Contents

Lecture 5

Externalities

A Model of One-sided Pollution

Ed smokes. Fiona, his neighbor, hates smoke. Ed and Fiona both love beans. Neither cares how many beans the other eats. Ed can get tobacco for free. Both have fixed incomes that can be used to buy beans. Ed's utility function is

$$U^E(S, B_E)$$

and Fiona's utility function is

$$U^F(S, B_F)$$

where S is the amount of smoking that Ed does and B_E and B_F are the amounts of beans consumed by Ed and Fiona respectively.

The set of allocations available to Ed and Fiona consists of all the combinations (S, B_E, B_F), of smoke and beans such that total beans consumed equal the total amount available. This requires that

$$B_E + B_F = W_E + W_F$$

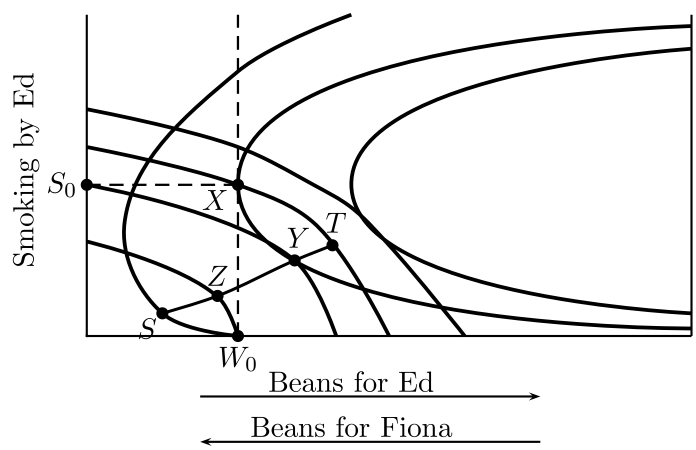
where W_E and W_F are the wealths of Ed and Fiona, measured in terms of the numeraire, beans.

Smoke in a Box

There is a nice way to show the set of possible allocations and the preferences of Ed and Fiona, using a diagram that looks like an Edgeworth box without a roof. The distance between the two vertical walls of the box in Figure 5.1

is constructed to be $W_E + W_F$, which is the total amount of beans to be allocated between Ed and Fiona. A point in the box represents an allocation in the following way. The horizontal distance of the point from the left side of the graph is beans for Ed. The distance from the right side is beans for Fiona. The vertical distance from the bottom of the graph is the total amount of smoking by Ed. Each point on the graph represents a feasible allocation since the sum of Ed's and Fiona's beans will always be $W_E + W_F$ and since we have assumed that there is no resource constraint on Ed's smoking. The point W_0 on the horizontal axis represents the allocation in which Ed and Fiona consume their initial allocations of beans and there is no smoking.

Figure 5.1: A One-Sided Externality



Ed's indifference curves are the curves bulging out from the right side. They bend back on themselves because even for Ed, too much smoking is unpleasant. Fiona's curves slope downwards away from the point 0. This gives her convex preferences and a preference for more beans and less smoke.

Property Rights

If there were no restrictions on smoking and no bargains were made between Ed and Fiona, then Ed and Fiona would each spend their own wealth on their own beans and Ed would smoke an amount, S_0. But the allocation

$$X = (S_0, W_E, W_F)$$

is not Pareto optimal. This can be seen by noticing that any point inside the football-shaped region whose tip is X designates a feasible allocation that is Pareto superior to X. They would both be made better off if Fiona would give Ed some of her beans in return for which Ed would smoke less. It is easy to see that the Pareto optimal allocations are points of tangency between Ed's and Fiona's curves. Those Pareto optimal allocations which are better for both Ed and Fiona than the allocation X are represented by the points on the line segment, YT. If Ed has a legal right to smoke as much as he likes and if Fiona and Ed bargain to reach a Pareto optimal point, the outcome would be somewhere on YT.

Alternatively, there might be a law that forbids Ed to smoke without Fiona's consent. If no deal were struck, the outcome would be the allocation marked by W_O on the box where there is no smoking and where Ed consumes W_E and Fiona consumes W_F. We see from Figure 5.1 that this allocation is not Pareto optimal. Both parties would benefit if Ed gave Fiona some beans in return for permission to smoke. The Pareto optimal allocations that are Pareto superior to the no-smoking allocation are represented by the line SZ in Figure 5.1.

