There are 9 questions. Answer any 8 of them. Good luck!

**Problem 1.** (True or False) “If a player has a dominant strategy in a simultaneous-move game, then she is sure to get her best possible outcome in any Nash equilibrium of the game.” Explain your answer and give an example of a game that illustrates your answer.

This is false. Consider the game Prisoners’ Dilemma. In this game, Defect is a dominant strategy for both players. The best possible outcome for either player is for this player to Defect and the other player to cooperate. But this outcome is not a Nash equilibrium. The only Nash equilibrium is one in which both defect. The payoff for a game depends on the actions of all players, so playing a dominant strategy does not guarantee a player his best possible outcome because the Nash equilibrium because the best outcome for Player 1 may occur only if Player 2 plays something that is not a best response for Player 2.

**Problem 2.**

Consider the game represented in the table below, where Player 1 chooses the row and Player 2 chooses the column.

<table>
<thead>
<tr>
<th></th>
<th>Swerve</th>
<th>Don’t Swerve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swerve</td>
<td>0,0</td>
<td>-1,1</td>
</tr>
<tr>
<td>Don’t Swerve</td>
<td>T,-1</td>
<td>-2,-2</td>
</tr>
</tbody>
</table>

A) Find all of the pure strategy Nash equilibrium strategy profiles for this game if \( T > 0 \).

There are two pure strategy Nash equilibria. In one of them Player 1 Swerves and Player 2 does not. In the other, Player 2 swerves and Player 1 does not.

B) Find all of the pure strategy Nash equilibrium profiles for this game if \( T < 0 \).

In this case, Swerve is a dominant strategy for Player 1 and the only Nash equilibrium is one in which Player 1 swerves and Player 2 does not swerve.

C) If \( T > 0 \), there is a mixed strategy Nash equilibrium strategy profile that is not a pure strategy Nash equilibrium. Find it and find the payoffs to each player in this equilibrium.

In the mixed strategy Nash equilibrium, Player 2 swerves with probability \( 1/(1 + T) \) and Player 1 swerves with probability \( 1/2 \).
D) In a mixed strategy Nash equilibrium with $T = 2$, which player is more likely to swerve? If $T = 2$, which player gets the higher expected payoff in equilibrium? Which player’s equilibrium mixed strategy depends on $T$.

Player 1 is more likely to swerve. (He swerves with probability $1/2$, while Player 2 swerves with probability $1/3$.) Player 2 gets the higher expected payoff. (His expected payoff is $-1/2$. Player 1’s expected payoff is $-2/3$.)

E) (extra credit) Is there anything paradoxical about the results in Parts B and C? If so, what?

If $T > 1$, Player 2 puts a higher value on “winning” the chicken game than Player 1. But it turns out that in the Nash equilibrium, the Player who cares less about winning plays the more aggressive strategy and in equilibrium has higher expected winnings. It works out this way because in a mixed strategy Nash equilibrium, each player’s mixed strategy has to make the other player indifferent between the two pure strategies. Since Player 2’s payoff does not change as $T$ changes, Player 1 does not change his equilibrium mixed strategy. As $T$ increases, Player 2 needs to increase the probability of not swerving in order for Player 1 to remain indifferent.

Problem 3.

An embezzler wants to hide some stolen money. An inspector is looking for the stolen money. There are two places that the embezzler can put the money. One place is difficult to access and one is easy to access. The inspector only has time to look in one of the two places. It is more costly to hide the money in the difficult place than in the easy place and also more costly for the inspector to look in the difficult place than in the easy case. The payoffs are as follows.

- If the embezzler hides the money in the difficult place and the inspector looks in the difficult place, the payoff is 0 for the embezzler and 2 for the inspector.
- If the embezzler hides the money in the difficult place and the inspector looks in the easy place, the payoff is 2 for the embezzler and 1 for the inspector.
- If the embezzler hides the money in the easy place and the inspector looks in the difficult place, the payoff is 3 for the embezzler and 0 for the inspector.
- If the embezzler hides the money in the easy place and the inspector looks in the easy place, the payoff is 1 for the embezzler and 3 for the inspector.

