1) Consider the following strategic form game in which Player 1 chooses the row and Player 2 chooses the column. Both players know that this is the payoff matrix and each knows that the other knows this.

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Center</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>1,2</td>
<td>2,1</td>
<td>1,0</td>
</tr>
<tr>
<td>Middle</td>
<td>0,5</td>
<td>1,2</td>
<td>7,4</td>
</tr>
<tr>
<td>Down</td>
<td>-1,1</td>
<td>3,0</td>
<td>5,2</td>
</tr>
</tbody>
</table>

A) Suppose that the two players move simultaneously. Assume that it is common knowledge to both players that neither player will ever play a strictly dominated strategy. What pure strategy outcome (outcomes) would you expect to see? Explain

B) Suppose that in the game with payoff matrix described above, Player 1 moves first and Player 2 moves after having observed Player 1’s move. List the possible strategies for Player 1. List the possible strategies for Player 2. Find the strategy profile that constitutes a subgame perfect Nash equilibrium.
2) Lucy asked Charlie to play the following game. “Let’s show pennies to each other. We can either show heads or tails. We will move simultaneously. If we both show heads, I will pay you $3. If we both show tails, I will pay you $1. If one of us shows heads and the other shows tails, then you will pay me $2.”

A) If Charlie agrees to play and if Lucy knows that he is equally likely to play heads or tails, what will Lucy do? Then what will Charlie’s expected profit or loss be?

B) Find a mixed strategy Nash equilibrium for the game that Lucy proposed. What strategies does each use? What will be Charlie’s expected profit or loss in this equilibrium? Explain your answer.
3) Alice and Bob are planning to take a trip together. They must choose one of three possible destinations: the mountains, the ocean, or the big city. Alice likes the mountains best, the ocean second, and the city third. Bob likes the ocean best, the mountains second, and the city third. They decide to select their destination by alternately vetoing possibilities until only one option remains. First Alice vetoes a destination. Then, knowing which destination Alice vetoed, Bob vetoes one of the destinations not vetoed by Alice. They will go to the destination that neither vetoed.

A) List the possible strategies for Alice? List the possible strategies for Bob.

B) Draw an extensive form representation of this game and assign payoffs that are consistent with the story.
C) Show this game in strategic form. Find all of the Nash equilibria. Are all of the Nash equilibria subgame perfect? Explain.

4) (Simplified Poker) Archie and Beth each put a dollar into the “pot”. Archie draws a card from a deck of cards. The card is equally likely to be high or low. Archie can see his card, but Beth can not. Archie can either “show” or “raise”. If Archie plays “show”, he shows his card to Beth. If Archie’s card is high, he gets all of the money in the pot and the game ends. If Archie’s card is low, Beth gets all of the money in the pot and the game ends. If Archie plays “raise”, he puts another dollar into the pot and Beth must choose whether to “pass” or “meet”. If Beth passes, Archie gets all of the money in the pot. If Beth meets, she adds a dollar to the pot and Archie then shows Beth his card. If the card is high, Archie gets all the money in the pot. If the card is low, Beth gets all the money in the pot.
A) Draw an extensive form representation of the game played by Archie and Beth.

B) List the possible strategies for Archie. List the possible strategies for Beth.

C) Describe this game in strategic form. (Payoffs will be the difference between the amount of money one takes out of the pot and the amount one puts in.)
D) How many pure strategy Nash equilibria does this game have? Explain your answer.

E) Find a mixed strategy equilibrium in which if Archie sees a high card, he is sure to raise, and if he sees a low card will show with some probability and raise with some probability. In this mixed strategy equilibrium, what is the probability that Archie raises? What is the probability that Beth meets?

5) Consider an infinitely repeated game where the stage game is displayed in the table below, with Player 1 choosing the row and Player 2 choosing the column.

<table>
<thead>
<tr>
<th></th>
<th>Strategy A</th>
<th>Strategy B</th>
<th>Strategy C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy A</td>
<td>6,6</td>
<td>3,8</td>
<td>1,4</td>
</tr>
<tr>
<td>Strategy B</td>
<td>8,3</td>
<td>3,3</td>
<td>0,1</td>
</tr>
<tr>
<td>Strategy C</td>
<td>4,1</td>
<td>1,0</td>
<td>2,2</td>
</tr>
</tbody>
</table>

Consider the symmetric strategy profile where each player plays Strategy A in the first round and continues to choose A, so long as neither player has ever chosen B or C. If B or C is ever chosen, then each player will play strategy C forever after. Find a condition on the discount factor so that this
strategy profile is a subgame perfect Nash equilibrium. Show that given this condition, this strategy is in fact a subgame perfect Nash equilibrium.