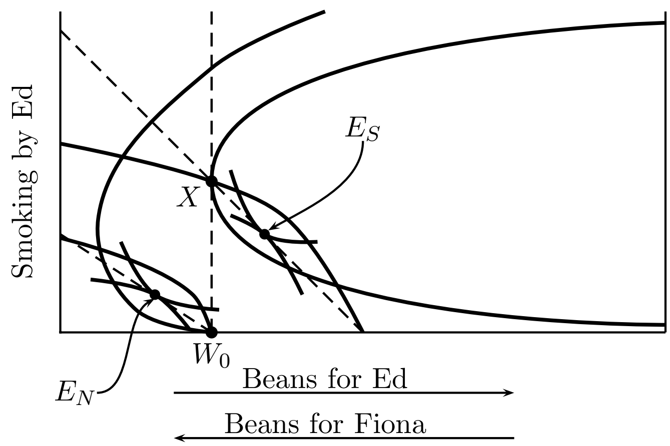
The set of all Pareto optimal allocations includes the entire line ST as well as points of tangency beyond S and T. We notice that the optimal amount of smoke is different at different points on the curve ST that is chosen.

Lindahl Equilibrium in a Smoky Box

To find the Lindahl equilibrium in Ed and Fiona's smoky box, we need to specify property rights. Consider first the case where initial property rights allow no smoking. Let beans be the numeraire, with price 1, let Ed's Lindahl price for Ed's smoking be p_E and let Fiona's Lindahl price for Ed's smoking be p_F. Recall that in Lindahl equilibrium, the allocation chosen must be an allocation that maximizes the the total value of output where public goods are evaluated at the sum of the Lindahl prices. Since smoking does not cost anything in terms of public goods, it must be that in Lindahl equilibrium, the amount of smoking S maximizes $(p_E + p_F)S$ over all possible values of S. This is possible for a finite positive S only if $p_E + p_F = 0$.[1] Thus we conclude that in Lindahl equilibrium, $p_F = -p_E$.

[1] The logic here is similar to the reasoning that tells us that in competitive equilibrium a firm that operates under constant returns to scale can be maximizing profits with a finite positive output only if it is making zero profits.

Figure 5.2: Lindahl Equilibrium



When no smoking is allowed in the initial allocation, Ed's Lindahl budget constraint must be

$$B_E + p_E S \le W_E. \tag{5.1}$$

His budget line is a straight line passing through the the point W_0 in Figure 5.2, with slope $-1/p_E$. Fiona's Lindahl budget constraint is

$$B_F + p_F S \le W_F. \tag{5.2}$$

Let $W = W_E + W_F$. Since Fiona's consumption is measured from the right side of the box, her budget constraint can also be written as $W - B_E + p_F S \le W - W_E$, which we see by rearranging terms is equivalent to $B_E - p_F S \ge W_E$. In Lindahl equilibrium, we must have $p_F = -p_E$. Therefore Fiona's budget constraint in equilibrium can be written as

$$B_E + p_E S \ge W_E. \tag{5.3}$$

Comparing the budget inequalities 5.1 and 5.3, we see that in Lindahl equilibrium, Ed is confined to choosing a point that is on or below a budget line passing through the initial allocation W_0 and Fiona is confined to choosing a point that is on or above the *same* budget line. If the price p_E is arbitrarily chosen, there is no reason to suspect that Ed would choose the same allocation that Fiona would choose. But, just as in the case of competitive equilibrium, it is possible to show that under quite weak assumptions there will be at least one point where their choices coincide. We have drawn the dashed budget line in Figure 5.2 to correspond to a price P_E at which Ed's preferred allocation is the same as Fiona's. This allocation is marked E_N in the figure. We see that in Lindahl equilibrium, Ed pays Fiona for permission to smoke. When Ed is paying the Lindahl equilibrium price, the amount of smoking that Ed demands is the same as the amount of smoking permission that Fiona is willing to grant at that price. In Lindahl equilibrium, Ed does not have to quit smoking altogether, but he smokes less than he would if he were free to smoke at no charge.

An alternative way to assign property rights is to allow Ed to smoke as much as he wishes. Fiona, of course, may choose to bribe him to smoke less. The corresponding Lindahl equilibrium is found by choosing a budget line that passes through the point X in Figure 5.2 with the property that Ed's favorite allocation from among those points that lie on or below this line is the same as Fiona's favorite allocation from among those points that lie on or above the line. We have drawn such a line in Figure 5.2 and marked the resulting Lindahl equilibrium allocation as E_S. In this Lindahl equilibrium, Fiona bribes Ed to reduce his smoking. The Lindahl price is the price at which Ed's demand for smoking is equal to the supply of smoking permission that Fiona is willing to grant. In Lindahl equilibrium, Ed smokes less than he would if there were no charge for smoking, but he consumes more beans than he would without trade.

