ANSWERS TO CHECK YOUR UNDERSTANDING

Chapter 2

2.1 The extensive form game in Figure 2.4a is legitimate even though player 2 has different actions at her two decision nodes—actions $d$ and $e$ when player 1 chooses $a$ and actions $d$ and $f$ when player 1 chooses $b$. Indeed, it is easy to come up with real-world settings in which what one player does affects the choices available to another player.

The extensive form game in Figure 2.4b is legitimate. Of particular note, the choice that a player makes is allowed to affect which player gets to move next. If player 1 chooses action $a$ then it is player 2's move, if player 1 chooses action $c$ then it is player 3's move, and if player 1 chooses action $b$ then the game is over.

The extensive form game in Figure 2.4c is not legitimate because it violates the rule that there is a unique path to each node. Consider player 3's decision node when player 1 has chosen action $a$ and player 2 has chosen $d$. That decision node is also reached by player 1 choosing $b$ and player 2 choosing $c$.

2.2

2.3
The strategy set for Vivica is the same as that derived for the extensive-form game in Figure 2.1, as she still has only one information set. She has two strategies: (1) If a kidnapping occurred, then pay ransom; and (2) if a kidnapping occurred, then do not pay ransom. In contrast, Guy's strategy template has changed, since he now has only two information sets (instead of three):

At the initial node, _______. [fill in kidnap or do not kidnap]

If a kidnapping occurred then _______. [fill in kill or release]

There are two possible actions at each of those two information sets, so Guy has four feasible strategies: (1) At the initial node, kidnap, and if a kidnapping occurred, then kill; (2) at the initial node, kidnap, and if a kidnapping occurred, then release; (3) at the initial node, do not kidnap, and if a kidnapping occurred, then kill; and (4) at the initial node, do not kidnap, and if a kidnapping occurred, then release. Although the choices faced by Guy are unchanged, altering the structure of his information sets affects his strategy set.

A strategy for the Republican candidate is a platform and her strategy set is composed of three policies: moderate (M), moderately conservative (MC), and conservative (C). A strategy for the Democratic candidate is a 3-tuple of actions: what position to take if the Republican candidate chooses M, what position to take if she chooses MC, and what position to take if she chooses C. For example, strategy M/L/M has the Democratic candidate taking a moderate position in response to her Republican opponent choosing a moderate or conservative platform and taking a liberal position when she takes a moderately conservative platform. The strategic form is:

<table>
<thead>
<tr>
<th>Republican candidate</th>
<th>M</th>
<th>M/C</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>M/M/M</td>
<td>4,4</td>
<td>6,3</td>
<td>8,2</td>
</tr>
<tr>
<td>M/L/M</td>
<td>4,4</td>
<td>4,9</td>
<td>8,2</td>
</tr>
<tr>
<td>M/M/L</td>
<td>4,4</td>
<td>6,3</td>
<td>7,7</td>
</tr>
<tr>
<td>M/L/L</td>
<td>4,4</td>
<td>4,9</td>
<td>7,7</td>
</tr>
<tr>
<td>L/M/M</td>
<td>2,8</td>
<td>6,3</td>
<td>8,2</td>
</tr>
<tr>
<td>L/L/M</td>
<td>2,8</td>
<td>4,9</td>
<td>8,2</td>
</tr>
<tr>
<td>L/M/L</td>
<td>2,8</td>
<td>6,3</td>
<td>7,7</td>
</tr>
<tr>
<td>L/L/L</td>
<td>2,8</td>
<td>4,9</td>
<td>7,7</td>
</tr>
</tbody>
</table>

The mugger has three strategies: gun and show; gun and hide; and no gun. Simon has two information sets, so a strategy for him is a pair of actions: what to do if the mugger shows a gun and what to do if he does not. There are then four strategies for Simon: R/R, R/DNR, DNR/R, and DNR/DNR (where R denotes resist, DNR denotes do not resist, and the first action refers to the information set in which the mugger shows a gun). The strategic form is as follows:

<table>
<thead>
<tr>
<th>Simon</th>
<th>R/R</th>
<th>R/DNR</th>
<th>DNR/R</th>
<th>DNR/DNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mugger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gun &amp; Show</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Gun &amp; Hide</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>No gun</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>
Chapter 3

3.1 It is useful to first note that, by the structure of payoffs, if it is possible for a player’s strategy to result in the sum of the three strategies being at least 50 then the optimal strategy is the one that makes the sum exactly equal to 50. If it is not possible for a player’s strategy to result in the sum of the three strategies being at least 50 (that
is, even if he chooses his largest strategy, the sum is less than 50) then he wants to choose his smallest strategy.

