

Name _____

Final Examination: Economics 210A
December, 2015

Answer Question 1 and any 5 of the remaining 6 questions.

1) The island nation of San Felicidad has N skilled workers and N unskilled workers. A skilled worker can earn $\$w_S$ per day if she works all the time and an unskilled worker can earn $\$w_U$ per day if he works all the time. A worker who can earn w per day and spends the fraction ℓ of his or her time NOT working will earn $(1 - \ell)w$ per day. There is a single consumption good, which is the *numeraire*. Preferences of all workers are represented by the utility function

$$U(c, \ell) = \ln c + \ln \ell \tag{1}$$

where c is the amount of the consumption good consumed per day and ℓ is the fraction of the consumer's time that is spent not working. Suppose that the government is able to recognize who is skilled and who is not skilled and collects a lump sum tax of t per day from each skilled worker and pays a lump sum subsidy of t per day to each unskilled worker.

A) Write down the daily budget constraint for a skilled worker and for an unskilled worker. If workers choose their labor effort so as to maximize the utility function in Equation 1, find the fraction of time that each type of worker will spend at leisure as a function of that type of worker's wage and the variable t .

B) A politician who opposes taxing the rich says that “Redistributive taxes harm the economy. An increase in such taxes will reduce the total supply of labor.” A politician who favors taxing the rich says that “Tax increases have no effect on the total value of labor earnings and will actually increase the “average productivity of labor”. To evaluate the claims of these politicians, do the following for the economy described in this problem: (i) Show how the total number of hours worked by San Felicidad workers of each type changes as t changes. (Remember that skilled workers pay a tax of t and unskilled workers get a subsidy of t .) As t increases, does the total number of hours worked by all workers increase, decrease or stay constant? (ii) Show how the total labor earnings of each type of worker changes as t changes. As t increases, does the total labor earnings of all laborers in the country increase, decrease or stay constant? (iii) Determine whether an increase in t increases, decreases, or leaves unchanged, the “average productivity of labor” as measured by the ratio of total labor earnings to total hours worked by San Felicidad workers.

C) Find the indirect utility function $v(w, \tau)$ where $v(w, \tau)$ is the utility of a worker with utility function given by Equation 1 who is paid a wage of w per day and who pays a tax of τ to the government, where τ could be either positive or negative and where $\tau > 0$ means a tax and $\tau < 0$ means a subsidy.

D) Suppose that young people in San Felicidad don't know whether they will be skilled or unskilled. Each of them thinks the probability of either outcome is $1/2$. Suppose that they are expected utility maximizers with von Neumann-Morgenstern utility functions given by Equation 1. If the government collects a tax of t from each skilled worker and pays a subsidy of t to each unskilled worker, write down an expression for the expected utility of a young person. Find the tax rate t that maximizes the expected utility of such young people.

2 A consumer has utility function

$$U(x_1, x_2) = \left(x_1^{1/2} + x_2^{1/2}\right)^2. \quad (2)$$

A) What is this consumer's elasticity of substitution between the two goods? Find this consumer's Marshallian demand function for each of the two goods. Then find a simple expression for the fraction of her income that she spends on each good.

B) Find this consumer's indirect utility function.

3 Consider a pure exchange economy with 200 consumers. All of them have the utility function given in Equation 2 of Question 2. There are 100 Type A consumers who have initial endowment of 90 units of good 1 and no units of good 2. There are 100 Type B consumers who have initial endowment of 10 units of good 2 and no units of good 1. Let Good 1 be the numeraire. Find the competitive equilibrium price of good 2 for this economy. Find the competitive equilibrium consumption of each good for each type of consumer. (Hint: Finding the competitive equilibrium price requires only a very simple calculation.)

4 **A)** If a decision-maker has constant absolute risk aversion, what functional form must that person's von Neumann Morgenstern utility function take? Show that your answer is correct.

B) Suppose that a decision-maker has constant absolute risk aversion and initial wealth w and prefers not to take between a gamble such that with probability $1/2$ he would win $\$x$ and with probability $1/2$ he would lose $\$y$. Is it possible that if this individual's wealth were much greater, he would want to accept the gamble? Prove your answer.

5 State Walras' Law. Provide some assumptions under which Walras' Law holds and prove that Walras' Law holds under these assumptions.

6 A) State (i) the Weak axiom of revealed preference, (ii) the Strong axiom of revealed preference and (iii) the generalized axiom of revealed preference. (*Be sure to define “is revealed preferred to.”*)

B) Show that a consumer who makes choices that maximize a continuous, transitive, strictly convex preference relation must satisfy the strong axiom of revealed preference.

7) Consider a pure exchange economy with two consumers and two goods. Consumer 1 has an initial endowment of ω^x units of good x and 0 units of good y . Consumer 2 has an initial endowment of ω^y units of good y and 0 units of good x . Consumer 1's utility function is $U(x_1, y_1) = \min\{x_1, y_1\}$ and consumer 2's utility function is $U(x_2, y_2) = x_2 y_2$, where x_i and y_i are consumptions of goods x and y respectively by consumer i .

A) Draw an Edgeworth box representation of this economy.

B) Suppose that $y_0 < 2x_0$. Where good 1 is the numeraire, find the competitive equilibrium price for good 2. Also find the competitive equilibrium quantities consumed by each consumer in competitive equilibrium.

For Extra credit—only if you have extra time. What can you say about competitive equilibrium for this economy if $y_0 > 2x_0$.