Problems with Uncertainty
UCSB Intro to Microeconomic Theory

1. Dilbert is an expected utility maximizer who cares about his consumption and about the state of his health. There are two possible events, either he gets sick or he doesn't. The probability that he gets sick is represented by Π. Let C_s be his consumption contingent on the event that he gets sick and C_w be his consumption contingent on not getting sick. He has an amount of money M that is available to spend on consumption goods and he can buy actuarially fair insurance, so that he can afford any combination (C_s, C_w) such that

$$\Pi C_s + (1 - \Pi)C_w = M.$$

His has a von Neuman-Morganstern utility function of the following form:

$$U(\Pi, C_s, C_w) = \Pi(u(C_s) - \alpha) + (1 - \Pi)u(C_w)$$

where $\alpha > 0$ and $u(\cdot)$ is an increasing function that is twice continuously differentiable and strictly concave. Dilbert has learned that by spending money on health care (which does not count as consumption in his utility), he can alter the probability of his getting sick. He realizes that the insurance companies will observe his new probability and will offer actuarially fair insurance whatever his probability of getting sick.

Dilbert calculates an "indirect utility function" $V(\Pi, M)$ which is the maximum expected utility that he can achieve if he has M to spend on contingent consumption and if his probability of getting sick is Π.

(i) Describe the constrained maximization problem that Dilbert needs to solve in order to determine the function V. What can you conclude about the relative amounts that he would consume contingent on being sick or well? (Prove your answer.) Your answer to the last question will allow you to write a simple expression for $V(\Pi, M)$. What is this expression?

(ii) If $u(C) = \ln C$ for all positive values of C, then write an explicit expression for Dilbert's marginal rate of substitution between Π and M (as a function of M and/or Π.)

(iii) Suppose that Dilbert has a total of W to allocate between expenditures on contingent consumption M and expenditures on health care H and that his probability of getting sick is given by a function $\Pi = g(H)$ where

$g(H) = 1/(1+H)$. Discuss first and second order conditions for an efficient choice of H. (Hint: The solution for Dilbert's best choice of H should be the solution to a quadratic equation.

2. Suppose that all is as in Problem 1, except that Dilbert's von Morgenstern utility function is of the form:

$$U(\Pi, C_s, C_w) = \Pi(u(C_s - \alpha) + (1 - \Pi)u(C_w).$$

Answer parts (i) and (ii) of Question 1 in this case.