What Is an Externality?

Pigou's Views

Economists are not entirely sure about how best to define externalities. Professor Arthur Cecil Pigou, one of the founders of modern public finance theory, devoted a chapter of his book *The Economics of Welfare* [3] to problems that most economists these days would call externalities. Pigou, however, doesn't use the word "externalities", he speaks of the *divergence between social and private product.*) q According to Pigou:

> "Here the essence of the matter is that one person A, in the course of rendering some service, for which payment is made, to a second person B, incidentally also renders services or disservices to other persons (not producers of like services), of such a sort that payment cannot be extracted from the benefited parties or compensation enforced on behalf of the injured parties." [3], page 183.

Pigou offers a list of examples of beneficial externalities, including the following ... Maintenance of a private forest may improve the environment for neighbors, lamps erected at the doors of private houses may illuminate the street, pollution abatement activities of firms improve air quality, resources devoted to fundamental research may in unexpected ways improve production processes. Pigou also lists some harmful externalities, "the game-preserving activities of one occupier involve the overrunning of a neighbor's land by rabbits," a factory in a residential neighborhood destroys the amenities of neighboring sites, motor cars congest and wear out roads, manufacturers produce noxious smoke as a biproduct.

Pigou suggests that appropriate taxes and subsidies may be useful for achieving efficiency in a competitive economy with externalities.[2] According to Pigou:

> "When competition rules and social and private net product at the margin diverge, it is theoretically possible to put matters right by imposition of a tax or the grant of a subsidy." [3], page 381.

Modern economists frequently refer to such interventions as "Pigovian" taxes or subsidies.

Externalities and Missing Markets

Walter P. Heller and David Starrett [2] propose and then (partially) renounce a definition that would seem to reasonably capture the "externality" found in the Ed-Fiona example and the examples suggest by Pigou. According to Heller and Starrett:

> "An externality is frequently defined to occur whenever a decision variable of one economic agent enters into the utility function or production function of another. We shall argue that this is not a very useful definition, at least until the institutional framework is given."

To understand Heller and Starett's point, it may be helpful to consider an example. Suppose that persons A and B both pick berries from a common berry-patch. As it happens, the more berries that B picks, the more difficult

[2] Pigou acknowledges that in practice, correction of externalities by means of taxes and subsidies may be difficult or impossible, and he discusses the alternative of using publicly-managed firms as an alternative.

it is for A to find berries and the harder A has to work to pick any given number of berries. In this case, A will care about the number of berries that B picks. According to our proposed definition, B's berry-picking generates an externality on A. If, on the other hand, the berry patch is owned by an owner who hires A and B to pick berries for an hourly wage and also sells berries to them, then the economy can be readily modelled as one in which there are no externalities; that is, neither A nor B cares about the berry-picking activities or the berry consumption of the other. As Heller and Starr suggest,

> "one of the prime attributes of the market system is that it isolates one individual from the influence of others' behavior (assuming of course that prices are taken by everyone as given.)"

Heller and Starr suggest that the definition proposed at the beginning of this paragraph should be modified to apply only if interdependencies exist in the framework of a competitive market system. Thus they propose to describe externalities as follows:

> "... one can think of externalities as nearly synonymous with nonexistence of markets. We define an externality to be a situation in which the private economy lacks sufficient incentives to create a potential market in some good and the nonexistence of markets results in losses in Pareto efficiency. "

Heller and Starrett suggest that when we observe situtations with apparent externalities, it is useful to focus our attention on the more fundamental question of why it is that the situation lacks markets which would eliminate the externality. Heller and Starr suggest the relevant considerations in this way.

> "We propose (roughly) that situations usually identified with "externality" have more fundamental explanations in terms of 1) difficulties in defining private property (2) noncompetitive behavior (3) absence of relevant economic information, or (4) nonconvexities in transaction sets."

Creating Markets for Externality Permits

The Case of One Polluter and One Victim

Let us pursue Heller and Starrett's suggestion that the externality in the case of Ed and Fiona might correspond to a "missing" market. In order to construct this market, however, we are going to have to introduce some legal institutions. In particular, let us suppose that the "government" introduces a new commodity called "smoking permits" along with a law that requires that for each unit of smoke that a person produces, he has to to present one smoking permit. The government prints a fixed supply $\bar{S}$ of smoking permits and distributes them in some way between Ed and Fiona.

