1. Charlie and Donna each own a hot dog cart. Charlie makes a $1 profit for every hot dog that he sells and Donna makes a $2 profit for every hot dog that she sells. Each of them can take his or her cart either to the beach or to the airport. If one of them goes to the airport, and the other to the beach, the one at the airport will sell 40 hot dogs and the one at the beach will sell 100 hot dogs. If they both go to the airport, they will each sell 20 hot dogs. If they both go to the beach, they will each sell 50 hot dogs. Charlie moves first and Donna decides where to go, knowing what Charlie has decided to do.

A) What are the possible strategies for Charlie? What are the possible strategies for Donna?

Charlie’s possible strategies are:
- Go to the Airport
- Go to the Beach

Donna’s strategies can depend on Charlie’s strategy. There are four possibilities.
- Go to the Airport no matter what Charlie does
- Go to the Airport if Charlie goes to the airport and go to the beach if Charlie goes to the beach.
- Go to the beach if Charlie goes to the airport and go to the airport if Charlie goes to the beach.
- Go to the beach, no matter what Charlie does

B) Describe this game in extensive form and in strategic form.

Drawing the game in extensive form is straightforward.

Here is the game in strategic form. Charlie’s strategies are the rows. Donna’s are the column’s. x/y means do x if Charlie goes to the airport and y if Charlie goes to the beach.

<table>
<thead>
<tr>
<th></th>
<th>A/A</th>
<th>A/B</th>
<th>B/A</th>
<th>B/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airport</td>
<td>20,40</td>
<td>20,40</td>
<td>40,200</td>
<td>40,200</td>
</tr>
<tr>
<td>Beach</td>
<td>100,80</td>
<td>50,100</td>
<td>100,80</td>
<td>50,100</td>
</tr>
</tbody>
</table>

C) Find all of the Nash equilibria and describe the strategies used by Charlie and by Donna in Nash equilibrium. Find the subgame perfect Nash equilibrium (or equilibria if there is more than one). What strategy does each use in the subgame perfect Nash equilibrium (or equilibria) that you found? What are the expected payoffs to each of them in the subgame perfect Nash equilibrium?

There are two Nash equilibria. In both of them, Charlie goes to the beach. In one of them Donna uses the strategy Go to the beach no matter what Charlie
does. In the other, Charlie goes to the beach and Donna uses the strategy “Go to the beach if Charlie goes to the beach and go to the airport if Charlie goes to the airport.”

In the subgame perfect Nash equilibrium, Donna uses the strategy “Go to the beach no matter what Charlie does” and Charlie uses the strategy “Go to the beach”. Payoffs are 50 for Charlie and 100 for Donna.

2. The setup is the same as in Problem 1 except that before Charlie decides where to go, Donna has the option of offering him a bribe of $15 if he will take his cart to the airport instead of the beach. If Charlie accepts the bribe, he will take his cart to the airport. Describe this game in extensive form. Find a subgame perfect equilibrium. What are the strategies used by Charlie and by Donna in this equilibrium?

Most people did this one right. A few people did not include Donna’s choice whether to bribe and Charlie’s decision to accept or reject the bribe in the game tree.

In the subgame perfect Nash equilibrium, Donna offers the bribe. Whether or not Charlie accepts the bribe, and no matter where he chooses to go, Donna will go to beach. Charlie will accept the bribe and go to the airport. To show that this is a SPNE, you need to show that if the bribe is offered then if Charlie accepts it, he will get a higher payoff in the subsequent subgame than in the subgame that begins where he rejects it. You also need to show that in SPNE, Donna’s payoff is higher if she offers the bribe than if she does not.

3. The setup is the same as in Problem 1 except that the number of hot dogs that can be sold at the beach will be 100 only if the day is sunny and will be 0 if the day is rainy. If just one goes on a sunny day, that person will sell 100 hot dogs. But if it is rainy, nobody will be able to sell any hot dogs at the beach. At the beginning of the day, Charlie believes that the probability is 1/2 that it will be sunny and 1/2 that it will be rainy. Both know that Donna will hear an accurate weather report about whether it will rain before she must decide whether to go to the airport or the beach. Charlie must decide where to go before hearing the weather report and without seeing where Donna goes. Donna will choose the airport or the beach after seeing where Charlie went and after she has heard the weather report. Describe this game in extensive form. Find a Bayes Nash subgame perfect equilibrium. What are the strategies used by Charlie and by Donna in this equilibrium? What are the expected payoffs to each of them in equilibrium?

