Midterm Examination: Economics 210A  
November 6, 2015

There are 3 questions. Use the attached blank pages to make calculations. For Questions 1 and 2, enter your answers and any clarifications on the front 3 pages.

Question 1.) A consumer has the utility function

$$u(x_1, x_2) = \left( \frac{1}{x_1} + \frac{1}{x_2} \right)^{-k}$$

where $0 < k < 1$.

A) What is the elasticity of substitution of this function?

B) Find this consumer’s Marshallian demand functions for Goods 1 and 2.

C) Find this consumer’s indirect utility function.

D) Find this consumer’s expenditure function.
Question 2.) A) A competitive firm has the production function

\[ f(x_1, x_2) = \left( \frac{1}{x_1} + \frac{1}{x_2} \right)^{-k} \]

where \(0 < k < 1\). The price of its output is \(p\) and the firm faces factor prices \((w_1, w_2)\). What is this firm’s cost function.

B) Suppose that for this firm, \(k = 1/2\). Suppose also that the firm faces factor prices \(w_1 = 1\) and \(w_2 = 4\) and the price it gets for its output is \(p = 90\). What is its cost function? How many units should it produce to maximize its profits?
Question 3. The function \( f(x_1, \ldots, x_n) \) is continuously differentiable and homogeneous of degree \( k \). Let
\[
f_i(x_1, \ldots, x_n) = \frac{\partial f(x_1, \ldots, x_n)}{\partial x_i}
\]
and let
\[
f_{ij}(x_1, \ldots, x_n) = \frac{\partial^2 f(x_1, \ldots, x_n)}{\partial x_i \partial x_j}
\]

A) Is the function \( f_i(x_1, \ldots, x_n) \) homogeneous of some degree? If so, what degree. Prove your answer.
B) Are the functions $f_{ij}(x_1, \ldots, x_n)$ homogeneous of some degree? If so, what degree. Prove your answer.
C) Let $G(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i f_i(x_1, \ldots, x_n)$. Given that $f$ is homogeneous of degree $k$, what can we say in general about the ratio $G(x_1, \ldots, x_n)/f(x_1, \ldots, x_n)$? Prove your answer.