

**Final Exam, Econ 210A, December, 2008**

Please use a blue book for your answers.

1) Firm A has production function

$$f(x_1, x_2) = \left(x_1^{1/2} + x_2^{1/2}\right)^{2k}$$

where  $k > 0$ .

a) For what values of  $k$  is  $f$  a quasi-concave function? For what values of  $k$  is  $f$  a concave function? Explain your answers.

Firm B has production function

$$g(x_1, x_2) = (x_1^2 + x_2^2)^{k/2}$$

where  $k > 0$ .

b) For what values of  $k$  is  $g$  a quasi-concave function? For what values of  $k$  is  $g$  a concave function? Explain your answers.

2) Firm A, described in Problem 1, is a price-taker in the factor market and must pay  $w_1$  per unit of factor 1 and  $w_2$  per unit of factor 2 that it uses.

a) Find its conditional input demand functions  $x_1(w_1, w_2, 1)$  and  $x_2(w_1, w_2, 1)$  for producing one unit of output.

b) Find its conditional input demand functions  $x_1(w_1, w_2, y)$  and  $x_2(w_1, w_2, y)$  for producing  $y$  units of output in the case where  $k = 1$ .

c) Find its conditional input demand functions  $x_1(w_1, w_2, y)$  and  $x_2(w_1, w_2, y)$  for producing  $y$  units of output for arbitrary  $k$ .

3) Firm B, described in Problem 1, is a price-taker in the factor market and must pay  $w_1$  per unit of factor 1 and  $w_2$  per unit of factor 2 that it uses.

a) Find its conditional input demand functions  $x_1(w_1, w_2, 1)$  and  $x_2(w_1, w_2, 1)$  for producing one unit of output.

b) Find its conditional input demand functions  $x_1(w_1, w_2, y)$  and  $x_2(w_1, w_2, y)$  for producing  $y$  units of output in the case where  $k = 1$ .

4) a) Find the cost function  $c(w_1, w_2, y)$  of Firm A for the case where  $k = 1$ .

b) Verify that Shephard's lemma is satisfied in the case of Firm A.

c) Find the cost function  $c(w_1, w_2, y)$  of Firm B for the case where  $k = 1$ .

5) a) There are two goods in the economy,  $X$  and  $Y$ . All consumers have identical utility functions of the form

$$U(x, y) = x + 2y^{1/2},$$

where  $x$  is the quantity of  $X$  consumed and  $y$  the quantity of  $Y$  consumed. There are  $n$  consumers and consumer  $i$  has income  $m_i$ . What is the demand function of consumer  $i$  for good  $Y$ ? For what prices does consumer  $i$  demand a positive amount of good  $Y$ ?

b) What is the aggregate demand for good  $Y$  at prices in the range for which all consumers demand positive amounts of  $Y$ ? What is the price elasticity of demand for good  $Y$  when all consumers are choosing positive consumptions of  $Y$ ?

c) Assume that all consumers demand positive amounts of good  $Y$  at all relevant prices. If  $Y$  is produced by a monopolist who has zero fixed costs and constant marginal costs of  $c$ , what price will the monopolist charge? How many units will he sell?

d) Suppose that the  $Y$  market is served by two Cournot duopolists, both of which have constant marginal costs of  $c$ . In Cournot equilibrium, what price will each charge and how many units will each sell.

6) There are three goods in the economy,  $X_0$ ,  $X_1$ , and  $X_2$ . Goods  $X_1$  and  $X_2$  are used by consumers as inputs into a “household production function” which produces a final consumer good  $Z$ . The household production function is

$$z = f(x_1, x_2) = \left( x_1^{1/2} + x_2^{1/2} \right)^2$$

where  $z$  is the amount of output of good  $Z$  and  $x_1$  and  $x_2$  are household inputs of goods 1 and 2. There are  $n$  consumers and each consumer has a utility function of the form

$$U(x_0, z) = x_0 + 2z^{1/2}.$$

The price of Good  $X_0$  is 1 dollar per unit. Consumers cannot buy  $Z$  but can buy goods  $X_1$  and  $X_2$  at prices  $p_1$  and  $p_2$  respectively and use these goods as inputs to produce  $Z$  in their households. Consumer  $i$  has income  $m_i$  dollars. Let  $x_0$  be the number of units of good  $X_0$  consumed and let  $y$  be the number of dollars spent on good  $Z$ . Then consumer  $i$ 's budget constraint is  $x_0 + y = m_i$ .

a) Suppose that the prices of goods  $X_1$  and  $X_2$  are  $p_1$  and  $p_2$ . If consumer  $i$  spends  $y$  dollars on these goods, how much good  $X_1$  should he buy? how much good  $X_2$ ?

b) Suppose we define  $v(p_1, p_2, y)$  to be the maximum amount of Good  $Z$  that a consumer can produce if the prices of  $X_1$  and  $X_2$  are  $p_1$  and  $p_2$  and if she spends a total of  $y$  dollars on goods  $X_1$  and  $X_2$ . Write down an explicit function for  $v(p_1, p_2, y)$ .

c) Given your solution to Part b), show how a consumer's utility maximization problem can be broken into two stages. In Stage A one decides how much money to spend on the inputs for good  $Z$ . In stage B one decides how to allocate this expenditures between goods  $X_1$  and  $X_2$ . Explain which stage you have to “solve first” and how you would proceed.

d) Carry out the procedure described in Part c) for the specific production function in this problem. (Hint: In part b) you found  $v(p_1, p_2, y)$  which was the total amount of  $Z$  that you could get if you spent  $y$  on the inputs for  $Z$ .) Assuming that the best choice is an interior solution, how much money  $y$  will a consumer with income  $m_i$  choose to spend on the inputs for Good  $z$ ? How much does he buy of each of the input goods? (Your answers will be functions of  $p_1$  and  $p_2$ .)

7) (extra credit—attempt only if you have done pretty well with problem 6.) Suppose that one monopolist controls the supply of  $X_1$  and another monopolist controls the supply of  $X_2$ . Suppose that both of these goods are produced at constant marginal cost  $c$ . Suppose that each assumes that the other's price is unaffected by his own choice of price. Set up the problem and say as much as you can about the solution.