In equilibrium for this game, it turns out that Donna will go to the airport if it is rainy and the beach if it is sunny, no matter what Charlie does. Charlie doesn’t know whether it will rain or shine when he has to decide where to go. Given Donna’s strategy and given that the probability of rain is 1/2, Charlie’s expected payoff is higher if he goes to the airport.
4. Archie and Bella each rent half of a duplex. The furnace is broken down and won’t be repaired unless somebody contacts the manager to get it fixed. Contacting the manager is difficult because he lives several miles away and doesn’t have a telephone. If at least one of them contacts the manager, he will fix the furnace. Each of them has to decide whether to contact the manager, without knowing whether the other is going to. The cost to either Archie or Bella of contacting the manager is $20. If both contact the manager, it will still cost $20 to each of them. Each of them places a value of $100 on getting the furnace fixed.

A) Write a strategic form payoff matrix for the game played between Archie and Bella. What are the pure strategy Nash equilibria of this game?

There are two pure strategy Nash equilibria. One in which Archie contacts the manager and Bella doesn’t and one in which Bella contacts the manager and Archie doesn’t.

B) Find the symmetric mixed strategy equilibrium for this game. In a symmetric mixed strategy equilibrium, what is the probability that exactly one of them contacts the manager? What is the probability that both of them contact the manager? In this equilibrium, what is the probability that the furnace gets fixed?

In a symmetric mixed strategy equilibrium, each of them will contact the manager with the same probability $p$. In such an equilibrium, each must be indifferent between the two pure strategies. The expected payoff to Archie from contacting the manager is sure to be $100 - 20 = 80$, since no matter what Bella does, the manager will be contacted and Archie will pay the cost of 20. The expected payoff to Archie from not contacting the manager is $p \times 100$, since he will have no costs and will get the benefit only if Bella contacts the manager and she does that with probability $p$. So in a mixed strategy equilibrium the probability that $p$ that Bella contacts the manager must satisfy the equation $80 = 100p$ or $p = 8/10$. Similarly, Bella will be indifferent between contacting and not contacting only if the probability that Archie contacts the manager is $8/10 = 4/5$. So there is a symmetric mixed strategy equilibrium where each contacts the manager with probability $4/5$. The probability that both contact the manager is $4/5 \times 4/5 = 16/25$. The probability that Archie contacts the manager and Bella doesn’t is $4/5 \times 1/5 = 4/25$ and the probability that Bella contacts the manager and Archie doesn’t is also $4/25$. So the probability that exactly one contacts the manager is $8/25$. The probability that neither contacts the manager is $1/5 \times 1/5 = 1/25$. The probability that the furnace gets fixed is $1 - 1/25 = 24/25 = 0.96$.

C) Suppose that Archie, Bella, and Clarissa live in a “triplex” with three apartments and the furnace breaks down. If at least one of the residents contacts the manager, the furnace will be fixed. Getting the furnace fixed is worth $100 to each resident. Contacting the manager costs $20 to anybody who does
so. The manager will fix the furnace if and only if at least one of the residents contacts him. Each resident has to decide whether to contact the manager without knowing what the others will do. Find the symmetric Nash equilibrium in mixed strategies. In this equilibrium, what is the probability that the furnace gets fixed?

In a symmetric mixed strategy Nash equilibrium, each has the same probability $p$ of contacting the manager and each is indifferent between contacting the manager or not. Where everyone contacts the manager with independent probability $p$, the payoff to any player from contacting the manager is certain to be $100 - 20 = 80$, since in this case the manager will certainly be contacted and the person who contacted the manager will certainly have a cost of 20. If a person does not contact the manager, he or she will not have to pay the cost of 20, and the probability that the manager is contacted is the probability that at least one of the other two residents contact the manager. The probability that neither of the other two contact the manager is equal to $(1 - p)^2$. So the probability that at least one of them contacts the manager is $1 - (1 - p)^2$. Therefore the expected payoff to the strategy of not contacting the manager is $100 (1 - (1 - p)^2)$. In Nash equilibrium it has to be that

$$80 = 100 (1 - (1 - p)^2).$$

Rearranging terms, we have

$$\frac{80}{100} = (1 - (1 - p)^2)$$

which implies that

$$(1 - p)^2 = \frac{2}{10}$$

and hence that

$$p = 1 - \sqrt{2/10}.$$

Since each contacts the manager with probability of $p = 1 - (2/10)^{1/2}$, the probability that none of the three contacts the manager is

$$(1 - p)^3 = (2/10)^{3/2}.$$

Notice that this is a special case of the example discussed in the textbook and in class lectures where there were $n$ people who could “contact the manager.”

