Final Exam, Econ 210A, December, 2010

Answer all five questions.

Question 1.
A) Define the Arrow-Pratt measure of absolute risk aversion.

B) Consider the following von Neumann Morgenstern utility function

\[ u(x) = -\frac{1}{\alpha} e^{-\alpha x}. \]

For what values of \( \alpha \) is a consumer with this utility function risk-averse? Does this consumer display increasing, decreasing, or constant absolute risk aversion? Explain.
C) Consider the following von Neumann Morgenstern utility function

\[ u(x) = \frac{1}{\alpha} x^\alpha. \]

For what values of \( \alpha \) is a consumer with this utility function risk-averse? Does this consumer display increasing, decreasing, or constant absolute risk aversion? Does this consumer display increasing, decreasing, or constant relative risk aversion? Explain

D) Ulrich and Virgil have twice-differentiable von Neumann Morgenstern utility functions \( u(x) \) and \( v(x) \). Virgil’s utility function is given by \( v(x) = f(u(x)) \) where \( f(\cdot) \) is a strictly increasing and strictly concave function. Prove that Virgil’s is strictly more risk averse than Ulrich by the Arrow-Pratt measure of risk aversion.
Question 2.
A) Calculate the cost function and the conditional input demands for the production function

\[ f(x) = \left( \sum_{i=1}^{n} a_i x_i \right)^{1/2}. \]

B) Calculate the cost function and the conditional input demands for the production function

\[ f(x_1, x_2) = A(x_1 x_2)^k. \]

C) Calculate the cost function and the conditional input demands for the production function

\[ f(x_1, x_2, x_3) = \min\{\sqrt{x_1 x_2}, x_3\}. \]
Question 3.

Consider a pure exchange economy with two consumers, Alice and Bob and two goods, \( X \) and \( Y \). Alice is endowed with \( \omega_x \) units of \( X \) and no good \( Y \). Bob is endowed with \( \omega_y \) units of \( Y \) and no good \( X \). Alice’s utility function is

\[
U_A(x_A, y_A) = x_A + C \ln y_A
\]

where \( C > 0 \) and \( x_A \) and \( y_A \) are her consumptions of \( X \) and \( Y \). Bob’s utility function is

\[
U_B(x_B, y_B) = x_B y_B
\]

where \( x_B \) and \( y_B \) are his consumptions of \( X \) and \( Y \). Let \( X \) be the numeraire and let \( p \) be the price of good \( Y \).

A) Solve for Alice’s demand for \( X \) as a function of \( p \).

B) Solve for Bob’s demand for \( X \) as a function of \( p \).

C) Write an equation that says that demand for good 1 equals supply of good 1 and solve for the price \( \bar{p} \) at which demand equals supply for good 1. At this price, what do we know about the excess demand for good 2? Explain

D) How much good \( X \) does Alice consume in competitive equilibrium? How much good \( Y \) does Alice consume in equilibrium?
Question 4.
A) Define the elasticity of substitution between two factors in a production function.

B) Write down a production function that has two goods, constant returns to scale, is concave, and has elasticity of substitution 1/2.

C) If the production function has constant elasticity of substitution $\sigma$, what can you say about the corresponding cost function?

D) Suppose the price of factor 1 rises relative to that of factor 2. Consider a cost-minimizing firm that uses both factors. For what values (if any) of the elasticity of substitution determine does the ratio of the firm’s total expenditures on factor 1 to its total expenditures on factor 2 go up and for what values (if any) does it go down?
Question 5. Gordon Grasshopper consumes two goods $X$ and $Y$ and he will survive for three periods. Let $x_t$ and $y_t$ be his consumptions in period $t$ of good $X$ and $Y$ respectively. Gordon’s preferences over time paths of consumption is given by the intertemporal utility function

$$U(x_1, x_2, x_3, y_1, y_2, y_3) = (x_1 y_1)^{k/2} + \frac{1}{2} (x_2 y_2)^{k/2} + \frac{1}{4} (x_3 y_3)^{k/2}$$

where $0 < k < 1$. Gordon has an endowment of $W$ in the first period and will receive no income in the other periods. He can, however, save money for future consumption. The interest rate is zero.

A) Find the indirect utility function in the case where there are just two goods and one time period for a person who has the utility function

$$u(x, y) = (xy)^{k/2} \text{ where } k > 0.$$ 

B) Suppose that the prices of goods $x$ and $y$ are $p_x(1) = p_y(1) = 1$ in period 1, $p_x(2) = 1/4$ and $p_y(2) = 4$ in period 2, and $p_x(3) = 1/4$, $p_y(3) = 4$ in period 3. Use your answer from part A to help you to write an expression for Gordon’s intertemporal utility as a function of the amounts of money $m_1$, $m_2$, and $m_3$ that he spends in each period.

C) Suppose that $k = 2/3$. Use your answer to Part B and Gordon’s budget constraint to solve for the amount of money that Gordon will spend in each period.

D) How would Gordon allocate his expenditures if $k = 1$?