A) The only pure strategies for Bob are Go to A and Go to B.

B) In a Bayes-Nash equilibrium for Alice and Bob, it must be that given Bob’s strategy, Alice, whichever type she turns out to be, will choose a best response to Bob’s action. Therefore if Bob goes to B, Alice will go to B if she loves him and A if she scorns him. If there is a Bayes-Nash equilibrium in which Bob goes to B, it must be that going to B maximizes Bob’s expected payoff if Alice goes to B if she loves Bob and to A if she scorns him. Suppose the probability that Alice loves Bob is $p$ and the probability that she scorns him is $1 - p$. Then if Alice goes to B if she loves Bob and A if she scorns him, Bob’s expected payoff from going to B is $3p + 1(1 - p) = 2p - 1$ and his expected payoff from going to A would be $2(1 - p) + p0 = 2 - 2p$. Going to B is Bob’s best response for Bob only if $2p + 1 \geq 2 - 2p$ which is the case only if $p \geq 1/4$. Therefore if $p < 1/4$, there will not be a Bayes-Nash equilibrium in which Bob is certain to go to B.

C) Suppose that $p < 1/4$ and Bob goes to A. Then Alice will go to A if she loves Bob and B if she scorns him. In this case, Bob’s expected payoff from going to A will be $2p + p0 = 2p$ and his expected payoff from going to B will be $3(1 - p) + 1p = 3 - 2p$. Going to A will be his best response to Alice’s behavior only if $2p > 3 - 2p$ which is the case only if $p > 3/4$. If $p > 1/4$, then certainly $p < 3/4$ and so there can’t be a Bayes-Nash equilibrium in which Bob is certain to go to A.

D) To find a mixed strategy equilibrium for the case where $p < 1/4$, let us first find a mixed strategy for Bob such that at least one of the Alice types will be indifferent about the two movies and hence willing to use a mixed strategy. Suppose that Bob goes to Movie A with probability $q$ and to B with probability $1 - q$. Let us find the $q$ such that if Alice scorns him, she will be indifferent about which movie to go to. her expected payoff from going to A will be $1q + 3(1 - q)$ and her expected payoff from going to B will be $2q + 0(1 - q)$. In this case, she would be indifferent between the two pure strategies if $q + 3(1 - q) = 2q$, which implies that $q = 3/4$.

Suppose that $q = 3/4$. Let’s check out what Alice would do if she loves Bob. If she loves Bob, her expected payoff from going to A would be $3q + 1(1 - q) = 2q + 1$ and her utility from going to B would be $2(1 - q) + 0q = 2 - 2q$. This means that her expected payoff from going to A would be greater than that from going to B whenever $2q + 1 > 2 - 2q$ or equivalently whenever $q = 1/4$.

So we see that if Bob randomizes with probability $q = 3/4$ of going to A, then if Alice loves Bob, she will go to A and if she scorns him, she will be indifferent between going to the two movies.

Let us see if we can find an equilibrium in which Alice uses a mixed strategy if she scorns Bob and a pure strategy if she loves Bob such that when Alice behaves in this way, Bob will be indifferent between going to A and going to B and hence will be willing to use a mixed strategy.
Suppose that Bob believes that the probability that Alice will be at movie A if \( q_A \). Then his expected payoff from going to A is \( 2q_A + 0(1 - q_A) = 2q_A \) and his expected payoff from going to B is \( 1q_A + 3(1 - q_A) = 3 - 2q_A \). Bob will be indifferent between the two movies if \( 2q_A = 3 - 2q_A \), which happens if \( q_A = 3/4 \).

Suppose that if Alice scorns Bob, she will go to movie A with probability \( q_s \) and that if she loves Bob, she will certainly go to movie A. Then Bob will believe that the probability that Alice loves him and goes to A will be \( p \) and the probability that she scorns him and goes to A will be \( (1 - p)q_s \). So he believes the probability that Alice will go to movie A is \( q_A = p + (1 - p)q_s \). Therefore Bob will be indifferent between which of the two movies to go to if \( p + (1 - p)q_s = 3/4 \). This will be the case when

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q_s = \frac{3 - p}{1 - p}.
\]

Thus when \( p < 1/4 \), there will be a mixed strategy Bayes-Nash equilibrium in which Bob goes to movie A with probability 3/4, Alice goes to movie A if she loves Bob, and Alice will use a mixed strategy in which she goes to movie A with probability

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\frac{3 - p}{1 - p}
\]

if she scorns Bob. For example, if the probability that Alice loves Bob is 1/8, we have a mixed strategy Nash equilibrium where Bob goes to movie A with probability 3/4, Alice goes to movie A if she loves Bob and Alice goes to movie A with probability 5/7 if she scorns Bob.

A Footnote: We can also check whether there is a mixed strategy Bayes-Nash equilibrium in which Bob uses a mixed strategy and Alice uses a mixed strategy if she loves Bob. If Bob goes to movie A with probability \( q \) and Alice loves Bob, she will be indifferent about which movie to go to if \( q = 1/4 \). If \( q = 1/4 \), then if Alice scorns Bob, she will go to movie A with certainty. We found that Bob would be indifferent between the two movies only if the probability that Alice goes to A is 3/4. If Alice goes to A for sure if she scorns Bob and goes to A with probability \( q_L \) if she loves him, then the probability that Alice will be at A is \( pq_L + (1 - p) \). At a mixed strategy equilibrium we must have \( pq_L + 1 - p = 3/4 \) or equivalently \( q_L = 1 - \frac{3}{4p} \). We see that in this case, \( 0 < q_L, 1 \) only if \( 4p > 1 \) or equivalently \( p > 1/4 \). So there is not such an equilibrium in the case where \( p < 1/4 \).