A) (True or false. Justify your answer.) If the inspector believes that the embezzler randomizes in choosing his hiding place and hides the money in the
hard place with probability 2/3, the inspector will maximize his expected payoff by looking in the hard place with probability 2/3.

This is false. The pure strategy "look in the easy place" is the best response to this strategy by the embezzler. This pure strategy has a higher expected payoff than the strategy of looking in the hard place with probability 2/3. If the embezzler hides the money in the hard place with probability 2/3, then the expected payoff to the inspector from looking in the hard place is $2 \times \frac{2}{3} + 0 \times \frac{1}{3} = \frac{4}{3}$ and the expected payoff to the inspector from looking in the easy place is $1 \times \frac{2}{3} + 3 \times \frac{1}{3} = \frac{5}{3}$. If the inspector looked in the hard place with probability 2/3, his expected payoff would be $\frac{2}{3} \times \frac{4}{3} + \frac{1}{3} \times \frac{5}{3} = \frac{13}{9} < \frac{5}{3}$.

B) Find a Nash equilibrium in mixed strategies for this game.

In the mixed strategy Nash equilibrium, the embezzler hides the money in the difficult place with probability 3/4 and in the easy place with probability 1/4. The inspector looks in the difficult place with probability 1/4 and in the easy place with probability 3/4.

C) In Nash equilibrium: What is the expected payoff for the embezzler? What is the expected payoff for the inspector? What is the probability that the inspector finds the money?

The expected payoff for each of them is 3/2. The probability that the inspector finds the money is 3/8.

Problem 4. Alice and Bob have differing tastes in movies, but they like to be together. There are two movies in town, Movie A and Movie B. Alice gets a payoff of 3 if she and Bob both go to Movie A. She gets a payoff of 2 if she and Bob both go to Movie B. She gets a payoff of 1 if she goes to Movie A and Bob goes to B. She gets a payoff of 0 if she goes to Movie B and Bob goes to Movie A. Bob gets a payoff of 3 if he and Alice both go to B. He gets a payoff of 2 if they both go to movie A. His payoff is 1 if he goes to B and Alice goes to A. His payoff is 0 if he goes to A and she goes to B. The last time they met, Alice and Bob agreed to go to a movie, but they didn’t get around to deciding each one. Each of them knows the other’s payoffs from the various outcomes. They have no way of communicating before the movie, and so they must make their choices simultaneously, without knowing the other’s choice.

A) Show this game in strategic form. Find a mixed strategy Nash equilibrium in which each of them has a positive probability of going to each of the movies. What is the expected payoff to each player in this mixed strategy Nash equilibrium?

In the strategic form is shown here, Alice chooses the row and Bob chooses the column.
Table 2: Movie Game

<table>
<thead>
<tr>
<th>Alice goes to A</th>
<th>Bob goes to A</th>
<th>Bob goes to B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,2</td>
<td>1,1</td>
<td></td>
</tr>
<tr>
<td>0,0</td>
<td></td>
<td>2,3</td>
</tr>
</tbody>
</table>

In the mixed strategy equilibrium for this game, Alice goes to Movie A with probability 3/4 and to Movie B with probability 1/4. Bob goes to Movie A with probability 1/4 and to B with probability 3/4. The equilibrium expected payoff to each of them is 3/2.

B) Suppose that the situation is as described above, except that Alice’s desire to go to Movie A is stronger. For some $T$ such that $0 < T < 1$, Alice’s payoff is $3+T$ if she goes to movie A and Bob goes there too. Her payoff is $1+T$ if she goes to Movie A and Bob goes to Movie B. All other payoffs are the same as described above. In the mixed strategy equilibrium of this game, does the probability that Alice goes to Movie A increase or decrease as $T$ increases? Does the probability that Bob goes to Movie A increase or decrease as $T$ increases? Does Alice’s expected payoff in this mixed strategy equilibrium increase or decrease as $T$ increases over the range between 0 and 1? What happens to Bob’s expected payoff as $T$ increases over this range?

