The Substitution Effects of Transportation Costs

John P. Gould and Joel Segall

University of Chicago

When a commodity is consumed at the production center and also at various distances from the center, it is alleged that the amount shipped for consumption will contain a greater proportion of high-quality output than will be found in the amount kept for consumption at the production center. It is said, for example, that New Yorkers consume a greater proportion of good oranges than do Californians, that Asians import a greater proportion of expensive American cars than do consuming areas closer to Detroit, that "seconds" are more heavily consumed near the place of manufacture than farther away, and that one must be more careful in buying Italian leather goods in Italy than in the United States (Alchian and Allen, 1964, p. 74). Students are frequently asked to explain the phenomenon in economics examinations, and the question has found its way into an advanced text on economic theory (Stigler, 1966*a*, p. 103).

The following quotation from *University Economics* provides what appears to be the "accepted" solution to the problem.

> Suppose that grapes are grown in California and that it costs 5 cents a pound to ship grapes to New York whether the grapes are "choice" or "standard" (poorer) and that the total production of grapes is 50 per cent "choice" and 50 per cent "standard." Suppose further that in California the "choice" grapes sell for 10 cents a pound and the standard for 5 cents a pound; that is in California 2 pounds of "standard" and one pound of "choice" grapes sell for the same price. If grapes are shipped to New York, the shipping costs will raise the costs of "choice" grapes to 15 cents and of "standard" grapes to 10 cents. In New York the costs of "choice" grapes are lower relative to "standard" grapes (1.5 to 1) than in California (2 to 1). To buy 1 pound of

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> "choice" grapes in New York would mean a sacrifice of 1.5 pounds of "standard" whereas in California it would cost two pounds of "standard." According to our law of demand, New Yorkers faced with a lower price of "choice" grapes relative to "standard" will consume relatively more "choice" grapes than Californians will. In California where "standard" grapes are cheaper relative to "choice" grapes a larger fraction of "standard" grapes will be consumed—and this is what actually happens [Alchian and Allen, 1964, p. 75].

Stigler (1966*b*, p. 11) provides the same explanation.

As with most propositions of economic theory, this proposition need not hold if one of the commodities has an unusual income elasticity. The burden of this note is to show that the proposition need not hold *even if income effects are eliminated* and substitution effects alone are considered. We begin with a two-commodity world, where the commodities are choice grade of X and standard grade of X.

In the two-commodity case, the individual has a budget line B_4A_4 (see Fig. 1), where the choice quality is assumed to sell at a higher price than the standard quality good so that the slope of the budget line is less than one in absolute value. The individual achieves a tangency with indifference curve I_1 when he consumes B_2 units of the choice quality good and A_1 units of the standard quality good. When a fixed transportation charge is added, an individual having the same money income and the same utility

