

CES Problems, November 2007

- 1) There are two factors and the production function is

$$F(x_1, x_2) = \text{Min}\left\{\frac{x_1}{a_1}, \frac{x_2}{a_2}\right\}$$

- A) Draw the unit isoquant in the x_1, x_2 plane.
B) Calculate $x(w_1, w_2, 1)$ and $c(w_1, w_2, 1)$ where $x(w_1, w_2, 1) = (x_1(w_1, w_2, 1), x_2(w_1, w_2, 1))$ is the conditional factor demand for producing one unit and $c(w_1, w_2, 1)$ is the minimum cost of producing one unit.
C) Verify that Shepherd's lemma holds in this case.

- 2) There are two factors and the production function is

$$F(x_1, x_2) = \text{Max}\left\{\frac{x_1}{a_1}, \frac{x_2}{a_2}\right\}$$

- A) Draw the unit isoquant in the x_1, x_2 plane.
B) Find $x(w_1, w_2, 1)$ and $c(w_1, w_2, 1)$.
3) There are two factors and the production function is

$$F(x_1, x_2) = a_1x_1 + a_2x_2.$$

- A) Draw the unit isoquant in the x_1, x_2 plane.
B) Calculate $x(w_1, w_2, 1)$ and $c(w_1, w_2, 1)$.
4) There are two factors and the production function is

$$F(x_1, x_2) = x_1^{1/2}x_2^{1/2}.$$

- A) Calculate the ratio of the marginal products of factors 1 and 2.
B) Calculate $x(w_1, w_2, 1)$ and $c(w_1, w_2, 1)$.
C) Verify that Shepherd's lemma holds in this case.

- 5) There are two factors and the production function is

$$F(x_1, x_2) = \frac{1}{\frac{1}{x_1} + \frac{1}{x_2}}.$$

- A) Show that this production function belongs to the CES family. What is the elasticity of substitution in this case?
B) Calculate $x(w_1, w_2, 1)$ and $c(w_1, w_2, 1)$.

C) Suppose that a competitive firm faces the wages w_1, w_2 for its inputs. How does the ratio of “factor shares”

$$\frac{w_1 x_1(w_1, w_2, 1)}{w_2 x_2(w_1, w_2, 1)}$$

vary with the ratio w_1/w_2 ?

5) There are two factors and the production function is

$$F(x_1, x_2) = \left(x_1^{1/2} + x_2^{1/2}\right)^2.$$

A) Show that this production function belongs to the CES family. What is the elasticity of substitution in this case?

B) Calculate $x(w_1, w_2, 1)$ and $c(w_1, w_2, 1)$.

C) Suppose that a competitive firm faces the wages (w_1, w_2) for its inputs. How does the ratio of “factor shares”

$$\frac{w_1 x_1(w_1, w_2, 1)}{w_2 x_2(w_1, w_2, 1)}$$

vary with the ratio w_1/w_2 ?