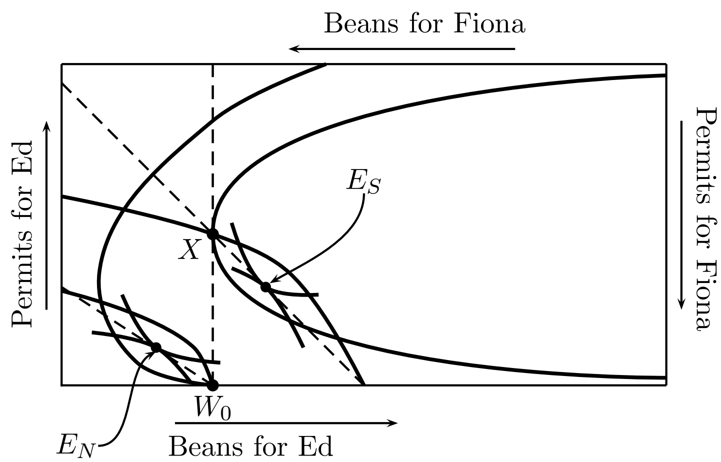
Although Fiona will not want to use a smoking ticket for permission to smoke, she will be willing to pay something for smoking tickets because she knows that there is a fixed supply of tickets and that every ticket that she acquires is one that Ed will not be able to use. If, for example, Fiona keeps all of the smoking tickets, then Ed will not be able to smoke at all.

We now have an economy with two private commodities, beans and smoking tickets. In our previous discussion, we defined Ed's and Fiona's utility functions $U_E(S, B_E)$ and $U_F(S, B_F)$ with the variable S representing Ed's smoking appearing in both people's utility functions. With the introduction of smoking permits, we can convert this economy into one with private goods only. In particular, if Ed always uses his smoking permits to get permission to smoke, his utility when he has S_E smoking permits will be $\tilde{U}_E(S_E, B_E) = U_E(S_E, B_E)$. If Fiona buys S_F smoking permits and hides them in her sock drawer, then Ed will have only $\bar{S} - S_F$ permits and hence will produce only $\bar{S} - S_F$ units of smoke. In this case, Fiona's utility will be $\tilde{U}_F(S_F, B_F) = U(\bar{S} - S_F, B_F)$, which depends only on her own consumption of beans and her own consumption of smoking permits. The economy that we have constructed in this way is a standard two-person, two-commodity pure exchange economy—the kind of economy that is found in Edgeworth boxes in all good intermediate price theory texts.

Let us now draw an Edgeworth box for this economy. This box turns out to look exactly like the Edgeworth box that we drew in Figures 5.1 and 5.2 except that we now put a roof on the Edgworth box. In particular, the box will be $\bar{S}$ units high, where $\bar{S}$ is the initial supply of tickets.

Before going further with our Edgeworth box construction, we need to decide who gets the smoking tickets initially. As you might guess, the question of to whom the permits are assigned initially is exactly the same question of property rights that we addressed in the case of Lindahl equilibrium. One possibility is that we assign a property right to clean air to Fiona. This could be accomplished by giving all of the smoking permits to Fiona initially. In this case the initial endowment corresponds to the point W_0 in the Edgeworth box. Alternatively, we could have given Ed an initial right to smoke as much as he wishes, given his initial holdings of beans. We could accomplish this assignment of rights by giving Ed an initial holding of per-

Figure 5.3: A Market for Smoking Permits



mits equal to the amount of smoking he would choose if he could smoke freely and by giving the rest of the permits to Fiona. In this case the initial allocation corresponds to the point X in the Edgeworth box. It would also be possible in principle to allocate initial holdings of permits in any other way such that the total number of permits adds to $\bar{S}$.

Figure 5.3 shows the competitive equilibrium budget lines and the competitive equilibrium allocations E_N and E_S corresponding to the two different initial allocations W_0 and X. Notice that these are the same as the Lindahl equilibria in our previous discussion.

You may also wonder what decides the total number $\bar{S}$ of smoking permits to be issued. In part, the answer is indeterminant. If we start from a situation in which $\bar{S}$ permits are issued and where in the resulting competitive equilibrium Fiona chooses to hold some permits, then we notice that if the government issued more permits, but gave them all to Fiona, the outcome would not be changed at all. Of course if the government wants to give Ed the right to produce at least S units of smoke, it will have to supply at least $\bar{S}$ permits.