The probability that Alice goes to A does not change. The probability that Bob goes to A decreases. Alice’s expected payoff increases as $T$ increases. Bob’s expected payoff does not change. In the mixed strategy Nash equilibrium where $0 < T < 1$, Bob goes to movie A with probability $(1-T)/4$ and to Movie B with probability $(3+T)/4$, while Alice goes to Movie A with probability $3/4$ and B with probability $1/4$. Alice’s payoff expected payoff is $(3+T)/2$.

C) Suppose that payoffs are as in part B, but with $T > 1$. Find all of the Nash equilibria in pure and or mixed strategies for this game.

The only Nash equilibrium is one in which Alice goes to A and Bob goes to A. In this case, going to A is a dominant strategy for Alice.

D) (extra credit) Try to give a convincing argument for each of your answers in Part B, without explicitly calculating the equilibrium mixed strategies.

—em In the mixed strategy equilibrium, Bob’s strategy must make Alice indifferent between going to A and going to B. Movie A is more attractive to Alice, the more likely Bob is to be there. As $T$ gets larger, in order to keep Alice indifferent, Bob would have to be less likely to go to A. Since Bob’s payoffs do
not change as $T$ changes, the equilibrium probabilities of Alice going to A can not change.

In equilibrium, Alice has to be indifferent between going to A and going to B. Since Bob is more likely to be at B when $T$ increases, going to B must be more attractive to Alice when $T$ increases. So it must be that in equilibrium, both pure strategies as well as all mixed strategies must give Alice a higher payoff as $T$ increases. Since Alice does not change her probabilities as $T$ increases, the payoff to both pure strategies and to all mixed strategies stays the same for Bob.

**Problem 5.** The duchess and the countess are invited to a ball. Each of them has two dresses suitable for the ball, a stunning red dress and a charming blue dress. Unfortunately, they have the same dress designer, and their dresses are duplicates. Both would be embarrassed if they wore identical dresses to the ball. Each of them prefers her red dress to her blue dress, but would rather wear the blue dress if the other is wearing the red dress. The duchess and the countess do not speak to each other and must decide on which dress to wear, without knowing what the other is wearing. If both wear the same color of dress, they both get payoffs of zero. If one wears red and the other wears blue, the one who wears red gets a payoff of 2 and the one who wears blue gets a payoff of 1.

A) Show the game played between the countess and the duchess in strategic form. Find a symmetric mixed strategy equilibrium for this game. In this equilibrium, what is the probability that both are wearing the same color of dress? What is the expected payoff to each of them in the symmetric mixed strategy equilibrium? bigskip

The strategic form looks like this:

Table 3: Dressing for the Ball

<table>
<thead>
<tr>
<th></th>
<th>C wears Red</th>
<th>C wears Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>D wears Red</td>
<td>0,0</td>
<td>2,1</td>
</tr>
<tr>
<td>D wears Blue</td>
<td>1,2</td>
<td>0,0</td>
</tr>
</tbody>
</table>

In the symmetric mixed strategy equilibrium, each wears Red with probability 2/3. The probability that both wear the same color dress is 5/9. The expected payoff to each of them is 2/3.

B) Suppose that the duchess and the countess are each able to send a single message to the other. The message can be either "I'll wear red" or "I'll wear blue". After they have received their messages, each of them chooses which dress to wear. Suppose that they both believe that if one of them sends message "I'll wear red" and the other says "I'll wear blue", then each will wear the color she said she would wear. Suppose that they also believe that if both said they would
wear the same color, then each will ignore the messages and use the equilibrium mixed strategy found in Part A. Given these beliefs, construct a strategic form table showing the expected payoffs that each would receive from saying “I’ll wear red” or “I’ll wear blue” given the other’s message. Find a symmetric mixed strategy Nash equilibrium for this game. What is the expected payoff to each of them in this Nash equilibrium?

