We all recognize that information technology has revolutionized the way we produce and consume. Some think that it is necessary to have a “new economics” to understand this New Economy. We think not. The economic tools that you have learned in this course can offer very powerful insights into the economics of information technology, as we illustrate in this set of problems.

1.1 (2) Bill Barriers, the president of MightySoft software company is about to introduce a new computer operating system called DoorKnobs. Because it is easier to swap files with people who have the same operating system, the amount people are willing to pay to have DoorKnobs on their computers is greater the larger they believe DoorKnobs’s market share to be.

The perceived market share for DoorKnobs is the fraction of all computers that the public believes is using DoorKnobs. When the price of DoorKnobs is \( p \), then its actual market share is the fraction of all computer owners that would be willing to pay at least \( \$p \) to have DoorKnobs installed on their computers. Market researchers have discovered that if DoorKnobs’s perceived market share is \( s \) and the price of DoorKnobs is \( \$p \), then its actual market share will be \( x \), where \( x \) is related to the price \( p \) and perceived market share \( s \) by the formula

\[
p = 256s(1 - x). \tag{1}
\]

In the short run, MightySoft can influence the perceived market share of DoorKnobs by publicity, advertising, giving liquor and gifts to friendly journalists, and giving away copies in conspicuous ways. In the long run, the truth will emerge, and DoorKnobs’s perceived market share \( s \) must equal its actual market share \( x \).

(a) If the perceived market share is \( s \), then the demand curve for DoorKnobs is given by Equation 1. On the graph below, draw the demand curve relating price to actual market share in the case in which DoorKnobs’s perceived market share is \( s = 1/2 \). Label this curve \( s = 1/2 \).

(b) On the demand curve that you just drew with \( s = 1/2 \), mark a red dot on the point at which the actual market share of DoorKnobs is \( 1/2 \). (This is the point on the demand curve directly above \( x = 1/2 \).) What is the price at which half of the computer owners actually want to buy DoorKnobs, given that everybody believes that half of all computer owners want to buy DoorKnobs? \( \$64 \)
(c) On the same graph, draw and label a separate demand curve for the case where DoorKnobs’s perceived market share \( s \) takes on each of the following values: \( s = 1/8, 1/4, 3/4, 7/8, 1. \)

\[ \text{Actual Market Share (in sixteenths)} \]
\[ \text{Willingness to Pay} \]

(d) On the demand curve for a perceived market share of \( s = 1/4 \), put a red dot on the point at which the actual market share of DoorKnobs is 1/4. (This is the point on this demand curve directly above \( x = 1/4 \).) If the perceived market share of DoorKnobs is 1/4, at what price is the actual market share of DoorKnobs also 1/4? \( \$48 \)

(e) Just as you did for \( s = 1/2 \) and \( s = 1/4 \), make red marks on the demand curves corresponding to \( s = 1/8, 3/4, 7/8, \) and 1, showing the price at which the actual market share is \( s \), given that the perceived market share is \( s \).

(f) Let us now draw the long-run demand curve for DoorKnobs, where we assume that computer owners’ perceived market shares \( s \) are the same as the actual market shares \( x \). If this is the case, it must be that \( s = x \), so the demand curve is given by \( p = 256x(1 - x) \). On the graph above, plot a few points on this curve and sketch in an approximation of the curve. (Hint: Note that the curve you draw must go through all the red points that you have already plotted.)

(g) Suppose that MightySoft sets a price of $48 for DoorKnobs and sticks with that price. There are three different perceived market shares such that the fraction of consumers who would actually want to buy DoorKnobs for $48 is equal to the perceived market share. One such perceived
market share is 0. What are the other two possibilities? \( s = 1/4 \) and \( s = 3/4 \)

