Problem 1. Two players, \( A \) and \( B \), have a chance to contribute effort to supplying a resource that is valuable to both of them. Where \( X_A \) is the effort level of Player \( A \) and \( X_B \) the effort level of Player \( B \), the amount of the resource that will be supplied is equal to \( \max\{X_A, X_B\} \), which is the maximum of \( X_A \) and \( X_B \). Each player can choose one of two effort levels, 0 or 5. If a player exerts effort level \( X \) and the amount of the resource supplied is \( Y \), the payoff to that player is \( 2Y - X \). The players must move simultaneously, without observing each others actions.

A) Show an extensive form representation of this game, labeling relevant information sets.

B) Show a strategic form representation of this game. Does this game have any strictly dominated strategies. If so, what are they? Write down the strategy profile(s) of all Nash equilibria for this game.

Table 1: Strategic Form

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0,0</td>
<td>10,5</td>
</tr>
<tr>
<td>5</td>
<td>5,10</td>
<td>5,5</td>
</tr>
</tbody>
</table>

Problem 2. A) The game is as in Problem 1, except that Player \( A \) moves first and Player \( B \) observes Player \( A \)'s effort level before deciding how much effort to provide. Draw an extensive form representation of this game, labeling relevant information sets.

Again, I will leave that to you.

B) Construct the strategic form representation of this game and list the strategy sets for each player.

Now Player \( B \) knows what Player \( A \) did before he moves. Thus he can make his move depend on Player \( A \)'s move. Player \( A \) still has only two strategies. He
can contribute effort 0 or effort 5. A strategy for B specifies what he does in each information set that he might reach. Possible strategies for A are

- $(0/0)$, which means contribute 0 if A contributes 0 and contribute 0 if A contributes 5.
- $(0/5)$ which means contribute 0 if A contributes 0 and contribute 5 if A contributes 5.
- $(5/0)$ which means contribute 5 if A contributes 0 and contribute 0 if A contributes 5.
- $(5/5)$ which means contribute 5 if A contributes 0 and contribute 5 if A contributes 5.

The strategic form for this game is

<table>
<thead>
<tr>
<th></th>
<th>0/0</th>
<th>0/5</th>
<th>5/0</th>
<th>5/5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0,0</td>
<td>0,0</td>
<td>10,5</td>
<td>10,5</td>
</tr>
<tr>
<td>5</td>
<td>5,10</td>
<td>5,5</td>
<td>5,5</td>
<td>5,10</td>
</tr>
</tbody>
</table>

C) How many strategies are possible for Player A and how many strategies are possible for Player B?

A has two possible strategies and B has 4.

D) List any strategies for this game that are strictly dominated. List any strategies for this game that are weakly dominated. Does this game have any Nash equilibria? If so, write down their strategy profiles. For Player B, the strategy $0/5$ is strictly dominated by the strategy $5/0$ and the strategies $5/5$ and $0/0$ are weakly dominated by $5/0$.

There are 3 Nash equilibrium profiles. Player A plays 5 and Player B plays $0/0$. Player A plays 0 and Player B plays $5/5$. Player A plays 0 and Player B plays $5/0$. 
Problem 3. Consider the game described in extensive form below.

A) List the strategies that are available to the mugger and list the strategies that are available to Simon.

The mugger can A) carry a gun and show it. B) carry a gun and conceal it C) carry no gun. Simon has two information sets. He sees a gun. He does not see a gun.

- Not resist if he sees a gun and not resist if he does not see a gun.
- Not resist if sees a gun and resist if he does not see a gun.
- Resist if he sees a gun and not resist if he does not see a gun.
- Resist if he sees a gun and resist if he does not see a gun.

B) What strategies, if any, can be eliminated by iterated deletion of strictly dominated strategies?

There are no strictly dominated strategies for either player.

C) Describe the game in Problem 3 in strategic form.

I leave this to you. It should be straightforward once you understand what the strategies are.

D) Find the Nash equilibrium or equilibria if there are any.

There is one Nash equilibrium. The strategy profile is: Mugger carries a gun and shows it. Simon does not resist if he sees a gun and resists if he does not see a gun.
Problem 4.

The game has three players, the Pope, the Grand Inquisitor, and Galileo. The extensive form of the game is as follows,

and the strategic form of this game is seen below.

Find and list all of the Nash equilibrium strategy profiles for this game.

There are 5 Nash equilibria. These are (C/C, torture, refer), (C/DNC, torture, refer), (DNC/C, DNT, do not refer), (DNC/DNC, torture, do not refer) and (DNC/DNC, DNT, do not refer).
Problem 5. Firm A currently has a monopoly. Firm B is considering entering the industry. If Firm B does not enter the industry, Firm A will have profits of $10 million and Firm B will have profits of 0. If Firm B does enter the industry, Firm A can choose either to start a price war or to share the market. If Firm B enters and Firm A starts a price war, Firm A will lose $1 million and firm B will lose $1 million. If Firm B enters and Firm A shares the market, Firm A will have profits of $4 million and Firm B will have profits of $2 million.

A) Describe this game in strategic form.

In this game Firm B chooses one of the two strategies: Enter, Do Not Enter. Firm A chooses one of the two strategies: Price war if B enters, Share the Market if B enters.

Table 3: Entry Game

<table>
<thead>
<tr>
<th></th>
<th>Enter</th>
<th>Do not Enter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price War</td>
<td>-1,-1</td>
<td>10,0</td>
</tr>
<tr>
<td>Share Market</td>
<td>4,2</td>
<td>10,0</td>
</tr>
</tbody>
</table>

B) What are the Nash equilibrium strategy profiles for this game?