Table 4: Dress Talk

<table>
<thead>
<tr>
<th></th>
<th>C says “Red”</th>
<th>C says “Blue”</th>
</tr>
</thead>
<tbody>
<tr>
<td>D says “Red”</td>
<td>2/3, 2/3</td>
<td>2, 1</td>
</tr>
<tr>
<td>D says “Blue”</td>
<td>1, 2</td>
<td>2/3, 2/3</td>
</tr>
</tbody>
</table>

In the symmetric Nash equilibrium, each says “I’ll wear red” with probability 4/5. Each now has an expected payoff of 14/15.

C) Suppose that all is as before, except that the messages that they send can be one of three things. “I’ll wear red”, “I’ll wear blue”, or “I’ll wear the opposite.” Suppose that they both believe that if one says red and one says blue, each will wear what she says she would. Suppose they also believe that if one names a color and the other says “I’ll wear the opposite” that they both do what they said they would do. Finally, suppose that they both believe that if the two of them give the same response, then they will play the symmetric mixed strategies found in Part A. Construct a strategic form table showing the expected payoffs that each would receive from each of the three messages, “I’ll wear red”, “I’ll wear blue”, and “I’ll wear the opposite.” Find a mixed strategy Nash equilibrium for this game. What is the expected payoff to each of them in this Nash equilibrium?

Table 5: Dressing with Larger Vocabularies

<table>
<thead>
<tr>
<th></th>
<th>C says “Red”</th>
<th>C says “Blue”</th>
<th>C says “Opposite”</th>
</tr>
</thead>
<tbody>
<tr>
<td>D says “Red”</td>
<td>2/3, 2/3</td>
<td>2, 1</td>
<td>2, 1</td>
</tr>
<tr>
<td>D says “Blue”</td>
<td>1, 2</td>
<td>2/3, 2/3</td>
<td>1, 2</td>
</tr>
<tr>
<td>D says “Opposite”</td>
<td>1, 2</td>
<td>2, 1</td>
<td>2/3, 2/3</td>
</tr>
</tbody>
</table>

To find a symmetric mixed strategy Nash equilibrium, let \( r \) be the probability that the counter says “Red”, let \( b \) be the probability that she says “Blue”, and \( 1 - r = b \) the probability that she says “Opposite”. Then solve two equations in two unknowns for \( r \) and \( b \), where one equation makes the expected payoff from
saying “Red” the same as that from saying “Blue” and the other makes the expected payoff from saying “Red” the same as that from saying “Opposite”. The solution is \( r = \frac{16}{21}, b = \frac{1}{21}, 1 - r - b = \frac{4}{21} \). Expected payoff for each of them is \( \frac{31}{32} \).

Problem 6. In South Carburetor Illinois, half of the used cars are good and half of them are lemons. The current owners of lemons would be willing to sell them for any price above $1000, while the current owners of good used cars would be willing to sell them if and only if the price is greater than $7,000. There are a large number of buyers who would be willing to pay $10,000 for a good used car, but would be willing to pay only $2000 for a lemon. Buyers cannot tell a good used car from a lemon. All used cars must therefore sell at the same price. The price of a used car will be the expected value of a used car, given the beliefs of buyers about the kinds of cars that are for sale.

A) Is there a pooling equilibrium in which buyers believe that all used cars will come on the market? If so, describe this equilibrium. If not, explain why not.

No. There is not. If all buyers believed that all used cars reached the market, they would believe that the average used car was worth $6000. At a price of $6000, the owners of good used cars would not be willing to sell. The only cars that would come on the market would be lemons. The buyer’s beliefs would not be confirmed.

B) Suppose that in South Carburetor there is a car inspection shop that will inspect used cars and certify them as good if they pass some tests. Good used cars always pass the tests, but lemons only pass the tests with probability 1/4. To have his car inspected, a used car owner must pay a fee of $C$, which he has to pay whether or not a car passes the test. For what values of $C$ would there be a Bayes-Nash separating equilibrium in which only the owners of good used cars would have their cars inspected? In this equilibrium, what would be the price of a car that has passed inspection? What would be the price of a car that failed inspection?