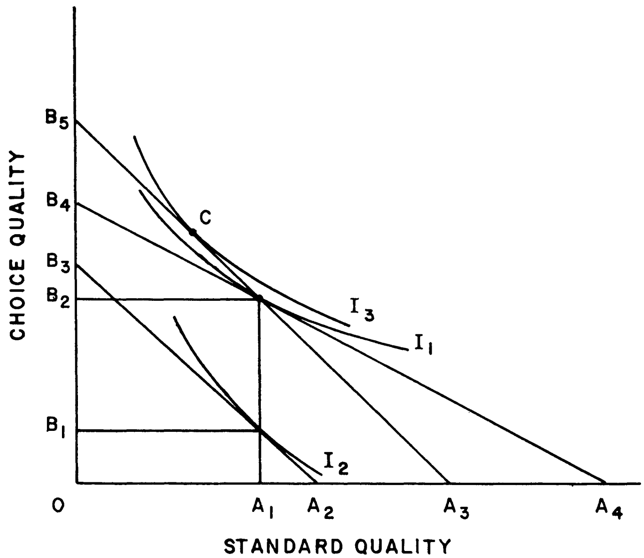

FIG. 1

function (but located away from the production center) will have budget line B_3A_2. This budget line lies below B_4A_4 (because real income is lower) and is steeper than B_4A_4 (because the ratio of the prices is closer to unity). Now the indifference map may be such that the tangency occurs on I_2, where B_1 units of choice and A_1 units of standard are consumed. Since there has been a reduction in the consumption of choice quality but no change in the consumption of standard quality, it follows that the proportion of choice quality goods consumed is *less* in the distant market. This phenomenon is always a possibility, particularly if the standard quality good is an inferior good, and the proposition need not hold if income effects are permitted. If, however, income is compensated so that the individual at the distant location has the option of purchasing the same bundle that the individual at the production center purchases (that is, if he is given additional money income so that his budget line is B_5A_3), then his tangency will occur at point C, where he consumes more of the choice quality good and less of the standard quality good. Indeed, it will be shown in the next section that it is only in this two-good, income-compensated case that one can unequivocally conclude that the proportion of choice quality goods consumed increases when transportation costs are added.

More than Two Goods with Income Compensation

An Algebraic Interpretation

Consider an individual maximizing the utility function

$$U(X_1, \ldots, X_n),$$

subject to the income constraint

$$I = \sum_1^n P_i X_i,$$

where P_i and X_i represent the price and quantity of the ith good, respectively. It is reasonably straightforward to show that the change in the equilibrium quantity of the jth good, given an income change dI and price changes dP_j ($j = 1 \ldots n$), is

$$dX_j = \left[\sum_{i=1}^n \mu\, D_{ij}\, dP_i + (dI - \sum_{k=1}^n X_k\, dP_k) D_{n+1,j}\right] / D, \tag{1}$$

where μ is the marginal utility of income (and is positive) and

$$D = \begin{vmatrix} U_{11} & \cdots & U_{1n} & P_1 \\ \vdots & & \vdots & \vdots \\ U_{n1} & \cdots & U_{nn} & P_n \\ P_1 & \cdots & P_n & 0 \end{vmatrix},$$

where U_{ij} is the cross-partial derivative $\partial^2 U/\partial X_i \, \partial X_j$. D_{ij} is the cofactor of the element in the ith row and jth column of the matrix of which D is the determinant.[1] The question of interest here is what happens to the amounts consumed of the choice and standard goods when the prices of these goods are increased by the *same* amount (this amount being transportation cost per unit). Let X_1 and X_2 be the amount consumed of the choice and standard good, respectively, and let P_1 and P_2 be the respective prices of these goods at the production center ($P_1 > P_2$). Let the transportation cost per unit be θ. The consumer at the production center spends

$$I = \sum_1^n P_i \hat{X}_i,$$

and since we want the consumer at the distant market to have the option of consuming the same quantities, his income must be

$$I^* = (P_1 + \theta)\hat{X}_1 + (P_2 + \theta)\hat{X}_2 + \sum_3^n P_i \hat{X}_i;$$

so

$$dI = I^* - I = \theta(\hat{X}_1 + \hat{X}_2).$$

Thus, since $dP_1 = dP_2 = \theta$, and $dP_i = 0$ for $i = 3, \ldots n$,

$$dX_1 = \theta\left[\frac{\mu D_{11} + \mu D_{12}}{D}\right]; \tag{2a}$$

$$dX_2 = \theta\left[\frac{\mu D_{22} + \mu D_{21}}{D}\right]. \tag{2b}$$

Note that from (1) we get

$$\frac{\partial X_j}{\partial P_i} = \frac{\mu D_{ij}}{D} - \frac{X_i D_{n+1,j}}{D} \quad (i, j = 1 \ldots n), \tag{3}$$

the Slutsky expression, where the term $\mu D_{ij}/D$ is the substitution effect of good j with respect to a change in the price of i.[2] This substitution

[1] For a complete exposition of this theory, see Samuelson (1948, pp. 90–124) and Hicks (1939, pp. 303–19). Samuelson's development and notation is used in this paper.

