Name ____________________

Midterm Examination: Economics 210A

October, 1016

1) Annie consumes two goods A and B. Her preferences over commodity bundles (x_A, x_B) are represented by a constant elasticity of substitution utility function with elasticity of substitution 1/2 and with the property that if the price of good A equals the price of good B and if her income is positive, the best bundle that she can afford has twice as many units of good A as of good B.

A Write down an explicit utility function $U(x_A, x_B)$ that has these properties.

$$U(x_1, x_2) = \left(4x_1^{-1} + x_2^{-1}\right)^{-1}$$

B Describe the family of all utility functions with these properties.

All functions of the form

$$F\left(\left(4x_1^{-1} + x_2^{-1}\right)^{-1}\right)$$

where F is a strictly increasing function.

2) Hermoine consumes two goods A and B. Her utility function is

$$U(x_A, x_B) = \left(x_A^{-1} + x_B^{-1}\right)^{-1}.$$

A Find Hermoine's Marshallian demand functions for goods A and B. Then find her indirect utility function.

Her Marshallian demand functions are

$$x_A(p, m) = \frac{m}{p_A + (p_A p_B)^{1/2}}$$

and

$$x_B(p, m) = \frac{m}{p_B + (p_A p_B)^{1/2}}.$$

Her indirect utility function is

$$\begin{aligned} v(p,m) &= \left(\frac{p_A + 2(p_A p_B)^{1/2} + p_B}{m}\right)^{-1} \\ &= \frac{m}{\left(p_A^{1/2} + p_B^{1/2}\right)^2} \end{aligned}$$

B Find Hermoine's Hicksian demand functions for goods A and B, and also find her expenditure function.

Her Hicksian demands are

$$x_A^h(p,u) = \left(\frac{p_A^{1/2} + p_B^{1/2}}{p_A^{1/2}}\right) u$$

and

$$x_B^h(p,u) = \left(\frac{p_A^{1/2} + p_B^{1/2}}{p_B^{1/2}}\right) u$$

Her expenditure function is

$$\begin{aligned} e(p,u) &= \left(p_A x_A h(p,u) + p_B x_B^h(p,u)\right) u \\ &= \left(p_a + p_B + 2(p_A p_B)^{1/2}\right) u \\ &= \left(p_A^{1/2} + p_B^{1/2}\right)^2 u. \end{aligned}$$

C Show that Hermoine's expenditure function is a constant elasticity of substitution function of the prices. (This may take a bit of algebraic manipulation of the expression you found in Part B.) What are the exponents of the prices in the expression you found?

The expenditure function as written in the last expression of Part B is of the constant elasticity form where the exponents of the prices are 1/2.

3) A What assumptions on the utility function will guarantee that the corresponding expenditure function $e(p,u)$ takes the special form $e(p,u) = e^*(p)u$ for some function e^* and the indirect utility function takes takes the special form $v(p,m) = v^*(p)m$ for some function v^*? Briefly explain your answer.

Let the utility function be homogeneous of degree one, continuous and locally non-satiated.

If utility is homogeneous of degree one, it is also homothetic. If utility is homothetic, then it must be that the Marshallian demand function satisfies $x(p, m) = mx(p, 1)$. (You should be able to prove this by the way.) But this implies that $v(p, m) = u(mx(p, 1))$. Since u is homogeneous of degree 1, $u(mx(p, 1)) = mu(x(p, 1))$. Therefore we have $v(p, m) = mv^*(p)$, where we define $v^*(p)$ as $v^*(p) = u(x(p, 1))$.

We have the duality result that $u = v(p, e(p, u))$. Since $v(p, m) = v^*(p)m$, it follows that $u = v(p, e(p, u)) = v^*(p)e(p, u)$. Therefore it must be that $e(p, u) = \frac{u}{v^*(p)}$. Thus if we let $e^*(p) = \frac{1}{v^*(p)}$, we have $e(p, u) = e^*(p)u$.

B If the assumptions needed for Part A are satisfied, and if you know the expenditure function, how can you find the indirect utility function?

The duality result states that $e(p, v(p, m)) = m$. We have seen that the expenditure function must be of the form $e(p, u) = e^*(p)u$. So the duality result implies that $e^*(p)v(p, m) = m$. Dividing both sides of this equation by $e^*(p)$, we find that $v(p, m) = \frac{m}{e^*(p)}$.