If only the good cars are inspected, then every car that passes inspection would be a good used car and would sell for $10,000 and every car that was not inspected would be a bad used car and would sell for $2000. If this is the case, good used car owners would keep their cars if they didn’t have them inspected. They would be willing to pay $C$ to have their cars inspected if $C < \$3000$. If the owner of a lemon has his car inspected, he must pay $C$, but with probability 1/4, he can sell it for $10,000 and with probability 3/4 it would fail inspection and would sell for $2000$. Therefore the expected payoff to a lemon owner who has his car inspected is \( (1/4)10,000 + (3/4)2000 - C = 4,000 - C \). A lemon owner who does not have his car inspected has an expected payoff of 2,000. So
lemon owners would choose not to have their cars inspected if \( C > \$2000 \). It follows that if \( \$2000 < C < \$3000 \), there will be a separating equilibrium in which only the good car owners have their cars inspected and hence all cars that pass inspection are good.

C) For what values of the inspection fee \( C \), would there be a Bayes-Nash equilibrium in which buyers believe that all used car owners have their cars inspected. In this case, what would be the expected value to buyers of a car that has passed inspection? (Hint: Calculating the conditional probability that a car is good, given that it has passed inspection is an application of Bayes’ rule.) What would be the expected value to buyers of a car that has not passed inspection? If used cars that pass inspection sell for their expected value to buyers and cars that have not passed inspection sell for their expected value to buyers, would it be in the interests of the current owners to act in the way that buyers believe?

Suppose that buyers believe that all used car owners have their cars inspected. In this case, the cars that have passed inspection include all of the good used cars and \( 1/4 \) of the lemons. Since half of the cars in town are good and half are lemons, this means that the fraction

\[
\frac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{2} + \frac{1}{2} \times \frac{1}{4}} = 1/5
\]

of the cars that have passed inspection are lemons and the remaining \( 4/5 \) of the cars that have passed inspection are good. Given these beliefs, the expected value to buyers of a car that has passed inspection is \( (4/5)10,000 + (1/5)2000 = \$8400 \). The expected value of cars that have not passed inspection would be \$2,000. An owner of a good car would be willing to have his car inspected and sell it for \$8,400 if \( C < \$1,400 \). The owner of a lemon would be willing to have his car inspected if \( (1/4)8,400 + 3/4(2000) - C > \$2,000 \). This is the case if \( C < \$1,600 \). Therefore these beliefs will be confirmed if \( C < \$1,400 \).

Problem 7. Everyone knows that in the city of Bent Crankshaft Ohio, \( 1/4 \) of the used cars are of high quality, \( 1/2 \) of the used cars are of medium quality, and \( 1/4 \) are of low quality. Their current owners know the quality of their cars. The high-quality used cars are worth an amount \( V_H \) to their current owners. The medium-quality used cars are worth \$18,000 to their current owners and the low-quality used cars are worth \$10,000 to their current owners. Potential buyers of used cars in Crankshaft can not tell whether a car is of high low or medium quality and they know the proportions of each quality that are present in the population. A high-quality used car is worth \$28,000 to buyers. A medium-quality used car is worth \$21,000 to buyers, and a low-quality used car is worth \$12,000 to buyers. Let us suppose that the market price of used cars is equal to the expected value of a random draw from the population of used cars that buyers believe will be available on the market.
A) For what values of $V_H$ would there be a pooling equilibrium in which buyers believe that all used cars in Bent Crankshaft are available on the market and their beliefs are confirmed by the outcome.

If these are the buyers’ beliefs, the market price of used cars will be their expected value, which is $20,500. For $V_H \leq 20,500$, all used car owners will want to sell at this price, so all used cars will be on the market.

B) Suppose there is a pooling equilibrium in which all used cars in Bent Crankshaft are available on the market. Is there also a semi-separating equilibrium in which buyers believe that medium and low quality used cars are on the market, but not high quality used cars? If so, what would be their expected value for of a used car. Explain.

If buyers believe that all of the medium and low quality used cars but not the high quality used cars are on the market, then they will believe that $2/3$ of the used cars on the market are of medium quality and $1/3$ will be of low quality. The price of a used car would then be $(2/3)21000 + (1/3)12000 = $18,000. At this price, owners of low quality used cars would be willing to sell and owners of medium quality used cars would be (just barely) willing to sell.

