Name ______________________________

Midterm Examination: Economics 210A

November 6, 2015

There are 3 questions. Use the attached blank pages to make calculations. For Questions 1 and 2, enter your answers and any clarifications on the front 3 pages.

Question 1.) A consumer has the utility function

$$u(x_1, x_2) = \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-k}$$

where $0 < k < 1$.

A) What is the elasticity of substitution of this function?

B) Find this consumer's Marshallian demand functions for Goods 1 and 2.

C) Find this consumer's indirect utility function.

D) Find this consumer's expenditure function.

Question 2.) A) A competitive firm has the production function

$$f(x_1, x_2) = \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-k}$$

where $0 < k < 1$. The price of its output is p and the firm faces factor prices (w_1, w_2). What is this firm's cost function.

B) Suppose that for this firm, $k = 1/2$. Suppose also that the firm faces factor prices $w_1 = 1$ and $w_2 = 4$ and the price it gets for its output is $p = 90$. What is its cost function? How many units should it produce to maximize its profits?

Question 3.) The function $f(x_1, \ldots, x_n)$ is continuously differentiable and homogeneous of degree k. Let

$$f_i(x_1, \ldots, x_n) = \frac{\partial f(x_1, \ldots, x_n)}{\partial x_i}$$

and let

$$f_{ij}(x_1, \ldots, x_n) = \frac{\partial^2 f(x_1, \ldots, x_n)}{\partial x_i \partial x_j}$$

A) Is the function $f_i(x_1, \ldots, x_n)$ homogeneous of some degree? If so, what degree. Prove your answer.

B) Are the functions $f_{ij}(x_1, \ldots, x_n)$ homogeneous of some degree? If so, what degree. Prove your answer

C) Let $G(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i f_i(x_1, \ldots, x_n)$. Given that f is homogeneous of degree k, what can we say in general about the ratio $G(x_1, \ldots, x_n)/f(x_1, \ldots, x_n)$. Prove your answer.