A Game with Altruistic Players

Two players play a single round of a game with two possible strategies, $C$ and $D$. They choose their strategies simultaneously. If both players play $C$, they each get a money payoff of $4$. If both players play $D$, they each get a money payoff of $3$. If one player plays $C$ and the other plays $D$, the player who played $D$ gets $6$ and the player who played $C$ gets $0$.

A) These players are not entirely “selfish.” Where $m_1$ and $m_2$ are the money payoffs of players 1 and 2, preferences of player 1 are represented by a utility function $m_1 + am_2$ and preferences of player 2 are represented by a utility function $m_2 + am_1$. The players act as if their payoffs are as given by their utility functions. Write down the strategic form description of the resulting game. For what range of values of $a$ is this game a Prisoners’ Dilemma? For values of $a$ where the game is not a Prisoners’ Dilemma, find the Nash equilibria.

B) Suppose that Player 2 cares only about his own money payoff, but Player 1 cares about the payoffs of both players. Player 1’s preferences are represented by the utility function $m_1 + am_2$. Player 2’s preferences are represented by the utility function $m_2$. Write down the strategic form description of the resulting game. For what range of values of $a$ is this game a Prisoners’ Dilemma? For values of $a$ where the game is not a Prisoners’ Dilemma, find the Nash equilibria.

C) Suppose that the game is as before, except that when both play $D$, each gets a payoff of $1$. Answer the questions in Parts A and B for this case.