5. Many handsome young fellows would like to marry Barbie, a beautiful princess. Barbie doesn’t care which one she marries as long as he is rich. But Barbie is not able to check her suitors’ bank balances. She can announce that she will surely marry a fellow if he spends $c$ on an ostentatious, but useless, gift for her. If a rich fellow gives Barbie a gift that costs $c$, she will marry him and his utility will be $1000 - c/2$. If a poor fellow gives Barbie a gift that costs $c$, she will marry him and his utility will be $1000 - c$. The utility of either type of
fellow if he doesn’t offer Barbie a gift and she doesn’t marry him will be 0. If Barbie knew that a fellow was rich, she would rather marry him and not have him waste money on the ostentatious gift. Her utility for marrying a rich fellow who gives her a gift that costs $c$ is $2000 - c$ and her utility for marrying a poor fellow who gives her a gift that costs $c$ is $500 - c$.

A) Barbie gets to choose the cost $c$ of the ostentatious gift that she demands. What values of $c$ would result in a separating equilibrium and what values of $c$ would result in a pooling equilibrium?

To get a separating equilibrium, we need $c$ to be big enough so that a poor fellow would not be willing to give her such a big gift in exchange for marriage, but small enough so a rich fellow would be willing to pay this price for a marriage. A poor fellow will choose to give her the gift and marry her if $c < 1000$ and will choose not to do so if $c > 1000$. A rich fellow would choose to give her the gift in order to marry her if $1000 - c/2 > 0$, which means that $c < 2000$. So there will be a separating equilibrium for any $c$ such that $1000 < c < 2000$. If $c < 1000$ there will be a pooling equilibrium where either type would give her the gift and marry her. If $c > 2000$ there will be a pooling equilibrium where nobody would give her the gift and marry her.

B) Before any suitor gives a gift, Barbie believes that the probability is $p$ that a handsome young fellow who would like to marry her is rich and the probability is $1 - p$ that he is poor. For what values of $p$ would she get a higher expected payoff from choosing $c = 0$ than from any other choice of $c$?

If she sets $c = 0$, she will get a rich fellow with probability $p$ and a poor fellow with probability $1 - p$. Her utility will be $2000$ if she gets a rich fellow and $500$ if she gets a poor fellow, so her expected utility will be $2000p + 500(1 - p) = 500 + 1500p$. What is the best she can do with $c > 0$? We see that her utility from either kind of husband is lower the greater $c$ is. But the advantage for Barbie of setting a high enough $c$ is that she can be assured of a rich husband. It is pretty clear that the best she can do with a positive $c$ is to set $c$ just barely large enough to get a separating outcome. She can accomplish this by setting $c$ just a tiny bit above 1000. If she does this, she gets a rich husband for sure and her utility will be $2000 - c$, which will be just a tiny bit less than 1000. This will be better than setting $c = 0$ and taking whoever asks her if $1000 < 500 + 1500p$. So she will get the highest payoff from setting $c = 0$ if $p > 1/3$ and will get the highest payoff from setting $c$ just a bit above 1000 if $p < 1/3$.

6. Employee Cog can either work hard or loaf on the job. If he works hard, his output will be worth $16 to his employer. If he loafs, his output will be worth $5 to his employer. His employer can either monitor Cog’s work or not. Mr. Cog gets a wage of $10 if he is not caught loafing and a wage of $0 if he is caught loafing. The employer will only catch Cog loafing if the employer monitors his effort. If the employer does monitor and Mr. Cog is loafing, he will be caught.
It costs the employer $1 to monitor Mr. Cog’s effort, regardless of whether he is working or loafing. If the employer does not monitor Mr. Cog’s work, the employer’s profit is equal to the total value of Cog’s output minus his wage and if the employer monitors Cog’s work, his profit is $1 less than the value of Cog’s output minus his wage.

**A)** Mr. Cog is an expected utility maximizer. Where \( w \) is his wage, his utility is \( w \) if he works hard and \( w+2 \) if he loafs. Is there a pure strategy Nash equilibrium for the game between Mr. Cog and his employer? If so, what is it? If not, show that there is no pure strategy Nash equilibrium.

