Building a Model of a Strategic Situation

1. The countries of Oceania and Eurasia are at war. As depicted in Figure PR2.1, Oceania has four cities—Argula, Betra, Carnat, and Dussel—and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra, or base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn’t know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate. Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. Write down the extensive form game.

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2. Player 1 moves initially by choosing among four actions: a, b, c, and d. If player 1 chose anything but d, then player 2 chooses between x and y. Player 2 gets to observe the choice of player 1. If player 1 chose d, then player 3 moves by choosing between left and right. Write down the extensive form of this setting. (You can ignore payoffs.)

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3. Consider a setting in which player 1 moves first by choosing among three actions: a, b, and c. After observing the choice of player 1, player 2 chooses among two actions: x and y. Consider the following three variants as to what player 3 can do and what she knows when she moves:

   a. If player 1 chose a, then player 3 selects among two actions: high and low. Player 3 knows player 2’s choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)

   b. If player 1 chose b, then player 3 selects among two actions: high and low. Player 3 does not know player 2’s choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)
ANSWER:  
FIGURE SOL2.3.2

If player 1 chose either a or b, then player 3 selects among two actions: high and low. Player 3 observes the choice of player 2, but not that of player 1. Write down the extensive form of this setting. (You can ignore payoffs.)

ANSWER:  
FIGURE SOL2.3.3

4. Return to the game involving the U.S. Court of Appeals in Section 2.2.
   a. Suppose, at the start of the game, it is known by all that Judge Z will read only the brief of Ms. Hasenfelder. Write down the corresponding extensive form game. You may exclude payoffs.
   b. Suppose, at the start of the game, it was known by all that Judge X would vote first and reveal his vote to Judges Y and Z before they vote simultaneously. Write down the corresponding extensive form game. You may exclude payoffs.
5. The city council is to decide on a proposal to raise property taxes. Suppose Ms. Tuttle is the chair and the council's other two members are Mr. Jones and Mrs. Doubtfire. The voting procedure works as follows: Excluding the chair, Mr. Jones and Mrs. Doubtfire simultaneously write down their votes on slips of paper. Each writes either for or against the tax increase. The secretary of the city council then opens the slips of paper and announces the vote tally. If the secretary reports that both slips say for, then the tax increase is implemented and the game is over. If both vote against, then the tax increase is not implemented and, again, the game is over. However, if it is reported that the vote is one for and one against, then Ms. Tuttle has to vote. If she votes for, then the tax increase is implemented, and if she votes against, then it is not. In both cases, the game is then over. As to payoffs, if the tax increase is implemented, then Mrs. Doubtfire and Mr. Jones each receive a payoff of 3. If the tax increase proposal fails, then Mrs. Doubtfire has a payoff of 4 and Mr. Jones's payoff is 1. As for Ms. Tuttle, she prefers to have a tax increase—believing that it will provide the funds to improve the city's schools—but would prefer not to be on record as voting for higher taxes. Her payoff from a tax increase when her vote is not required is 5, her payoff from a tax increase when her vote is required is 2, and her payoff from taxes not being increased is zero (regardless of whether or not she voted). Write down the extensive form of the game composed of Ms. Tuttle, Mr. Jones, and Mrs. Doubtfire.

**ANSWER:**

6. Consider a contestant on the legendary game show *Let's Make a Deal*. There are three doors, and behind two doors is a booby prize (i.e., a prize of little value), while behind one door is a prize of considerable value, such as an automobile. The doors are labeled 1, 2, and 3. The strategic situation starts when, prior to the show, host Monty Hall selects one of the three doors behind which to place the good prize. Then, during the show, a contestant selects one of the three doors. After its selection, Monty opens up one of the two doors not selected by the contestant. In opening up a door, a rule of the show is that Monty is prohibited from opening the door with the good prize. After Monty Hall opens a door, the contestant is then given the opportunity to continue with the door originally selected or switch to the other unopened door. After the contestant's decision, the remaining two doors are opened.
a. Write down an extensive form game of let’s Make a Deal up to (but not including) the stage at which the contestant decides whether to maintain his original choice or switch to the other unopened door. Thus, you are to write down the extensive form for when (1) Monty Hall chooses the door behind which the good prize is placed; (2) the contestant chooses a door; and (3) Monty Hall chooses a door to open. You may exclude payoffs.

ANSWER:  

FIGURE SOL2.6.1

b. For the stage at which the contestant decides whether or not to switch, write down the contestant’s collection of information sets. In doing so, denote a node by a triple, such as 3/2/1, which describes the sequence of play leading up to that node. 3/2/1 would mean that Monty Hall put the good prize behind door 3, the contestant initially selected door 2, and Monty Hall opened door 1.