[2] Note from (1) that

$$\frac{\partial X_j}{\partial I} = \frac{D_{n+1,j}}{D}$$

so that the term

$$-X_i \frac{D_{n+1,j}}{D} = -X_i \frac{\partial X_j}{\partial I}$$

in (3) is the "income effect" of a change in the price of good i on the demand for good j.

component will be denoted

$$K_{ji} = \frac{\mu D_{ij}}{D}.^{3}$$

Thus substituting into (2a) and (2b)

$$dX_1 = \theta[K_{11} + K_{12}]; \tag{4a}$$

$$dX_2 = \theta[K_{22} + K_{12}]; \tag{4b}$$

since from symmetry $K_{12} = K_{21}$. Hicks (1939, p. 310) has shown that $K_{ii} < 0$ and that

$$\sum_{\substack{j=1 \\ j \neq r}}^{n} P_j K_{rj} = -P_r K_{rr}. \tag{5}$$

In a two-commodity world we get from (5) that

$$-P_1 K_{11} = P_2 K_{12},$$

and

$$-P_2 K_{22} = P_1 K_{12},$$

so that substituting these results into (4a) and (4b)

$$dX_1 = \theta\left[K_{11}\left(1 - \frac{P_1}{P_2}\right)\right]; \tag{6a}$$

$$dX_2 = \theta\left[K_{22}\left(1 - \frac{P_2}{P_1}\right)\right]. \tag{6b}$$

Since $1 - (P_1/P_2) < 0$, $K_{11} < 0$, and $\theta > 0$, it follows from (6a) that $dX_1 > 0$. Similarly, $dX_2 < 0$, so that in a two-commodity, income-compensated world the proposition holds as asserted. Assume, however, that there are three commodities.[4] In this case,

$$-P_1 K_{11} = P_2 K_{12} + P_3 K_{13}$$

$$-P_2 K_{22} = P_1 K_{12} + P_3 K_{23}$$

so that

$$K_{12} = -\left(\frac{P_3}{P_2} K_{13} + \frac{P_1}{P_2} K_{11}\right) = -\left(\frac{P_3}{P_1} K_{23} + \frac{P_2}{P_1} K_{22}\right);$$

and

$$dX_1 = \theta\left[K_{11}\left(1 - \frac{P_1}{P_2}\right) - \frac{P_3}{P_2} K_{13}\right]; \tag{7a}$$

$$dX_2 = \theta\left[K_{22}\left(1 - \frac{P_2}{P_1}\right) - \frac{P_3}{P_1} K_{23}\right]. \tag{7b}$$

[3] Note that this is *not* an elasticity since it has not been multiplied by P_i/X_j.

[4] Actually this is perfectly general. Since only the prices of goods 1 and 2 change, we can lump all other commodities into a group and treat this group as a single commodity (see Hicks, 1939, pp. 312–13).

The most plausible assumption is that the good in question is a substitute with respect to all other commodities so that K_{13} and K_{23} are positive. Thus, it is possible that (7a) and (7b) will be negative, in which case consumption of *both* X_1 and X_2 will be reduced, and this may result in a smaller proportion of the quality good being consumed at the distant market. Indeed, a smaller proportion of the choice good will be consumed at the distant market if

$$\frac{X_1}{X_2} > \frac{X_1 + dX_1}{X_2 + dX_2}.$$

That is, if

$$X_1\, dX_2 > X_2\, dX_1,$$

or, given that $dX_2 < 0$, if

$$\frac{X_1}{X_2} < \frac{dX_1}{dX_2} = \frac{K_{11} + K_{12}}{K_{22} + K_{12}}. \tag{8}$$

Now, for example, if K_{11} and K_{22} are both large relative to K_{12} and if they are about the same order of magnitude, then dX_1/dX_2 will be about one. Hence, in this case, if $X_1 < X_2$, a *smaller* proportion of the choice quality good will be consumed. Other cases can of course be imagined, but the main point has been made; it is not possible on theoretical grounds to rule out the case in which a smaller proportion of the choice good will be consumed at the distant market.

A Geometric Interpretation

It has been established algebraically that transportation costs may result in proportionally greater consumption of the standard grade in distant markets. The geometric analogue of this result is more difficult, since it is awkward to draw three-dimensional utility functions on a two-dimensional graph. It is fairly straightforward, however, to represent three-dimensional budget planes in two dimensions and then rely on revealed preference arguments to describe the results of the mathematical analysis.