Consider player 1. The sum of numbers for players 2 and 3 is at least as great as 40 which means that player 1's payoff is $100 - x_1$ when $x_1 \approx 10$. Thus, a strategy of 10 yields a payoff of 90 which is strictly higher than that for any higher strategy. Hence, if $x_1 > 10$ then $x_1$ is strictly dominated by 10. Next, note that if $40 \leq x_2 + x_3 \leq 45$ then $x_1 = 50 - x_2 - x_3$ is player 1's optimal strategy. Thus, if $5 \leq x_1 \leq 10$ then $x_1$ is not strictly dominated because it can actually be the best strategy for the other two players. In sum, all strategies for player 1 larger than 10 are strictly dominated and all strategies less than or equal to 10 are not. The set of strictly dominated strategies for player 1 is $\{11, \ldots , 20\}$.

Next consider player 2. The sum of numbers for players 1 and 3 is at least as great as 35 which means that player 2's payoff is $100 - x_2$ when $x_1 \approx 15$. Thus, a strategy of 15 yields a payoff of 85 which is strictly higher than that for any higher strategy. Hence, if $x_2 > 15$ then $x_2$ is strictly dominated by 15. Next note that if $35 \leq x_1 + x_3 \leq 40$ then $x_2 = 50 - x_1 - x_3$ is player 2's optimal strategy. Thus, if $10 \leq x_2 \leq 15$ then $x_2$ is not strictly dominated. In sum, the set of strictly dominated strategies for player 2 is $\{16, \ldots , 20\}$.

Finally, consider player 3. The sum of numbers for players 1 and 2 is at least as great as 15 which means that player 3's payoff is $100 - x_3$ when $x_1 \approx 35$. Thus, a strategy of 35 yields a payoff of 65 which is strictly higher than that for any higher strategy. Hence, if $x_3 > 35$ then $x_3$ is strictly dominated by 35. Next, note that if $15 \leq x_1 + x_2 \leq 20$ then $x_3 = 50 - x_1 - x_2$ is player 3's optimal strategy. Thus, if $30 \leq x_3 \leq 35$ then $x_3$ is not strictly dominated. In sum, the set of strictly dominated strategies for player 3 is $\{36, \ldots , 40\}$.

3.2 For player 1, strategy $a$ strictly dominates $c$, and $b$ weakly dominates $d$. For player 2, $y$ strictly dominates $w$ and $x$, and $w$ weakly dominates $x$.

3.3 The payoff function for player $i$ can be presented as:

$$\text{Payoff to player } i = \begin{cases} 
0 & \text{if } q_i = 0 \\
8 - q_i & \text{if } q_i = 1 \\
12 - 2q_i & \text{if } q_i = 2 \\
12 - 3q_i & \text{if } q_i = 3' \\
8 - 4q_i & \text{if } q_i = 4 \\
-5q_i & \text{if } q_i = 5
\end{cases}$$

In deriving the strictly dominated strategies, first note that strategy 0 is strictly dominated by strategies 1 and 2 because $q_i = 0$ yields a payoff of 0 and those other two strategies yield a positive payoff for all strategies of the other player. Next, note that strategy 4 is strictly dominated by strategies 2 and 3 because, for any value of $q_i$, $12 - 3q_i > 8 - 4q_i$ and $12 - 2q_i > 8 - 4q_i$. Strategy 5 is strictly dominated by strategies 1, 2, 3, and 4. Strategy 1 is not strictly dominated as it is an optimal strategy when the other player chooses 4 or 5. Strategy 2 is not strictly dominated as it is an optimal strategy when the other player chooses 1, 2, 3, or 4. Strategy 3 is not strictly dominated as it is an optimal strategy when the other player chooses 0. Thus, strategies 0, 4, and 5 are strictly dominated.

