

Name _____

Final Exam, Economics 210A, December 2014

Answer any 7 of these 8 questions Good luck!

1) For each of the following statements, state whether it is true or false. If it is true, prove that it is true. If it is false, show that is false by providing a counterexample. Your answers should include a definition of each of the technical terms used.

A) If preferences are strictly monotonic, they must be locally non-satiated.

B) If preferences are locally non-satiated, they must be strictly monotonic.

C) If preferences are transitive, convex, and continuous, they must be locally non-satiated.

D) If preferences are non-satiated and strictly convex, they are locally non-satiated.

2) Consider a consumer with preference relation \succeq over an n -dimensional commodity space. This consumer has a utility function $u(x)$ and a Marshallian demand correspondence $X(p, m)$. Let $e(p, u)$ be the expenditure function for this consumer.

A) Let $x \in X(p, m)$. If the preference relation \succeq is locally non-satiated, prove that $px = m$.

B) Give an example where \succeq is not locally non-satiated and where $x \in X(p, m)$ and $px < m$.

C) What assumptions do you need in order to prove that if $x \in X(p, m)$, then $e(p, u(x)) \leq px$? Give a proof, using whatever assumptions are needed.

D) Prove that if $x \in X(p, u)$, then $e(p, u(x)) = px$. Use whatever assumptions are needed and show that it takes stronger assumptions to prove this result than to prove the result in Part C.

3) A consumer consumes two goods. For each of the following utility functions, find this consumer's indirect utility function and this consumer's expenditure function.

A) $U(x_1, x_2) = x_1 + x_2$.

B) $U(x_1, x_2) = \min\{x_1, x_2\}$.

C) $U(x_1, x_2) = x_1 + \ln x_2$.

D) $U(x_1, x_2) = x_1 x_2$

4) A consumer consumes two goods. For each of the following utility functions, find this consumer's indirect utility function and this consumer's expenditure function.

A) $U(x_1, x_2) = \ln(x_1 + x_2)$

B) $U(x_1, x_2) = \ln(\min\{x_1, x_2\})$

C) $U(x_1, x_2) = \ln(x_1 + \ln x_2)$

5) A consumer consumes two goods. His preferences over lotteries of consumption bundles are represented by a von Neumann Morgenstern expected utility function.

A) Suppose that this consumer's preferences over lotteries of consumption bundles are represented by the expected value of $U(x_1, x_2) = \ln(x_1 + x_2)$. If this consumer's income is sure to be m , would she prefer that the price vector be $(2, 2)$ with certainty, or that the price vector will be $(1, 3)$ with probability $1/2$ and $(3, 1)$ with probability $1/2$ or would she be indifferent? Explain your answer.

B) Suppose that this consumer's preferences over lotteries of consumption bundles are represented by the expected value of $U(x_1, x_2) = \ln(\min\{x_1, x_2\})$. If this consumer's income is sure to be m , would she prefer that the price vector be $(2, 2)$ with certainty, or that the price vector will be $(1, 3)$ with probability $1/2$ and $(3, 1)$ with probability $1/2$ or would she be indifferent? Explain your answer.

6) Part A) Consider a pure exchange economy with two goods and 100 Type A consumers and 100 Type B consumers.

Type A's have preferences represented by the utility function

$$U^B(x_1, x_2) = x_1 x_2.$$

Each type A has an initial endowments of 0 units of good 1 and 4 units of good 2. Type B's have preferences represented by the utility function

$$U^A(x_1, x_2) = x_1 + 2x_2^{1/2}.$$

Each type B has an initial endowment of 5 units of good 1 and 3 units of good 2. Let good 1 be the numeraire. Find a competitive equilibrium price for good 2. Also find the equilibrium consumption of each good for each consumer.

Part B) Consider a pure exchange economy with two goods and 100 Type A consumers and 100 Type B consumers. Type A's have preferences represented by the utility function

$$U^A(x_1, x_2) = x_1 + 2x_2^{1/2}.$$

Each type A has an initial endowments of 0 units of good 1 and 4 units of good 2. Type B's have preferences represented by the utility function

$$U^B(x_1, x_2) = x_1 + x_2^{1/2}.$$

Each type B has an initial endowment of 6 units of good 1 and 2 units of good 2. Let good 1 be the numeraire. Find a competitive equilibrium price for good 2. Also find the equilibrium consumption of each good for each consumer.

7) A poor fellow finds a lottery ticket that pays X ducats with probability p and 0 with probability $(1 - p)$. This poor fellow has initial wealth of W_0 ducats, which he will be able to keep regardless of what the lottery ticket pays. Suppose that the poor fellow is an expected utility maximizer with von Neumann Morgenstern utility function

$$U(W) = \frac{1}{\gamma} W^\gamma$$

where W is his wealth.

A) Write down the certainty equivalent for the gamble that he faces if he keeps the lottery ticket. (This will be a function of X , p , and W_0 .)

B) For someone with the risk preferences of this poor fellow, calculate the Arrow Pratt measure of absolute risk aversion, as a function of his wealth. Also calculate his measure of relative risk aversion.

C) If $X = 100$, $W_0 = 100$, and $p = 1/4$, what is the smallest price at which the poor fellow would be willing to sell this lottery ticket.

D) Suppose that a richer guy has initial wealth of 1500 ducats and the same preferences as the poor fellow, if $X = 100$ and $p = 1/4$, what is the most that this richer guy would offer for this lottery ticket?

8 Peter Habit consumes two goods, apples and bananas. He will live for two periods. At the beginning of period 1, he has wealth W and he will not receive any more income other than interest on savings. Let a_t denote his consumption of apples in period t and b_t his consumption of bananas in year t . The price of apples is p_a in both periods and the price of bananas is p_b in both periods. At the beginning of period 1, his preferences over lifetime consumption streams are represented by the utility function

$$U(a_1, b_1, a_2, b_2) = \ln a_1 + \ln b_1 + \beta (\ln a_2 + \ln b_2) .$$

If Peter spends less than W in period 1, he will receive interest on his savings at the interest rate r .

A) If $\beta = 1/(1+r)$, how much money will he save and how much of each good will he consume in each period?

B) Peter discovers that apples are addictive in the sense that the more apples you consume in period 1, the more apples it takes in period 2 to give you the same “apple rush.” To simplify calculations, let us assume that $\beta = 1$, $r = 0$, and $p_a = p_b = 1$. Taking the addictive effect of apples into account, his preferences are represented by the utility function

$$U(a_1, b_1, a_2, b_2) = \ln a_1 + \ln b_1 + \ln(a_2 - \lambda a_1) + \ln b_2$$

where $\lambda > 0$. Suppose Peter makes his decisions about saving and consumption so as to maximize this utility subject to his budget constraint. Compare Peter’s apple consumption in periods 1 and 2 when $\lambda > 0$ to the case of no addiction, where $\lambda = 0$. Does his total apple consumption increase or decrease as λ increases? Does his savings increase or decrease as λ increases?

C) (For extra credit if time allows:) Construct some alternative models of addiction or habit. For example: What if consuming more apples today makes you like bananas less tomorrow? What if experience with consuming apples in the past makes them taste better in the future? Or suggest any other effect that seems interesting.