The payoff matrix for the game between Cog and his boss is

<table>
<thead>
<tr>
<th></th>
<th>Monitor</th>
<th>Don’t Monitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Hard</td>
<td>10,5</td>
<td>10 ,6</td>
</tr>
<tr>
<td>Loaf</td>
<td>2,4</td>
<td>12,-5</td>
</tr>
</tbody>
</table>

There is no pure strategy Nash equilibrium. In a Nash equilibrium each would be doing a best response to what the other is doing. If the boss does not monitor, Cog’s best response is to loaf. But if Cog loafs, the boss’s best response is to monitor. (since he could then pay a wage of 0 instead of $10. So there can be no Nash equilibrium in which the boss does not monitor. If the boss monitors, Cog’s best response is to work hard. But if Cog works hard, the boss’s best response is not to monitor (since he would save the cost of monitoring.) So there can be no Nash equilibrium in which the boss monitors. Thus there are no possible pure strategy Nash equilibria.

**B)** Is there a mixed strategy Nash equilibrium in which Mr. Cog and his employer both randomize their actions. If so, what probabilities do they use in mixed strategy Nash equilibrium? In this equilibrium, what are Mr. Cog’s expected profits?

Let \( p \) be the probability that the boss monitors. Cog will be willing to use a mixed strategy if he is indifferent between loafing and working hard. His utility from working hard is 10. His utility from loafing is \( 2p + 12(1 - p) \). Solving the equation \( 2p + 12(1 - p) = 10 \), we have \( p = 1/5 \). So in the mixed strategy equilibrium, the boss monitors with probability 1/5.

Let \( q \) be the probability that Cog works hard. The boss will be willing to use a mixed strategy only if his expected payoff from monitoring is the same as that from not monitoring. The boss’s expected payoff if he monitors is \( 5q + 4(1 - q) \) and his expected payoff if he doesn’t monitor is \( 6q - 5(1 - q) \). Solving the equation \( 5q + 4(1 - q) = 6q - 5(1 - q) \) we have \( q = 9/10 \). So in the mixed strategy equilibrium, Cog works hard with probability 9/10.

7. All is the same as in Problem 6, except that the employer believes that Mr. Cog might be one of two types. With probability \( p \) he will be a workaholic, in which case if he works hard and is paid \( w \), his utility will be \( w + 1 \), and if he
loafs and is paid $w$, his utility will be $w - 1$. With probability $1 - p$ he will be a lazy guy with utility as in Problem 6.

A) For some values of $p$, there will be a pure strategy Nash equilibrium. Describe this pure strategy equilibrium and find the values of $p$ for which this is an equilibrium?

In a Bayes Nash equilibrium, if the employer does not monitor, Mr. Cog will loaf if he is of the lazy types and would work hard if he is a workaholic. The expected payoff to the employer from the strategy do n’t monitor would therefore be $p \times 6 + (1 - p) \times (-5) = 11p - 5$. The employer’s expected payoff if he monitors would be $5p + 4(1 - p) = p + 4$. Therefore there will be a Bayes Nash equilibrium in which the employer does not monitor and Mr. Cog loafs if he is a lazy type and works hard if he is a workaholic if $11p - 5 \geq p + 4$. This occurs if and only if $p \geq 9/10$. If $p < 9/10$, then we know that there is not a pure strategy Bayes Nash equilibrium in which the employer does not monitor. Suppose that the employer always monitors. Then in a Bayes-Nash equilibrium, workers of both types would work hard. But if workers of both types work hard, the employer’s best response is to not monitor. So there is no pure strategy Nash equilibrium if $p < 8/9$.

B) Suppose that $p = 1/2$. Find a mixed strategy Nash equilibrium. In this mixed strategy equilibrium, what is the probability that the employer monitors? What is the probability that the workaholics loaf? What is the probability that the lazy guys loaf?

In a mixed strategy equilibrium, the employer would have to use a mixed strategy such that the lazy type would be willing to use mixed strategies. We found in Problem 6 that the lazy type is indifferent between loafing and working hard if the employer monitors with probability $1/5$. We also found that the employer is willing to use a mixed strategy if the probability that Mr. Cog will work hard is $9/10$. Mr. Cog will surely work hard if he is a workaholic, so if the probability that a lazy type works hard is $q$, the probability that Mr. Cog will work hard is $p + (1 - p)q$ where $p$ is the probability that he is a workaholic. So the employer will be willing to use a mixed strategy if $p + (1 - p)q = 9/10$. For this problem, we have $p = 1/2$. Therefore in a mixed strategy equilibrium, the probability that a lazy type works hard is $q$ where $1/2 + (1/2)q = 9/10$, which implies that $q = 4/5$. 