The Lazy Housekeepers’ Problem

The Symmetric Case

Let $x$ be the amount of housekeeping effort that Alice supplies and $y$ the amount that Bob supplies. Alice has utility function $U_A(x,y)=x+y-x^2$ and Bob has utility function $U_B(x,y)=x+y-y^2$. If we draw a box with $x$ on the horizontal axis and $y$ on the vertical axis, Alice’s indifference curves are shown as the U-shaped curves below. Regardless of what Bob does, she would prefer to do 1/2 unit of housework. But the more housework Bob does, the happier she is.

```math
\text{In[428]} = \text{apl} = \text{ContourPlot}[x + y - x^2, \{x, 0, 2\}, \{y, 0, 2\}, \text{Axes} \rightarrow \text{True}]
```

![Contour Plot](image)

We can draw Bob’s indifference curves on the same axes. This is shown in the figure below. I used the command `Parametric plot` to draw a bunch of indifference curves each of which holds $u$ constant at some value between -4 and 4 and letting $x$ vary from 0 to 10. I restrict the range of $x$’s and $y$’s that are displayed to values between 0 and 2.
Next I draw the contract curve. As shown in our notes, this has the equation \( y = x^2 x - 1 \).
Now we display the contract curve on the labelled Edgeworth box.

\[
\text{In}[454]= \text{Show[BoxSymPlot, ContCurveSym, Frame} \rightarrow \text{True,}
\text{FrameLabel} \rightarrow \{\text{"Cleaning by Alice", "Cleaning by Bob"}\}]
\]

---

**Bob hates housekeeping more than Alice**

Let utilities be \( u_A = x + y - x^2 \) and \( u_B = x + y - 2y^2 \). We first plot the Edgeworth box. Then we add the contract curve and display them both together.
In[452]:= box12plot = ParametricPlot[
{(x, u + x^2 - x), {u + 2 * x^2 - x, x}, {0.50, x}, {x, 0.25}},
{u, -10, 10}, {x, 0, 10}, AspectRatio -> 1 / 1, PlotRange -> {{0, 2}, {0, 2}},
Mesh -> 60, MeshStyle -> {Directive[Blue], Directive[White, Dashed]}]

Out[452]=

In[442]:= contcurve12 = ParametricPlot[
{x, x/ (2 * (2 * x - 1))}, {x, .51, 10},
AspectRatio -> 1 / 1, PlotRange -> {{0, 2}, {0, 2}}, PlotStyle -> Thick]

Out[442]=
We construct the utility possibility frontier for the

**Utility possibility frontiers**

We construct the utility possibility frontier for the symmetric and asymmetric cases. To do this, I use ParametricPlot, varying \( x \) from 1/2 to 2, using the efficiency conditions to determine \( y \) from \( x \) and then plotting the utilities for each of the two people as \( x \) is varied while \( y \) moves with \( x \) according to the efficiency condition. In the symmetric the symmetri case, we have \( y=x/(2x-1) \) along the contract curve. In the asymmetric case, \( y=x/(4x-2) \) along this curve.
Utility possibility frontier: Symmetric Case

\begin{align*}
\text{In}[445] &= \text{upossym} = \text{ParametricPlot}[
\{x + (x / (2 \times x - 1)) - x^2, x + (x / (2 \times x - 1)) - (x^2) / (2 \times x - 1)^2),
\{x, .51, 2\}, \text{PlotRange} \rightarrow \{(0, 2), (0, 2)\}, \text{AspectRatio} \rightarrow 1 / 1, \text{PlotStyle} \rightarrow \text{Thick}]
\end{align*}
Utility possibility frontier: Case where Bob hates housekeeping

```math
\text{uposs12 = ParametricPlot[}
  \{x + (x / (4 x - 2)) - x^2, x + (x / (4 x - 2)) - 2 * ((x^2) / (4 x - 2)^2)},
  \{x, .51, 2},
  \text{PlotRange \to \{(0, 2), (0, 2)}},
  \text{AspectRatio \to 1/1},
  \text{PlotStyle \to Directive[Thick, Dashed]}\}
```

![Plot of Utility possibility frontier](lazyhousekeepers.nb)
Both cases together

In[447]:= Show[upossym, uposs12, ListLinePlot[{{2.01, 0}, {0, 2.01}}],
          ListLinePlot[{{1.5, 0}, {0, 1.5}}], ListPlot[{{1, 1}, {1/2, 1}}, PlotMarkers -> Automatic],
          Frame -> True, FrameLabel -> {"Utility for Alice", "Utility for Bob"}]

Out[447]=

Utility for Bob

Utility for Alice