Having shown the way in which markets can "privatize" the smoking externality in our model of Ed and Fiona, it is useful to return to the focus suggested by Heller and Starrett. Why did it seem natural for us to model the effects of Ed's smoking on Fiona, without immediately assigning ownership rights and without introducing a corresponding market for transfer of such rights? It may be fruitful to turn this question around and ask why it seemed entirely natural to assign initial property rights to beans. Certainly it is physically possible for Ed to steal Fiona's beans and *vice versa*. In many societies, but certainly not in all, institutions and norms have evolved that make theft relatively rare. It is possible in principle to regard inflicting tobacco smoke on another person without that person's permission as the legal and moral equivalent of theft. Indeed norms in the United States appear to be shifting in that direction. This is undoubtedly in part a response to relatively new scientific information about actual damage that smokers inflict on non-smokers and in part due to an increase in the proportion of non-smokers in the population.

As Heller and Starrett point out, even where market equilibrium exist, as in the case of Ed and Fiona, introduction of market institutions is likely to have costs. If there is to be a market, then somehow Ed has to be prevented from smoking without a permit. For violations to be enforceable, they must be relatively cheaply observable. In realistic circumstances, it may not be so easy to tell whether Ed is secretly puffing a cigar, or whether the nasty smell that plagues Fiona comes instead from a burning tire or a flatulent canine.

A fundamental difficulty in the establishment of property rights in the face of "externalities" is that it is easy for people to claim damage from the actions of others and difficult to verify that actual damage has been done. It would certainly be impractical to force everyone to buy permission for each publicly observable action that he or she might take. In every society, people are willing to accept occasional annoyance from others without compensation, knowing that some of their own actions will also cause offense. It seems to me that a free society must be one whose members are relatively tolerant of annoyance that does not cause objectively measurable harm. As science develops new methods of detecting, measuring, and pricing harmful externalities, however, new market forms and new forms of property rights are quite likely to evolve. Conspicuous examples of this kind include markets for emissions of pollutants into the air and water, and for congestion of highways, streets and other public areas.

One Pollutant, Many Polluters and Neighbors

We found a simple assignment of property rights that leads to Pareto efficient allocation for the two-person example of Ed and Fiona. This seems to be very encouraging news. Can it be that "externalities" like pollution can be tamed by simple adjustments of property law, so that the "invisible hand" of the market will guide the economy to a competitive equilibrium with little government interference?

Does the permit market that worked so well for Ed and Fiona would work equally well if more than two people are involved? Let us imagine that Ed is not a seedy little guy who smokes cheap cigars, but a power company with a large smokestack whose smoke afflicts thousands of people. For good measure, let us add some more firms, each of which pumps the same kind of smoke into the same neighborhood. The total amount of smoke in the neighborhood is the sum of that produced by the firms. Profits of each firm depend on how much smoke it produces and on cash payments that it spends or receives.

Pareto efficiency for this economy would require that the marginal profitability of an extra bit of smoke for any polluting firm is the same as that for any other. In addition, efficiency would require that this marginal profitability would be equal to the sum of the neighbors' marginal willingness to pay for reduced pollution.

Suppose that pollution is initially unregulated and that firms produce a total of S_0 units of smoke. The government decides to improve air quality by requiring a pollution control permit for each unit of smoke emitted. It prints a total of S_0 permits and gives each of the n neighbors S_0/n marketable permits, which they can sell to the firms if they wish. A competitive market develops in which firms can buy permits that allow them to produce smoke. Since the market is competitive, all firms pay the same price for permits and so they all produce smoke up to the point where their marginal profit from producing a unit of smoke is equal to the price of a permit. The price of a permit will be determined by supply and demand. Let us assume that each firm has a downward-sloping demand curve for producing smoke. Then the market inverse demand curve that intersects the horizontal axis at S_0. The supply of permits comes from neighbors who are willing to sell their permits, even though they realize that each permit sold will increase smoke by one unit. We can draw a supply curve, showing for each quantity S, the lowest price at which consumers would be willing to sell a total of S permits. The equilibrium price and quantity of pollution permits will be determined by the intersection of supply and demand.

In a large community, this equilibrium price will be far too low. In equilibrium, some consumer(s) is just indifferent between selling one an additional pollution permit and keeping it. For this consumer, the marginal willingness to pay for reduced smoke is equal to the price of permits. In contrast, firms will produce a Pareto efficient amount of smoke only if the marginal profitability is equal the *sum of all* consumers' marginal willingness to pay. In the case where there are n consumers have identical wealth and preferences, this would mean that the price of permits would be only $1/n$th of the price needed for Pareto efficiency.