ANSWER: There are six information sets for the contestant at the point when he has to decide whether or not to switch: (1) nodes 1/1/2 and 3/1/2; (2) nodes 1/1/3 and 2/1/3; (3) nodes 1/2/3 and 2/2/3; (4) nodes 1/3/2 and 3/3/2; (5) nodes 2/2/1 and 3/2/1; and (6) nodes 2/3/1 and 3/3/1. For example, the first information set comprises nodes 1/1/2 and 3/1/2. At node 1/1/2, Monty put the prize behind door 1, the contestant chose door 1, and Monty opened door 2. At node 3/1/2, Monty put the prize behind door 3, the contestant chose door 1, and Monty opened door 2. The contestant cannot discriminate between those two nodes since they entail the same sequence of observed actions—the contestant chose door 1 and Monty opened door 2—and differ only in terms of where Monty put the prize. That the information set includes both nodes 1/1/2 and 3/1/2 means that the contestant doesn’t know whether the good prize is behind door 1 or door 3.

7. For the Iraq War game in Figure 2.10, write down the strategy sets for the three players.

ANSWER: Iraq has three information sets: (1) the initial node; (2) the set in which it does not have WMD and the UN requested inspections; and (3) the set in which it does have WMD and the UN requested inspections. A strategy for Iraq is then a triple of actions. At two of these information sets it has two feasible actions and at the other one it has three actions. The total number of strategies for Iraq is then 12 strategies. The United States has four information sets: (1) the UN did not request inspections; (2) the UN requested inspections and Iraq rejected the request; (3) the UN requested inspections, Iraq acquiesced to the request, and WMD were not found (that is, either Iraq doesn’t have them or has them and hid them); and (4) the UN requested inspections, Iraq acquiesced to the request, and WMD were found (that is, Iraq had them and did not hide them). The final three information sets each comprise two nodes, one corresponding to Iraq’s having WMD and one to its not having WMD. The final information set is a singleton because of the implicit assumption that a UN inspection will reveal that Iraq has WMD when Iraq does not attempt to hide them. A strategy for the U.S. is then a 4-tuple of actions. Since at each of its four information sets the U.S. has two feasible actions—attacking Iraq or not attacking Iraq—the U.S. has 16 strategies. Finally, the UN has one information set. Like the U.S., it has two nodes, with one corresponding to Iraq’s having WMD and one not. A strategy for the UN is then a single action; its strategy set is composed of request inspections and do not request inspections.

8. For the extensive form game in Figure PR2.8, derive its corresponding strategic form.

FIGURE PR2.8

ANSWER: Player 1 has two information sets, the initial node and the information set associated with a1 and a2 having been played. Let s1y denote a strategy for player 1 that assigns action x to the initial node and action y to the other information set. Player 1’s strategy set contains four elements: s1a1, s1a2, s2a1, and s2a2. Player 2 also has two information sets, the singleton associated with 1 having used s1 and the information set with two nodes—one when the path is a1 → a2 → c1 (read as "a1 is chosen then a2 is chosen then c1 is chosen") and one when the path is a1 → a2 → d1. If strategy s1y assigns action x to the first information set and action y to the second one, then player 2 has four strategies: s1a1, s1a2, s2a1, and s2a2. The payoff matrix associated with these strategies is shown in Figure SOL2.8.1.

FIGURE SOL2.8.1

Player 2

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Player 1
9. Write down the strategic form game for the extensive form game in Figure PR2.9.

**FIGURE PR2.9**

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Player 1: 1 2 3 4
Player 2: a b c d
Player 2: x y
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**ANSWER:** Player 1 has only one information set, which is the initial node. Player 2 has two information sets. Her first information set is the information set associated with player 1 having chosen either a or b. Her second information set is associated with player 1 having chosen c or d. Strategy x/y for player 2 assigns action x to the first information set and action y to the second information set. The strategic form game for this game is shown in Figure SOL2.9.1.

**FIGURE SOL2.9.1**

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<td>c</td>
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10. Write down the strategic form game for the extensive form game in Figure PR2.10.

**FIGURE PR2.10**

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Player 1: 1 2 3
Player 2: a b c d
Player 2: x y
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11. Three extensive form games are shown in Figure PR2.11. State which, if any, violate the assumption of perfect recall. Explain your answer.
ANSWER: Only game (b) satisfies perfect recall. In game (a), consider the information set for player 1 that includes two nodes. One node is associated with 1 having chosen $a$ and 2 having chosen $y$. The other is associated with 1 having chosen $b$ and 2 having chosen $x$. At this information set, 1 is unsure whether she chose $a$ or $b$. This violates perfect recall. As to game (c), the information set for player 1, which includes four nodes, captures the property that, when 1 chooses between actions $c$ and $d$, she doesn't know what player 2 chose (which is not in violation of perfect recall) nor what she originally chose at the initial node (which is in violation of perfect recall). Game (b) satisfies perfect recall. When 1 chooses between actions $c$ and $d$, she cannot discriminate between the nodes in which play was $b \rightarrow x$ and play was $a \rightarrow x$, nor between the nodes in which play was $b \rightarrow y$ and play was $b \rightarrow y$. The former reflects 1's uncertainty over 2's action but knowledge that she originally chose $a$. The latter reflects 1's uncertainty over 2's action but knowledge that she originally chose $b$. 