C) Is it possible for there to be three different perfect Bayes-Nash equilibria, one in which all used cars reach the market, one in which only the low and medium quality used cars reach the market and one in which only the low quality used cars reach the market? Explain.

Yes, we have already shown two possibilities. There is also an equilibrium where everybody believes that only lemons are available, the price is $2000 and only lemon owners want to sell.

Problem 8. Consider a stage game with the strategic form found below, where Player 1 chooses the row and Player 2 chooses the column. Suppose that this game is repeated 20 times. After each round of play, both players are informed of all previous plays. The total payoff to each player in this repeated game is the sum of the payoffs received in the 20 repetitions of the stage game.

A) Suppose that $T > 5$. Find all of the Nash equilibria for the stage game.

There are two Nash equilibria. In one of them, Player 1 plays $c$ and Player 2 plays $y$. In the other, Player 1 plays $d$ and Player 2 plays $z$.

B) For what values of $T > 5$, if any, is there a subgame perfect Nash equilibrium for this repeated game, such that in equilibrium, Player 1 plays $a$ and
Player 2 plays \( w \) in the first 19 rounds of play? If this can be done, find a strategy for each player that results in this outcome.

Suppose that Player 1 has the strategy "Play \( a \) in the first round and for the next 18 rounds, play \( a \) if Player 2 has played \( w \) in every round so far and Player 1 has played \( a \) in every round so far. If either player ever plays anything else in any of the first 19 rounds, play \( c \) on all future rounds. If Player 2 has always played \( w \) and Player 1 has always played \( a \) on the first 19 rounds, then play \( d \) on the last round." Suppose that Player 2’s strategy is “Play \( w \) on the first round and for the next 18 rounds, play \( w \) so long as Player 1 has played \( a \) and Player 2 has played \( w \) on every round so far. If either player deviates from these actions on the first 19 rounds, then play \( y \) in all future rounds. If Player 1 has always played \( a \) on the first 19 rounds, then play \( z \) in the 20th round.”

We need to check whether it pays anybody to deviate from this strategy, given the other player’s strategy. Let’s see if it pays Player 1 to deviate on the 19th round. Suppose that Player 1 chooses \( b \) on round 19. Then in Round 19, he will get a payoff of \( T \) and in Round 20, he knows that Player 2 will play \( y \). If Player 2 is playing \( y \) in round 20, the best that 1 can do is play \( c \) and get a payoff of 1. So if he deviates in Round 19, he will get a payoff of \( T + 1 \) for the last two rounds. If he does not deviate in Round 19, then he will receive a payoff of 5 in Round 19 and Player 2 will play \( z \) in Round 20. If Player 2 plays \( z \) in round 20, the best that Player 1 can do in this round is play \( d \) which gives him a payoff of 4 for a total of 5+4 in the last two rounds. So it will not pay for Player 1 to deviate in round 19 if \( T + 1 < 9 \) and it will pay him to deviate if \( T + 1 > 9 \).

Therefore, we need \( T < 8 \) if the strategies we propose are a Nash equilibrium. In fact, it is not hard to see that if it doesn’t pay to deviate in round 19, it certainly doesn’t pay to deviate earlier, since the “punishment” is greater, the earlier you deviate. By the symmetry of the game, the same argument that shows that it doesn’t pay for Player 1 to deviate if \( T < 8 \) also implies that it doesn’t pay for Player 2 to deviate if \( T < 8 \).

C) For what values of \( T > 5 \), if any, is there a subgame perfect Nash equi-
librium for this repeated game, such that in equilibrium, Player 1 plays $a$ and Player 2 plays $w$ in the first 18 rounds of play? If this can be done, find a strategy for each player that results in this outcome.

The strategies proposed here will be like the ones in Part B, except that if both players have played their parts ($w$ and $a$) on the first 18 rounds, then Player 1 will play $d$ on the 19th and 20th rounds and Player 2 will play $z$ on the 19th and 20th rounds. If either player deviates from this course of play, then Player 1 will play $c$ on all remaining rounds and Player 2 will play $y$ on all remaining rounds.