The problem with this permit scheme is that the polluters only need to get permission from one citizen to emit pollution that afflicts them all. Perhaps we can do better with a scheme that requires firms to obtain permits from every citizen for each unit of pollution that they produce. Thus the government might issue personalized permits to each individual and require that for each unit of smoke produced, it must have a personalized permit from each citizen. In our discussion of Lindahl equilibrium, we found that a Lindahl equilibrium is equivalent to a competitive equilibrium with markets for personalized public goods. Similar reasoning shows that a competitive equilibrium in an economy where all of these personalized permits are marketed would be a Lindahl equilibrium, and hence would be Pareto efficient.

This seems promising, but there are some difficulties. One problem arises if each neighbor is a monopoly holder of personalized permissions to pollute himself. With so many monopolists in the economy, it is hard to believe that the outcome will be competitive. A market in which separate monopolists supply factors of production used in fixed coefficients by a competitive buyer was presented by A. A. Cournot [1] in his great book, *Researches into the Mathematical Principles of the Theory of Wealth.* Cournot tells the story of a copper monopolist and a zinc monopolist who sell their products to a competitive brass industry.[3] In Cournot's model, the equilibrium price of brass is not only higher than the competitive price, but also higher than the price would be if a single monopolist controlled the supplies of copper and zinc. Similarly, in a model of monopoly suppliers of personalized permits, we would find that the equilibrium cost of assembling a full set of permits in order to produce a unit of smoke would be far higher than the cost in competitive equilibrium and would be much higher than what is needed for Pareto efficiency.

We can escape the problem of monopoly sellers if we spread the endowments around. Suppose that for each person i, we issue S_0 personalized tickets with i's name on them, but we initially distribute the tickets so that everyone gets S_0/n tickets with each person's name on them. What would happen then? (To be continued...)

[3] His assumption that the factors are used in fixed coefficients is an exercise of economists' license-to-abstract, much like Alfred Marshall's fixed-proportions sheep. In actually many variations of brass are produced, using copper and zinc in differing proportions.

The Case of Many Polluters and Many Victims

Suppose that instead of just two people, Ed and Fiona, we have a community in which there are many polluters and many pollution victims. We will not assume that polluters and pollutees are separate people, but allow the possibility recognized by Walt Kelly's Pogo, who said "We have met the enemy and it is us."

In this economy, there are n consumers, m private goods, and k *non-private activities*. Consumer i cares about i's own vector of private goods x_i but does not care about the private goods consumption of others. Consumers also care about their own vectors of non-private activities as well as the *sum* of the vectors of non-private activities of others. Thus Consumer i has a utility function $u_i(x_i, , y_i, z)$ where y_i is the vector of non-private activities performed by i and where $z = \sum_{s=1}^{n} y_s$.

For simplicity of notation, let us confine our attention to a pure exchange economy without production of private goods. Each consumer i has an initial endowment vector of private goods, $\hat{x}_i$ and we define $\hat{x} = \sum_{i=1}^{n} x_i$.[4] This formulation account for pollution activities in the following way. Consumer i may take pleasure in releasing pollutant j but, holding constant his own release of pollutant, every consumer may regard the total amount of pollutant j in the atmosphere as a "bad". In this case, u_i is an increasing function of y_{ij}, but a decreasing function of $z_j = \sum_{s=1}^{n} y_{sj}$. Suppose that for each polluting activity j, the government issues a fixed number $\hat{z}_j$ transferable permits, where consumer i is given $\hat{z}_{ij}$ permits and where we define $\hat{z}_j = \sum_{i=1}^{n} \hat{z}_{ij}$. Consumers are allowed to trade these permits for private goods or for other kinds of tickets. A consumer is not allowed to release more pollution than the amount for which he has permits.

The formulation can also account for positive externalities. For example, there may be a service activity, like picking up trash or beautifying the environment, which is unpleasant to perform, but where the total amount of this activity perform is regarded as a good by all consumers. For such an activity, j, u_i would be a decreasing function of the amount y_{ij} of the service performed by i but an increasing function of the total amount z_j of service j that is performed by community members. The government could issue an initial endowment of marketable service obligations, such that the holder of

[4] This model can be interpreted as a production economy, in which we allow some consumers to own firms (or parts of firms). These consumers may engage in "production" which is treated as negative consumption of output goods, along with positive consumption of input goods. The consumers' budget equations then apply to net purchases positive or negative.

each unit of obligations is required to perform a corresponding unit of the service.

