1. Charlie and Donna each own a hot dog cart. Charlie makes a $1 profit for every hot dog that he sells and Donna makes a $2 profit for every hot dog that she sells. Each of them can take his or her cart either to the beach or to the airport. If one of them goes to the airport, and the other to the beach, the one at the airport will sell 40 hot dogs and the one at the beach will sell 100 hot dogs. If they both go to the airport, they will each sell 20 hot dogs. If they both go to the beach, they will each sell 50 hot dogs. Charlie moves first and Donna decides where to go, knowing what Charlie has decided to do.

A) What are the possible strategies for Charlie? What are the possible strategies for Donna?

B) Describe this game in extensive form and in strategic form.

C) Find all of the Nash equilibria and describe the strategies used by Charlie and by Donna in Nash equilibrium. Find the subgame perfect Nash equilibrium (or equilibria if there is more than one). What strategy does each use in the subgame perfect Nash equilibrium (or equilibria) that you found? What are the expected payoffs to each of them in the subgame perfect Nash equilibrium?
2. The setup is the same as in Problem 1 except that before Charlie decides where to go, Donna has the option of offering him a bribe of $15 if he will take his cart to the airport instead of the beach. If Charlie accepts the bribe, he will take his cart to the airport. Describe this game in extensive form. Find a subgame perfect equilibrium. What are the strategies used by Charlie and by Donna in this equilibrium?
3. The setup is the same as in Problem 1 except that the number of hot dogs that can be sold at the beach will be 100 only if the day is sunny and will be 0 if the day is rainy. If both go to the beach on a sunny day, each will sell 50 hot dogs. If just one goes on a sunny day, that person will sell 100 hot dogs. But if it is rainy, nobody will be able to sell any hot dogs at the beach. At the beginning of the day, Charlie believes that the probability is 1/2 that it will be sunny and 1/2 that it will be rainy. Both know that Donna will hear an accurate weather report about whether it will rain before she must decide whether to go to the airport or the beach. Charlie must decide where to go before hearing the weather report and without seeing where Donna goes. Donna will choose the airport or the beach after seeing where Charlie went and after she has heard the weather report. Describe this game in extensive form. Find a Bayes Nash subgame perfect equilibrium. What are the strategies used by Charlie and by Donna in this equilibrium? What are the expected payoffs to each of them in equilibrium?
4. Archie and Bella each rent half of a duplex. The furnace is broken down and won't be repaired unless somebody contacts the manager to get it fixed. Contacting the manager is difficult because he lives several miles away and doesn't have a telephone. If at least one of them contacts the manager, he will fix the furnace. Each of them has to decide whether to contact the manager, without knowing whether the other is going to. The cost to either Archie or Bella of contacting the manager is $20. If both contact the manager, it will still cost $20 to each of them. Each of them places a value of $100 on getting the furnace fixed.

A) Write a strategic form payoff matrix for the game played between Archie and Bella. What are the pure strategy Nash equilibria of this game?

B) Find the symmetric mixed strategy equilibrium for this game. In a symmetric mixed strategy equilibrium, what is the probability that exactly one of them contacts the manager? What is the probability that both of them contact the manager? In this equilibrium, what is the probability that the furnace gets fixed?

C) Suppose that Archie, Bella, and Clarissa live in a “triplex” with three apartments and the furnace breaks down. If at least one of the residents contacts the manager, the furnace will be fixed. Getting the furnace fixed is worth $100 to each resident. Contacting the manager costs $20 to anybody who does so. The manager will fix the furnace if and only if at least one of the residents contacts him. Each resident has to decide whether to contact the manager without knowing what the others will do. Find the symmetric Nash equilibrium in mixed strategies. In this equilibrium, what is the probability that the furnace gets fixed?
5. Many handsome young fellows would like to marry Barbie, a beautiful princess. Barbie doesn’t care which one she marries as long as he is rich. But Barbie is not able to check her suitors’ bank balances. She can announce that she will surely marry a fellow if he spends $c$ on an ostentatious, but useless, gift for her. If a rich fellow gives Barbie a gift that costs $c$, she will marry him and his utility will be $1000 - c/2$. If a poor fellow gives Barbie a gift that costs $c$, she will marry him and his utility will be $1000 - c$. The utility of either type of fellow if he doesn’t offer Barbie a gift and she doesn’t marry him will be 0. If Barbie knew that a fellow was rich, she would rather marry him and not have him waste money on the ostentatious gift. Her utility for marrying a rich fellow who gives her a gift that costs $c$ is $2000 - c$ and her utility for marrying a poor fellow who gives her a gift that costs $c$ is $500 - c$.

A) Barbie gets to choose the cost $c$ of the ostentatious gift that she demands. What values of $c$ would result in a separating equilibrium and what values of $c$ would result in a pooling equilibrium?

B) Before any suitor gives a gift, Barbie believes that the probability is $p$ that a handsome young fellow who would like to marry her is rich and the probability is $1 - p$ that he is poor. For what values of $p$ would she get a higher expected payoff from choosing $c = 0$ than from any other choice of $c$?
6. Employee Cog can either work hard or loaf on the job. If he works hard, his output will be worth $16 to his employer. If he loaf, his output will be worth $5 to his employer. His employer can either monitor Cog’s work or not. Mr. Cog gets a wage of $10 if he is not caught loafing and a wage of $0 if he is caught loafing. The employer will only catch Cog loafing if the employer monitors his effort. If the employer does monitor and Mr. Cog is loafing, he will be caught. It costs the employer $1 to monitor Mr. Cog’s effort, regardless of whether he is working or loafing. If the employer does not monitor Mr. Cog’s work, the employer’s profit is equal to the total value of Cog’s output minus his wage and if the employer monitors Cog’s work, his profit is $1 less than the value of Cog’s output minus his wage.

A) Mr. Cog is an expected utility maximizer. Where \( w \) is his wage, his utility is \( w \) if he works hard and \( w + 2 \) if he loaf. Is there a pure strategy Nash equilibrium? If so, what is it? If not, show that there is no pure strategy Nash equilibrium.

B) Is there a mixed strategy Nash equilibrium in which Mr. Cog and his employer both randomize their actions. If so, what probabilities do they use in mixed strategy Nash equilibrium? In this equilibrium, what are Mr. Cog’s expected profits?
7. All is the same as in Problem 6, except that the employer believes that Mr. Cog might be one of two types. With probability \( p \) he will be a workaholic, in which case if he works hard and is paid \( w \), his utility will be \( w + 1 \), and if he loaf and is paid \( w \), his utility will be \( w - 1 \). With probability \( 1 - p \) he will be a lazy guy with utility as in Problem 4.

A) For some values of \( p \), there will be a pure strategy Nash equilibrium. Describe this pure strategy equilibrium and find the values of \( p \) for which this is an equilibrium?

B) Suppose that \( p = 1/2 \). Find a mixed strategy Nash equilibrium. In this mixed strategy equilibrium, what is the probability that the employer monitors? what is the probability that the workaholics loaf? What is the probability that the lazy guys loaf?