Three Problems from Midterm to be done as Homework

5. Many handsome young fellows would like to marry Barbie, a beautiful princess. Barbie doesn’t care which one she marries as long as he is rich. But Barbie is not able to check her suitors’ bank balances. She can announce that she will surely marry a fellow if he spends $c$ on an ostentatious, but useless, gift for her. If a rich fellow gives Barbie a gift that costs $c$, she will marry him and his utility will be $1000 - c/2$. If a poor fellow gives Barbie a gift that costs $c$, she will marry him and his utility will be $1000 - c$. The utility of either type of fellow if he doesn’t offer Barbie a gift and she doesn’t marry him will be 0. If Barbie knew that a fellow was rich, she would rather marry him and not have him waste money on the ostentatious gift. Her utility for marrying a rich fellow who gives her a gift that costs $c$ is $2000 - c$ and her utility for marrying a poor fellow who gives her a gift that costs $c$ is $500 - c$.

A) Barbie gets to choose the cost $c$ of the ostentatious gift that she demands. What values of $c$ would result in a separating equilibrium and what values of $c$ would result in a pooling equilibrium?

B) Before any suitor gives a gift, Barbie believes that the probability is $p$ that a handsome young fellow who would like to marry her is rich and the probability is $1 - p$ that he is poor. For what values of $p$ would she get a higher expected payoff from choosing $c = 0$ than from any other choice of $c$?

6. Employee Cog can either work hard or loaf on the job. If he works hard, his output will be worth $16$ to his employer. If he loafs, his output will be worth $5$ to his employer. His employer can either monitor Cog’s work or not. Mr. Cog gets a wage of $10$ if he is not caught loafing and a wage of $0$ if he is caught loafing. The employer will only catch Cog loafing if the employer monitors his effort. If the employer does monitor and Mr. Cog is loafing, he will be caught. It costs the employer $1$ to monitor Mr. Cog’s effort, regardless of whether he is working or loafing. If the employer does not monitor Mr. Cog’s work, the employer’s profit is equal to the total value of Cog’s output minus his wage and if the employer monitors Cog’s work, his profit is $1$ less than the value of Cog’s output minus his wage.

A) Mr. Cog is an expected utility maximizer. Where $w$ is his wage, his utility is $w$ if he works hard and $w + 2$ if he loafs. Is there a pure strategy Nash equilibrium for the game between Mr. Cog and his employer? If so, what is it? If not, show that there is no pure strategy Nash equilibrium.

B) Is there a mixed strategy Nash equilibrium in which Mr. Cog and his employer both randomize their actions. If so, what probabilities do they use in mixed strategy Nash equilibrium? In this equilibrium, what are Mr. Cog’s expected profits?
7. All is the same as in Problem 6, except that the employer believes that Mr. Cog might be one of two types. With probability $p$ he will be a workaholic, in which case if he works hard and is paid $w$, his utility will be $w + 1$, and if he loaf s and is paid $w$, his utility will be $w - 1$. With probability $1 - p$ he will be a lazy guy with utility as described in Problem 6.

A) For some values of $p$, there will be a pure strategy Nash equilibrium. Describe this pure strategy equilibrium and find the values of $p$ for which this is an equilibrium?

B) Suppose that $p = 1/2$. Find a mixed strategy Nash equilibrium. In this mixed strategy equilibrium, what is the probability that the employer monitors? what is the probability that the workaholics loaf? What is the probability that the lazy guys loaf?