With this assignment of property rights, the total amount of each non-private activity that will be performed must equal the total number of permits or obligations for that activity that are issued by the government. Trades of permits and obligations will determine the ultimate distribution of non-private activities, but will not affect the vector $\hat{z}$ of total amounts of pollution and of service activities. In the resulting economy, each consumer's utility takes the form $u_i(x_i, y_i, \hat{z})$ where $\hat{z}$ is fixed. The only variables that i chooses are x_i and y_i. When $\hat{z}$ is held constant, nobody other than person i cares about either x_i or y_i. Thus when $\hat{z}$ is fixed, we have a model that is formally the same as a pure exchange model with private goods only where any feasible allocation of x's and y's must satisfy the equations $\sum_{i=1}^{n} x_i = \hat{x}_i$ and $\sum_{i=1}^{n} y_i = \hat{z}$.

As is well known from competitive equilibrium theory, a competitive equilibrium will exist for this economy if all individuals have continuous, convex preferences and if a few other relatively weak technical assumptions are satisfied. If, however, the vector of permits and obligations $\hat{z}$ is arbitrarily selected, there is no reason to expect that the outcome will be Pareto optimal. Although the competitive equilibrium with $\hat{z}$, may not be the optimal, it will be true that this outcome will be Pareto optimal *conditional on the aggregate vector* $\hat{z}$. That is to say, any allocation that is Pareto superior to this competitive equilibrium must either be infeasible or must be one in which the aggregate vector of non-private activities is different from $\hat{z}$. To say this yet another way, although this competitive equilibrium may not have the right total amount of non-private activities, allocation of these activities among individuals is done efficiently.

Exercises

5.1 Suppose that Ed's utility function is $U_E(B_E, S) = B_E S$ for $0 \leq S \leq 4$, and $U(B_E, S) = 0$ for $S > 4$. Suppose that Fiona's utility function is $U_F(B_F, S) = B_F - S^2$. Assume that the initial allocations of beans are W_E and W_F, where $W_E + W_F = 16$.

a). Sketch an Edgeworth diagram, showing Ed's and Fiona's preferences over possible allocations.

b). Write algebraic expression(s) to describe all of the Pareto optimal allocations for Ed and Fiona.

c). Write an equation for the utility possibility frontier and sketch it.

d). Find the Lindahl equilibrium prices and quantities as a function of W_E where initial property rights forbid smoking.

e). Find the Lindahl equilibrium prices and quantities as a function of W_E where initial property rights allow one to smoke as much as one wishes.

5.2 Jim and Tammy are partners in business and in Life. As is all too common in this imperfect world, each has a little habit that annoys the other. Jim's habit, we will call Activity X and Tammy's habit, activity Y. Let x be the amount of activity X that Jim pursues and y be the amount of activity Y that Tammy pursues. Jim must choose an amount of activity X between 0 and 50. Tammy must choose an amount of activity Y between 0 and 100. Let c_J be the amount of money that Jim spends on consumption goods and let c_T be the amount that Tammy spends on consumption goods. Jim and Tammy have only \$1,000,000 per year to spend on consumption goods. Jim's habit costs \$40 per unit. Tammy's habit also costs \$100 per unit. Jim's utility function is

$$U_J = c_J + 500 \ln x - 20y$$

and Tammy's utility function is

$$U_T = c_T + 500 \ln y - 10x.$$

a). Find the set of Pareto optimal allocations of money and activities in this partnership.

b). Suppose that Jim has a contractual right to half of the family income and Tammy has a contractual right to the other half. Find a Pareto optimal outcome in which each spends the same total amount.

c). If they make no bargains about how much of the externality generating activities to perform, how much x will Jim choose and how much y will Tammy choose?

d). Find Lindahl equilibrium prices and quantities if the initial property rights specify that neither activity X nor activity Y can be performed without ones partner's consent.

e). Find Lindahl equilibrium prices and quantities if Jim has a right to perform X as much as he is able to and Tammy has a right to perform activity Y as much as she is able to.

5.3 The cottagers on the shores of Lake Invidious are an unsavoury bunch. There are 100 of them and they live in a circle around the lake. Each cottager has two neighbors, one on his right and one on his left. There is only one commodity and they all consume it on their front lawns in full view of their two neighbors. Each cottager likes to consume the commodity, but is envious of consumption by the neighbor on his left. Nobody cares what the neighbor on his right is doing. Every consumer has a utility function $U(c, l) = c - l^2$, where c is her own consumption and l is consumption by her neighbor on the left.

a). Suppose that every consumer owns 1 unit of the consumption good and consumes it. Calculate the utility of each individual.

b). Suppose that every consumer consumes only 3/4 of a unit. What will be the utility of each of them?

c). What is the best possible consumption if all are to consume the same amount?

d). Suppose that everybody around the lake is consuming 1 unit, can any two persons make themselves both better off either by redistributing consumption between them or by throwing something away?

e). How about a group of three persons?

f). How large is the smallest group that could cooperate to benefit all of its members.