This strategy will “work” when the “temptation” $T$ is higher than the strategy outlined in Part B. We see this as follows. Suppose that up until the 18th round, Player 1 had been playing $a$ and Player 2 had been playing $w$. If Player 1 chooses to deviate on the 18th round, he would get a payoff of $T$ in that round and then would get payoffs of 1 in each of the two remaining rounds, giving him a total of $T + 2$ for the last 3 rounds. If instead he plays $a$ in the 18th round, he gets a payoff of 5 in that round and 4 in rounds 19 and 20, giving him a total payoff of 13 for the last 3 rounds. Therefore this deviation would reduce his total payoff if $T < 11$.

**Problem 9.** Doc and Slim are playing a simplified version of poker. Each puts an initial bet of one dollar in a "pot". Doc draws a card, which is either a King or a Queen with equal probabilities. Doc knows what he drew, but Slim does not.

After looking at his card, Doc decides whether to Fold or Bet. If Doc chooses to Fold, the game ends and Slim gets all of the money in the pot. If Doc chooses to Bet, he puts another dollar in the pot.

If Doc decides to bet, Slim must decide whether to Fold or Call. If Slim Folds, Doc wins the pot (which now contains 3 dollars, two of which he contributed himself.) If Slim Calls, Slim must add another dollar to the pot. Then Doc shows his card. If Doc has a King, Doc gets all the money (4 dollars) that is now in the pot. If Doc has a Queen, Slim gets all the money that is in the pot.

A) List the possible strategies for Doc. List the possible strategies for Slim.

**Strategies for Doc are:**

- Bet if King, Bet if Queen
- Bet if King, Fold if Queen
- Fold if King, Bet if Queen
- Fold if King, Fold if Queen
Slim has only two strategies, Fold or Call.

B) Show this game in extensive form. Be careful about the information sets. Note that the payoffs to each player are the amounts of money that player received from the pot minus the amount of money the player put into the pot.

C Suppose that Doc bets if he draws a king and folds if he draws a queen and that Slim always folds. What is Doc’s expected payoff, given these two strategy choices? Hint: (Given these strategy choices, what is Doc’s payoff if he draws a king? What is Doc’s payoff if he draws a queen?)

His expected payoff is zero.

D) Show this game in strategic form where the payoffs in each cell are expected payoffs given each player’s strategy.

In the table below, Doc chooses the row and Slim chooses the column.

<table>
<thead>
<tr>
<th>Table 7: Poker Payoffs</th>
<th>Call</th>
<th>Fold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bet if King, Bet if Queen</td>
<td>0,0</td>
<td>1,-1</td>
</tr>
<tr>
<td>Bet if King, Fold if Queen</td>
<td>.5,-.5</td>
<td>0,0</td>
</tr>
<tr>
<td>Bet if Queen, Fold if King</td>
<td>-1.5,1.5</td>
<td>0,0</td>
</tr>
<tr>
<td>Fold if Queen, Fold if King</td>
<td>-1,1</td>
<td>-1,1</td>
</tr>
</tbody>
</table>
D) Eliminate strictly dominated strategies if any. Find a Nash equilibrium in mixed strategies. What is the expected payoff to Doc in this equilibrium? What is the expected payoff to Slim in this equilibrium?

The strategies Fold if Queen, Fold if King and Bet if Queen, Fold if King, are strictly dominated. The remaining game looks like this.

Table 8: Poker Payoffs (undominated strategies)

<table>
<thead>
<tr>
<th></th>
<th>Call</th>
<th>Fold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bet if King, Bet if Queen</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Bet if King, Fold if Queen</td>
<td>.5,.5</td>
<td>0,0</td>
</tr>
</tbody>
</table>

This game has no pure strategy Nash equilibria. It has a mixed strategy Nash equilibrium in which Slim Calls with probability 2/3 and folds with probability 1/3 and Doc plays Bet if King, Bet if Queen (the bluffing strategy) with probability 1/3 and plays Bet if King, Fold if Queen with probability 2/3. The expected payoff for Doc is 1/3 and the expected payoff for Slim is -1/3.