6) Find all of the Evolutionarily Stable Strategies for the following game, where Player 1 chooses the row and Player 2 chooses the column. Explain your answer.

<table>
<thead>
<tr>
<th></th>
<th>Strategy A</th>
<th>Strategy B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy A</td>
<td>0,0</td>
<td>2,1</td>
</tr>
<tr>
<td>Strategy B</td>
<td>1,2</td>
<td>1,1</td>
</tr>
</tbody>
</table>
7) In a symmetric game, a pure strategy is an Evolutionarily Stable Strategy if it is a strict symmetric Nash equilibrium. True or False? If true, explain why this is true. If false, show a counterexample.

8) An employer has a large workforce, some of whom are self-motivated and some of whom are not. The employer can not tell which workers are self-motivated and which are not, but he knows that the fraction who are self-motivated is $p$. A worker who works hard produces output that is worth $20 to the employer. A worker who loafs produces output worth $5 to the employer. The employer can get a monitor to determine whether a worker is working hard at a cost of $5 per worker.

The employer pays wages of $10 to any worker who is not caught loafing and will pay a wage of $0 to any worker who is caught loafing. Self-motivated workers prefer working hard to loafing, even if the employer doesn’t monitor. A worker who is not self-motivated and gets paid $w$ has a payoff of $w - 2$ if he or she works hard and $w + 5$ if he or she loafs.

A) For what range of values of $p$ is there a Bayes-Nash equilibrium in which the employer does not monitor the workers? Explain your answer.
B) Suppose that $p = 1/3$. Find a mixed strategy Bayes-Nash equilibrium. In this equilibrium, what is the probability that the employer monitors? What is the probability that a worker that is not self-motivated will work hard? What is the probability that a self-motivated worker will work hard?

9) Suppose that members of a large population are randomly matched to play a game of repeated prisoners’ dilemma. In the stage game, if both cooperate, they each get a payoff of 1. If both defect, they each get a payoff of 0. If one cooperates and the other defects, the one who cooperated gets a payoff of $-1$ and the one who defected gets a payoff of $T$. After each round of play, each gets to observe how they other played. A fair coin is then tossed. If the coin comes up heads, they will play again. If the coin comes up tails, the game stops. The game continues to be repeated until the first time that the coin comes up tails.

A) Suppose that the population consists almost entirely of individuals who use the following tit-for-tat strategy, *Cooperate on the first round of play. In all later rounds, use the same strategy that the other player used on the previous round.* What is the expected payoff to a tit-for-player who is matched with another tit-for-tat player?

B) What is the expected payoff of a player who plays *Defect, no matter what the other player does* if matched against a tit-for-tat player?

C) What is the expected payoff to a player who plays *Defect on odd-numbered rounds, cooperate on even numbered rounds* if matched against
a tit-for-tat player?

D) For what values of $T > 1$ would it be true that in a population of tit-for-tat players, rare mutants of types the types mentioned in parts B and C would get lower expected payoffs than tit-for-tat players?

E) Suppose in this large population, matching is not entirely random, but with probability $1/2$ an individual will be matched with someone who plays the same strategy as oneself and with probability $1/2$, one will be matched with a random selection of the remaining population. For what values of $T$ would it be true that if the population is made up almost entirely of individuals playing *always cooperate* the always cooperate players would get higher expected payoffs than “mutants” who play *always defect*?

10) Alice and Bob have differing tastes in movies, but like to be together. If they both go to movie A, Alice will get a payoff of 3 and Bob will get a payoff of 2. If they both go to Movie B, Bob will get a payoff of 3 and Alice will get a payoff of 2. If Alice goes to A and Bob to B, Alice and Bob will each get a payoff of 1. If Alice goes to B and Bob goes to A, they will each get a payoff of 0.

A) Find a symmetric mixed strategy equilibrium for this game. What is the probability that they arrive at the same movie.

B) Suppose that Alice and Bob have not settled on which movie to go to
and will not get a chance to discuss it at length. Each can leave a single text message for the other, saying to which movie he or she plans to go. They must choose their messages simultaneously, without knowing the other’s message. If the two messages both name the same movie, they will both go to that movie. If the two messages name different movies, they will both ignore the messages and play an equilibrium symmetric mixed strategy to the original game. Find a symmetric mixed strategy Nash equilibrium to the game in which Alice and Bob each choose a message to send to the other.

C) Suppose that the situation is as in Part B, except that if the first messages between Alice and Bob name different movies, they each get one more chance to send messages about their intentions. Find a symmetric mixed strategy Nash equilibrium to this game and find the probability that they will both go to the same movie.