5.4 The town of Puuey has 10 firms and 1000 people. The profits of firm i's manufacturing operations are given by $A_i S - (1/2)S^2$ where S is the amount of smoke that i produces. The people of Puuey have identical linear utility functions $U(x, S) = x - dS$, where x is private consumption and where $d > 0$ is the marginal (and average cost) of smoke to one individual. It happens that $A_i > 1000d$ for every firm i in Puuey and that $\sum A_i = 100,000$.

a). If there are no penalties for pollution and the firms choose their amount of smoke to maximize profits, how much smoke will there be in Puuey?

b). At a Pareto optimum, how many units of smoke should firm i produce? How many units of smoke should be produced in total?

c). Suppose that the city council of Puuey requires that every firm have a permit for each unit of smoke that it emits. The council issues 100,000 permits and gives 100 of them to every citizen. A permit market opens where firms purchase permits in order to be able to produce smoke. In competitive equilibrium, what is the price of a ticket? How much smoke is produced in total? In equilibrium, how much revenue does each citizen get from selling permits to firms and what is the total cost of smoke to each citizen?

d). Suppose that each citizen is given 100,000 personalized permits with his name on each one. For each unit of smoke that a firm produces it must have a personalized permit from each of the 1000 citizens. Markets are opened for each of the 1000 types of permit. If each citizen sets a price at which he will sell permits and believes that his actions have no effect on the prices set by the others, in equilibrium, what price would we expect each citizen to set? What would be the total amount of smoke produced in Puuey?

5.5 Romeo loves Juliet and Juliet loves Romeo. Besides love, they consume only one good, spaghetti. Romeo likes spaghetti, but he also likes Juliet to be happy and he knows that spaghetti makes her happy. Juliet likes spaghetti, but she also likes Romeo to be happy and she knows that spaghetti makes Romeo happy. Romeo's utility function is $U_R(S_R, S_J) = S_R^a S_J^{1-a}$ and Juliet's utility function is $U_J(S_J, S_R) = S_J^a S_R^{1-a}$, where S_J and S_R are the amount of spaghetti for Romeo and the amount of spaghetti for Juliet respectively. There is a total of 24 units of spaghetti to be divided between Romeo and Juliet.

a). Suppose that $a = 2/3$. If Romeo got to allocate the 24 units of spaghetti exactly as he wanted to, how much would he give himself and how much would he give Juliet? If Juliet got to allocate the spaghetti exactly as she wanted to, how much would she take for herself and how much would she give Romeo?

b). What are the Pareto optimal allocations?

c). When we have to allocate two goods between two people, we draw an Edgeworth box with indifference curves in it. When we have just one good to allocate between two people, all we need is an "Edgeworth line" and instead of indifference curves, we will just have indifference dots. Draw an Edgeworth line. Let the distance from left to right denote spaghetti for Romeo and the distance from right to left denote spaghetti for Juliet. On the Edgeworth line, show Romeo's favorite point and Juliet's favorite point. Also show the locus of Pareto optimal points.

d). Suppose that $a = 1/3$. If Romeo got to allocate the spaghetti, how much would he choose for himself? If Juliet got to allocate the spaghetti, how much would she choose for herself? Draw another "Edgeworth line" below, showing the two people's favorite points and the locus of Pareto optimal points. When $a = 1/3$, describe the nature of disagreements between Romeo and Juliet at the Pareto optimal allocations.

5.6 If we treat "spaghetti for Romeo" and "spaghetti for Juliet" as public goods, we would have an economy with two public goods and no private goods. We can find a Lindahl equilibria by finding personalized Lindahl prices where p_{ij} is the price that person i pays per unit of j's consumption. In Lindahl equilibrium the Lindahl prices must be chosen in such a way that given their personalized prices, consumers all agree on the quantity that should be consumed by every consumer and such that for each j, the sum of the prices p_{ij} over all consumers i is one.

a). Suppose that Romeo in the previous problem has an intital endowment of 18 units of spaghetti and Juliet has an initial endowment of 6 units. Find the Lindahl equilibrium prices and the Lindahl equilibrium quantities of spaghetti for Romeo and Juliet.

b). Suppose there are n consumers and one commodity. Consumer i has an initial endowment of W_i units of this commodity consumer i's utility function is given by

$$U_i(X_1, \ldots, X_n) = \sum_{j=1}^{n} \alpha_{ij} \ln X_j.$$

Find the Lindahl equilibrium prices and quantities for this economy, expressed as a function of the α_{ij}'s and the W_i's.

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