 10.11 and since Expression 10.14 equals
Expression 10.9, we see that moving to Lindahl equilibrium with this system
of bribes benefits all toads and leaves all dudes as well as before. If we made
the bribes slightly larger, everyone would be better off than in the current system.

The dudes and the toads were all impressed by this argument. The bribes were paid, and the entire community agreed to switch to the Lindahl system. There was one small hitch. You can’t always tell by looking, whether a person is a toad or a dude. To solve this problem, the mayor asked everyone to come down to the town hall and answer the simple question:

“Are you a dude?”

To his amazement almost everyone sauntered in and said

“Yeah, man.”

How did this happen? Toads being a thoughtful lot, each toad asked itself, would I be better off pretending to be a dude? If everybody else is telling the truth, then if I confess to being a toad, my share of the cost of bribing the dudes to accept the Lindahl system will be $N_D$. Since my Lindahl tax will be 0 and I don’t care at all about the circus, my utility will be $W_i - N_D$. But what if I claim to be a dude? Then the number of dudes registered in the town hall will be $N_D + 1$ and the number of registered toads will be $N_T + 1$. In this case, my Lindahl tax will be $(N_D + 1)^2/(N_D + 1) = N_D + 1$ and I will receive a bribe from the other toads for accepting the Lindahl system. The bribe that I get will also be equal to $N_D + 1$. Since my bribe is equal to my Lindahl tax, I will be able to consume $W_i$ hot dogs and my utility will be $W_i$. So if the others all tell the truth, I am best off claiming to be a dude.\(^1\)

The mayor calculated and provided the Lindahl equilibrium amount of circus and the distribution of taxes, given the reported number of dudes and toads. Since everyone claimed to be a dude, the Lindahl quantity of circus was $N^2$, just as it had been before the reforms were introduced. Also, just as before, each individual in town paid a tax of $N^2/N = N$. Curiously enough, even though the folks in Our Town took the economists’ advice to

\(^1\)One or two of the deeper-thinking toads thought further along these lines. If it pays me to claim to be a dude, then perhaps some of the other toads will also notice that it pays them to claim to be dudes. And for that matter, how do I know that the real dudes will want to admit to be dudes? Such a toad would think as follows: Suppose that $M_D$ residents claim to be dudes and $N - M_D = M_T$ claim to be toads. Then if I admit to being a toad, I will pay $M_D$ as my share of the cost of bribing the alleged dudes to accept the Lindahl mechanism so my utility will be $W_i - M_D$. If I claim to be a dude, I will have Lindahl taxes of $M_D^2/M_D = M_D$ and I will receive a bribe of $M_D$ from the alleged toads for accepting the Lindahl system. Since my bribe is equal to my Lindahl tax, my utility if I claim to be a dude will be $W_i$, which is greater than my utility if I confess to being a toad.
heart and acted on it, the outcome was no different than it had been before the economist rode into town.\footnote{A tactless political scientist might remark that this is one of the rare occasions when economists’ advice did no harm.}

As a result of this sad experience, dudes in Our Town are inclined to look at economists (and at each other) with suspicion. True toads, of course, are pleased and amused with the outcome.

It is time, I think, to draw the curtain on the sordid situation in Our Town, while we seek aid from some more general analysis. So far, we have learned the following lessons which apply not only in Our Town but quite generally.

1. For an arbitrary distribution of taxes, majority voting will not in general lead to a Pareto optimal supply of public goods.

2. Lindahl equilibrium is Pareto optimal. However imposition of a Lindahl equilibrium requires the central authority to know individual preferences.

3. If people are asked to state their preferences, knowing that their statements will be used to calculate a Lindahl equilibrium that is then imposed, the situation where everyone tells the truth is not a best response (Nash) equilibrium.

The difficulty alluded to in (3) is often called the “free-rider problem”. It is representative of a fascinating class of problems of the firm. “How do you get someone else to tell you the truth about something that only he knows?” A related question is “When can you design a system of rewards and punishments such that when selfish people who are willing to lie will act in such a way as to yield a Pareto optimal outcome?”

**Eliciting the Truth via the Pivotal Mechanism**

A philosopher who dabbles in economics, Alan Gibbard, and an economist who dabbles in philosophy, Mark Satterthwaite, independently showed that in general it is not possible to design such mechanisms.