(h) Suppose that by using its advertising and media influence, MightySoft can temporarily set its perceived market share at any number between 0 and 1. If DoorKnobs’s perceived market share is \( x \) and if MightySoft charges a price \( p = 256x(1-x) \), the actual market fraction will also be \( x \) and the earlier perceptions will be reinforced and maintained. Assuming that MightySoft chooses a perceived market share \( x \) and a price that makes the actual market share equal to the perceived market share, what market share \( x \) should MightySoft choose in order to maximize its revenue and what price should it charge in order to maintain this market share? (Hint: Revenue is \( px = 256x^2(1-x) \).) Use calculus and show your work. \( x = 2/3 \). The first-order condition is

\[
\frac{d}{dx} 256(x^2 - x^3) = 0. \text{ This implies } 2x = 3x^2, \]

which implies that \( x = 2/3 \) or \( x = 0 \). The second order condition is satisfied only when \( x = 2/3 \). Price should be \( $256 \times 1/3 \times 2/3 = $56.89 \).

1.2 (1) Suppose that demand for DoorKnobs is as given in the previous problem, and assume that the perceived market share in any period is equal to the actual market share in the previous period. Then where \( x_t \) is the actual market share in period \( t \), the equation \( p = 256x_{t-1}(1-x_t) \) is satisfied. Rearranging this equation, we find that \( x_t = 1 - (p/256x_{t-1}) \) whenever \( p/256x_{t-1} \leq 1 \). If \( p/256x_{t-1} \leq 0 \), then \( x_t = 0 \). With this formula, if we know actual market share for any time period, we can calculate market share for the next period.

Let us assume that DoorKnobs sets the price at \( p = $32 \) and never changes this price. (To answer the following questions, you will find a calculator useful.)

(a) If the actual market share in the first period was 1/2, find the actual market share in the second period .75 , the third period .833 . Write down the actual market shares for the next few periods .8529 , 8534 . Do they seem to be approaching a limit? If so, what? .853553 .
Notice that when price is held constant at \( p \), if DoorKnobs's market share converges to a constant \( x \), it must be that \( x = 1 - (p/256\bar{x}) \). Solve this equation for \( x \) in the case where \( p = $32 \). What do you make of the fact that there are two solutions? This equation implies \( x^2 - x + 1/8 = 0 \). Solutions are \( x = 0.85355 \) and \( x = 0.14645 \). Both are equilibrium market shares with a price of $32.

1.3 (1) A group of 13 consumers are considering whether to connect to a new computer network. Consumer 1 has an initial value of $1 for hooking up to the network, consumer 2 has an initial value of $2, consumer 3 has an initial value of $3, and so on up to consumer 13. Each consumer’s willingness to pay to connect to the network depends on the total number of persons who are connected to it. In fact, for each \( i \), consumer \( i \)’s willingness to pay to connect to the network is \( i \) times the total number of persons connected. Thus if 5 people are connected to the network, consumer 1’s willingness to pay is $5, consumer 2’s willingness to pay is $10 and so on.

(a) What is the highest price at which 9 customers could hook up to the market and all of them either make a profit or break even? $45

(b) Suppose that the industry that supplies the computer network is competitive and that the cost of hooking up each consumer to the network is $45. Suppose that consumers are very conservative and nobody will sign up for the network unless her buyer value will be at least as high as the price she paid as soon as she signs up. How many people will sign up if the price is $45?

0

(c) Suppose that the government offers to subsidize “pioneer users” of the system. The first two users are allowed to connect for $10 each. After the first two users are hooked up, the government allows the next two to connect for $25. After that, everyone who signs up will have to pay the full cost of $45. Assume that users remain so conservative that will sign up only if their buyer values will be at least equal to the price they are charged when they connect. With the subsidy in place, how many consumers in toto will sign up for the network?

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1.4 (2) Professor Kremepuff has written a new, highly simplified economics text, Microeconomics for the Muddleheaded, which will be published by East Frisian Press. The first edition of this book will be in print for two years, at which time it will be replaced by a new edition. East Frisian
Press has already made all its fixed cost investments in the book and must pay a constant marginal cost of $c$ for each copy that it sells.

Let \( p_1 \) be the price charged for new copies sold in the first year of publication and let \( p_2 \) be the price charged for new copies sold in the second year of publication. The publisher and the students who buy the book are aware that there will be an active market for used copies of *Microeconomics for the Muddleheaded* one year after publication and that used copies of the first edition will have zero resale value two years after publication. At the end of the first year of publication, students can resell their used textbooks to bookstores for 40% of the second-year price, \( p_2 \).