The relevant budget planes are illustrated in Figure 2. This figure shows the attainable commodity bundles in (X_1, X_2, X_3) space for given prices and given incomes. Assume that at the production center prices and income are such that the consumer's budget plane is given by $A_1A_2A_3$ and that this budget plane is tangent to the consumer's iso-utility surface at the point F. The consumer thus purchases X_1^0 units of the choice quality good and X_2^0 units of the standard quality good. Now consider an identical consumer located at some distance from the production center. Transportation costs are such that the choice quality good relative to the standard quality good is cheaper than at the production center so that

$$\frac{A_2}{A_1} = \frac{P_1}{P_2} > \frac{P_1 + \theta}{P_2 + \theta} = \frac{B_2}{B_1}.$$

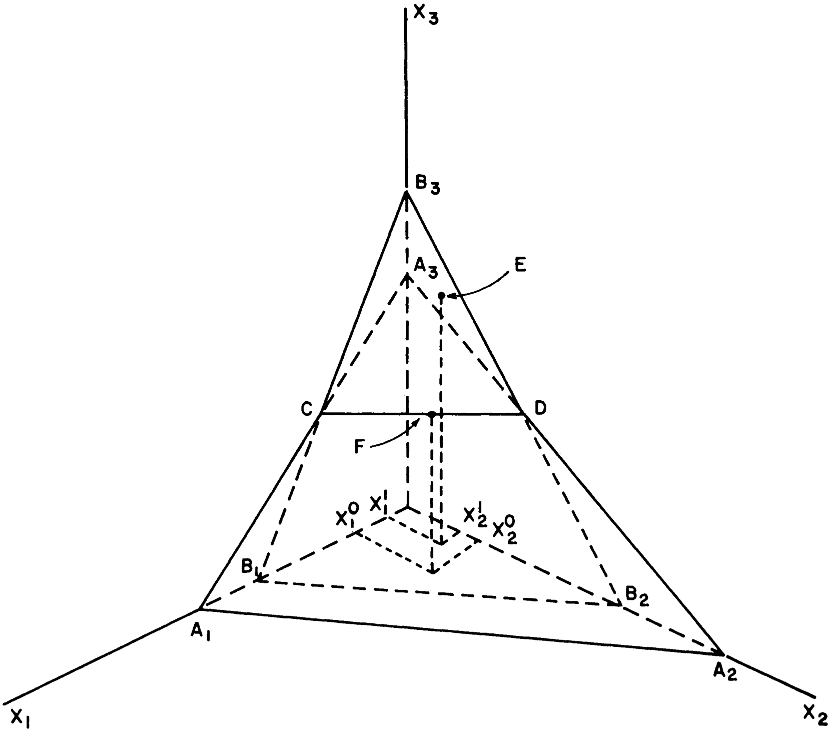

FIG. 2

At the same time, we give this distant consumer an increment in income so that he could, if he wished, purchase the bundle *F*. As may be seen in Figure 2, the resulting budget plane $B_1B_2B_3$ intersects the previous budget plane along the line *CFD*. As a result, the consumer now has a new set of options given by the triangle CDB_3. On the basis of revealed preference theory we know that he will in fact choose his new consumption bundle on this triangle. If he chooses the bundle given by point *E* (there is no theoretical reason to rule this choice out), his consumption of X_1 and X_2 will be given by (X_1^1, X_2^1). It is easy to see that $(X_2^1/X_1^1) > (X_2^0/X_1^0)$ so that he consumes proportionally more of the standard good.[5] It is of course possible that he would choose some other point on CDB_3 such that the inequality goes the other way, but there is no theoretical reason which assures such choice.

Conclusion

It has been shown that the allegation stated in the first sentence of this paper cannot be defended on strictly theoretical grounds. It may be argued

[5] Since the analysis of the last section applies to homothetic functions (linear Engel curves), a similar result holds if a Hicksian income compensation is used.

that the phenomenon in question is an empirical reality and that the theory is being used to explain why the phenomenon occurs. In such a circumstance, however, the theory explains the opposite phenomenon equally well; and hence without prior knowledge of the magnitude of the relevant variables, the theory cannot predict which phenomenon will occur. Moreover, only casual empirical evidence is given in support of the assertion, and there are similar kinds of informal observations on the other side of the issue. How often is it heard, for example, that the way to get really good farm produce is to drive out to the country and buy it at a roadside stand or that one must go to Maine to get truly delectable lobsters?

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