There are, however, some interesting special cases where the truth can be elicited even though the answers are used to choose a Pareto optimal policy.
A Second-price Sealed Bid Auction

One method that selects a Pareto efficient outcome is found by applying William Vickrey’s idea of a second-price sealed bid auction to public decision making. Recall the way the second-price sealed bid auction works. There are \( n \) people and one object to be allocated among them. Let \( V_i \) be the maximum amount that person \( i \) would be willing to pay for the object. Pareto efficient allocations would have the object go to the person with the greatest willingness to pay. If a sealed-bid auction were held, with the object going to the highest bidder at his bid price, it would not be wise for anyone to bid his true valuation. In the second-price sealed bid auction, the object is awarded to the highest bidder who pays the bid made by the second highest bidder. With this system, it turns out that bidding one’s true valuation is the best thing to do no matter what other people bid. A strategy that is best no matter what others do is known as a dominant strategy. A social outcome where everyone is using a dominant strategy is a dominant strategy equilibrium. In Vickrey’s auction, the outcome where everyone bids his true valuation and the object goes to the person with the highest valuation at price equal to the second highest valuation is a dominant strategy equilibrium. Let’s see why this is so. Suppose that you bid more than your true evaluation. If your bid is not the highest bid, you are no better (or worse) off than if you had told the truth. If your bid is the highest bid, then there are two possible cases. If your true valuation would also have been the highest bid, then you are no better (or worse) off than if you had bid the truth. If your true valuation is lower than the second highest bid, then you get the object but you must pay more than it is worth to you. You would have been better off bidding the truth and not getting the object. Thus we see that you can not gain but you can lose by overbidding. You should be able to construct a similar argument to show that you can not gain and may lose by underbidding. Therefore, bidding the truth is a dominant strategy.

Extension to a Yes-No Social Choice

The idea of Vickerey’s auction can be extended to other kinds of discrete choices. Of particular interest are all-or-nothing choices on public issues, such as whether to allow public nudity or public rugby playing, or the sale of handguns. Let us consider the Springbok issue faced by New Zealan-
Define $V_i$ to be person $i$’s willingness to pay to have the Springboks allowed into New Zealand. More formally $i$’s utility when his wealth is $X_i$ and the Springboks are allowed or not allowed to tour, and let $\bar{X}_i$ denote $i$’s current wealth. Then $V_i$ is the solution to the equation

$$U_i(\bar{X}_i + V_i, 0) = U_i(\bar{X}_i, 1).$$

Thus $V_i$ is positive for people who want them to come and negative for people who don’t want them.

One possible decision mechanism is to decide the issue by majority vote. The weakness of this mechanism is that it may not be Pareto optimal. The minority may be intensely concerned, while members of the majority each care very little. In this case it might be possible to find a Pareto superior outcome which reverses the result since the minority cares enough to buy off the majority.

To make an efficient decision, we need to compute the sum over the entire population, $\sum_i V_i$. If $\sum_i V_i > 0$, then with the current allocation of wealth, allowing the Springboks to come is Pareto optimal and not allowing them to come is not. If $\sum_i V_i < 0$, the story is reversed.

If we just asked people to state $V_i$, and then decided on the Springbok issue by the sign of $\sum V_i$, they would have an incentive to overstate the intensity of their preferences. We need a more subtle device. Here is one that works. Each $i$ is asked to state $V_i$. The state then calculates $\sum_i V_i > 0$. In addition some taxes are assessed in the following way. If person $j$’s answer does not affect the outcome, that is if the sign of $\sum_{j \neq i} V_j$ is the same as the sign of $\sum_i V_i$, then he pays no tax. If person $j$’s answer does make a difference, then he pays the amount $\sum_{j \neq i} V_j$. Any revenue from this scheme is thrown away. Using exactly the same kind of reasoning that we did in the case of the Vickrey auction, we can show that the outcome where everyone tells the truth is a dominant strategy equilibrium. Furthermore, the resulting decision is “Pareto optimal”. There is, however, some waste in the process, since the tax revenue is thrown away. In large economies, it can be shown that under reasonable assumptions the amount of waste of this type will be small.

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3The Springboks are the South African national rugby time. When this lecture was delivered South Africa still practiced apartheid and its athletic teams were international pariahs. Although they were traditional rugby rivals, the Springboks had not played in New Zealand for some years. Tempers ran hot over this issue in New Zealand.