In deriving the weakly dominated strategies, strategies 0, 4, and 5 are weakly dominated because they are strictly dominated. Next note that strategy 3 is weakly dominated by strategy 2; strategy 2 yields a strictly higher payoff when $q_i > 0$ and the same payoff when $q_i = 0$. Strategy 1 is not weakly dominated because it is the
unique optimal strategy when \( q_i = 5 \); and strategy 2 is not weakly dominated because it is the unique optimal strategy when \( q_i = 1, 2, \) or 3. Therefore, strategies 0, 3, 4, and 5 are weakly dominated.

3.4 Given that strategies 0, 4, and 5 are strictly dominated then no rational player will use them. Furthermore, if a player believes the other player is rational then she knows the other player will not deploy 0, 4, or 5. Having eliminated strategy 0, strategy 2 now strictly dominates strategy 3. (Recall that strategy 2 weakly dominated strategy 3 and had identical payoffs only when the other player chose 0 which has now been eliminated.) Strategy 2 also strictly dominates strategy 1 as they yield payoffs of 10 and 7, respectively, when \( q_i = 1 \); payoffs of 8 and 6, respectively, when \( q_i = 2 \); and payoffs of 6 and 5, respectively, when \( q_i = 3 \). The only strategy "standing" is then strategy 2. Hence, a player who is rational and believes the other player is rational will use strategy 2.

3.5 For player 1, strategy \( a \) strictly dominates \( c \); for player 2, \( y \) strictly dominates \( w \) and \( x \). Because players are rational, those strategies can be eliminated. Since player 1 knows that player 2 is rational, it follows that player 1 knows that player 2 will not use \( w \) and \( x \). With those strategies for player 2 eliminated, \( a \) strictly dominates \( b \) for player 1. Analogously, since player 2 knows that player 1 is rational, it follows that player 2 knows that player 1 will not use \( c \). With that strategy for player 1 eliminated, \( z \) strictly dominates \( y \) for player 2. The answer is then strategies \( a \) and \( d \) for player 1 and strategy \( z \) for player 2. Because this is the same game used in CYU 3.2, we can continue on to step 3. After step 2, strategies \( a \) and \( d \) remain for player 1 and strategy \( z \) for player 2. Strategy \( d \) then strictly dominates \( a \). Accordingly, this game is dominance solvable, as there is a unique strategy pair that survives the IDSDS: Player 1 uses \( d \) and player 2 uses \( z \).

3.6 In round 1 of the IDSDS, \( flat \) strictly dominates \( down \) for player 1; \( fast \) strictly dominates \( slow \) for player 2; and neither strategy is strictly dominated for player 3. After eliminating those strategies, the reduced game is

\[
\begin{array}{c|cc}
\text{Player 3: Left} & \text{Fast} \\
\hline
\text{Player 2} \\
\hline
\text{Up} & 3, 12 \\
\text{Flat} & 1, 5, 0 \\
\end{array}
\]

\[
\begin{array}{c|cc}
\text{Player 3: Right} & \text{Fast} \\
\hline
\text{Player 2} \\
\hline
\text{Up} & 3, 15 \\
\text{Flat} & 5, 2, 1 \\
\end{array}
\]

In round 2, \( right \) strictly dominates \( left \) for player 3. Neither \( up \) nor \( flat \) are strictly dominated for player 1. The reduced game after two rounds is:

\[
\begin{array}{c|cc}
\text{Player 3: Right} & \text{Fast} \\
\hline
\text{Player 2} \\
\hline
\text{Up} & 3, 15 \\
\text{Flat} & 5, 2, 1 \\
\end{array}
\]

In round 3, \( flat \) strictly dominates \( up \) for player 1. In conclusion, we find that player 1 chooses \( flat \), player 2 chooses \( fast \), and player 3 chooses \( right \).