There are two Nash equilibria. In one of these the strategy profiles, Firm A has the strategy, Share the market if B enters, and Firm B has the strategy Enter, In the other profile, Firm A has the strategy Fight a price war if B enters and Firm B has the strategy, Do not Enter.

C) What strategy profiles remain after Iterated Deletion of Weakly Dominated Strategies?

The strategy Fight a price war if B enters is weakly dominated for Player A. If Player B believes that Player A will not play weakly dominated strategies, then the strategy Enter is dominant for Player B. So the only strategy profile that survives iterated deletion of weakly dominated strategies is Player A shares the market if Player B enters, Player B enters.
Problem 6. As in the previous problem, Firm A currently has a monopoly and Firm B is considering entering the industry. But this game has an extra stage. Before Firm B decides whether to enter the industry, Firm A offers a bribe of $1 million to Firm B to stay out of the industry. If Firm A does not offer a bribe, then the game is as in the previous problem. Firm B can either enter or not and if firm B does enter, Firm A can either fight or share. As in Problem 5 if Firm B enters and Firm A fights, both firms will lose $1 million. If Firm B enters and Firm A shares, Firm A has profits of $4 million and Firm B has profits of $2 million. If Firm A offers the bribe, Firm B can do one of two things. He can either accept the bribe and stay out of the industry or can reject the bribe and enter the industry. If the bribe is accepted, Firm A’s profits are $9 million and Firm B’s profits are $1 million. If the bribe is rejected and Firm B enters, Firm A can either fight or share. If Firm A fights, both lose $1 million. If Firm A shares, Firm A will have profits of $4 million and Firm B will have profits of $2 million.

A) Draw an extended form representation of this game.
I will leave this for you.
B) Show the strategic form payoff matrix for this game.

We see from the extended form for this game that Firm A has 3 information sets and thus a strategy must specify what he does at each information set that he might reach. One is at the beginning of the game where he decides whether to offer a bribe or not. The second is if he offers a bribe and the bribe is rejected and B enters, he must decide whether to fight or share. A third is if he does not offer a bribe and B enters, he must decide to fight or share. Since there are 3 information sets and two things he could do at each, Player B has $2 \times 2 \times 2 = 8$ possible strategies. These include

- (O/F/F) Offer a bribe. Fight a price war if the bribe is rejected and B enters. Fight a price war if no bribe is offered and B enters.
- (O/S/S) Offer a bribe. Share the market if the bribe is rejected and B enters. Share the market if no bribe is offered and B enters.
- (O/S/F) Offer a bribe. Share the market if the bribe is rejected and B enters. Fight a price war if no bribe is offered and B enters.
- (O/F/S) Offer a bribe. Fight a price war if the bribe is rejected and B enters. Share the market if no bribe is offered and B enters.
- (N/F/F) Don’t offer a bribe. Fight a price war if a bribe is rejected and B enters. Fight a price war if no bribe is offered and B enters.
- (N/S/S) Don’t offer a bribe. Share the market if a bribe is rejected and B enters. Share the market if no bribe is offered and B enters.
- (N/S/F) Don’t offer a bribe. Share the market if a bribe is rejected and B enters. Fight a price war if no bribe is offered and B enters.
• (N/F/S) Don’t offer a bribe. Fight a price war if a bribe is rejected and B enters. Share the market if no bribe is offered and B enters.

Firm B has two information sets. At each of these he must decide whether to enter or not. Thus a strategy for B specifies what he will do if offered a bribe and what he will do if not offered a bribe. So B has 4 possible strategies.

• (A/E) Accept bribe if it is offered, Enter if no bribe is offered.
• (A/D) Accept bribe if it is offered. Do not enter if no bribe is offered.
• (R/E) Reject bribe if it offered. Enter if no bribe is offered.
• (R/D) Reject bribe if it is offered. Do not enter if no bribe is offered.

The strategic form for this game looks like this:

Table 4: Entry Game with Bribe

<table>
<thead>
<tr>
<th></th>
<th>A/E</th>
<th>A/D</th>
<th>R/E</th>
<th>R/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>O/F/F</td>
<td>9,1</td>
<td>9,1</td>
<td>-1,-1</td>
<td>-1,-1</td>
</tr>
<tr>
<td>O/S/S</td>
<td>9,1</td>
<td>9,1</td>
<td>4,2</td>
<td>4,2</td>
</tr>
<tr>
<td>O/S/F</td>
<td>9,1</td>
<td>9,1</td>
<td>-1,-1</td>
<td>-1,-1</td>
</tr>
<tr>
<td>O/F/S</td>
<td>9,1</td>
<td>9,1</td>
<td>-1,-1</td>
<td>-1,-1</td>
</tr>
<tr>
<td>N/F/F</td>
<td>-1,-1</td>
<td>10,0</td>
<td>-1,-1</td>
<td>10,0</td>
</tr>
<tr>
<td>N/S/S</td>
<td>4,2</td>
<td>10,0</td>
<td>4,2</td>
<td>10,0</td>
</tr>
<tr>
<td>N/S/F</td>
<td>-1,-1</td>
<td>10,0</td>
<td>-1,-1</td>
<td>10,0</td>
</tr>
<tr>
<td>N/F/S</td>
<td>4,2</td>
<td>10,0</td>
<td>4,2</td>
<td>10,0</td>
</tr>
</tbody>
</table>

C) What are the Nash equilibrium strategy profiles for this game?

The Nash equilibrium profiles are:

• O/F/F, A/E
• O/S/S, R/E
• O/S/F, R/E
• O/F/S, A/E
• N/F/F,A/D
• N/F/F,R/D
• N/S/S, R/E
• N/S/F,A/D
• N/S/F, R/D
• N/F/S, R/E