The net cost to a student of buying the book in the first year, using it for class, and reselling it at the end of the year is \( p_1 - 0.4p_2 \). The number of copies demanded in the first year of publication is given by a demand function, \( q_1 = D_1(p_1 - 0.4p_2) \).

Some of the students who use the book in the first year of publication will want to keep their copies for future reference, and some will damage their books so that they cannot be resold. The cost of keeping one’s old copy or of damaging it is the resale price \( 0.4p_2 \). The number of books that are either damaged or kept for reference is given by a “keepers” demand function, \( D^k(0.4p_2) \). It follows that the number of used copies available at the end of the first year will be \( D_1(p_1 - 0.4p_2) - D^k(0.4p_2) \).

Students who buy *Microeconomics for the Muddleheaded* in the second year of publication will not be able to resell their used copies, since a new edition will then be available. These students can, however, buy either a new copy or a used copy of the book. For simplicity of calculations, let us assume that students are indifferent between buying a new copy or a used copy and that used copies cost the same as new copies in the bookstore. (The results would be the same if students preferred new to used copies, but bookstores priced used copies so that students were indifferent between buying new and used copies.) The total number of copies, new and used, that are purchased in the second year of publication is \( q_2 = D_2(p_2) \).

(a) Write an expression for the number of new copies that East Frisian Press can sell in the second year after publication if it sets prices \( p_1 \) in year 1 and \( p_2 \) in year 2. \( D_2(p_2) - D_1(p_1 - 0.4p_2) + D^k(0.4p_2) \).

(b) Write an expression for the total number of new copies of *Microeconomics for the Muddleheaded* that East Frisian can sell over two years at prices \( p_1 \) and \( p_2 \) in years 1 and 2. \( D_1(p_1 - 0.4p_2) + D_2(p_2) - D_1(p_1 - 0.4p_2) + D^k(0.4p_2) = D_2(p_2) + D^k(0.4p_2) \).

(c) Would the total number of copies sold over two years increase, decrease, or remain constant if \( p_1 \) were increased and \( p_2 \) remained constant?

It would remain constant.
Write an expression for the total revenue that East Frisian Press will receive over the next two years if it sets prices \( p_1 \) and \( p_2 \).

\[
p_1 D_1(p_1 - .4p_2) + p_2(D_2(p_2) - D_1(p_1 - .4p_2) + D^k(.4p_2)) = (p_1 - p_2)D_1(p_1 - .4p_2) + p_2(D_2(p_2) + D^k(.4p_2)).
\]

To maximize its total profits over the next two years, East Frisian must maximize the difference between its total revenue and its variable costs. Show that this difference can be written as

\[
(p_1 - p_2)D_1(p_1 - .4p_2) + (p_2 - c)[D_2(p_2) + D^k(.4p_2)].
\]

Variable cost is \( c(D_2(p_2) + D^k(.4p_2)) \).

Subtract this from previous answer.

Suppose that East Frisian has decided that it must charge the same price for the first edition in both years that it is sold. Thus it must set \( p = p_1 = p_2 \). Write an expression for East Frisian’s revenue net of variable costs over the next two years as a function of \( p \).

\[
(p - c)(D_2(p) + D^k(.4p))
\]

Suppose that East Frisian Press, discussed in the previous problem, has a constant marginal cost of \( c = \$10 \) for each copy of *Microeconomics for the Muddleheaded* that it sells and let the demand functions be

\[
D_1(p_1 - 0.4p_2) = 100(90 - p_1 + 0.4p_2)
\]

\[
D_2(p_2) = 100(90 - p_2).
\]

The number of books that people either damage or keep for reference after the first year is

\[
D^k(0.4p_2) = 100(90 - 0.8p_2).
\]

(This assumption is consistent with the assumption that everyone’s willingness to pay for keeping the book is half as great as her willingness to pay to have the book while she is taking the course.) Assume that East Frisian Press is determined to charge the same price in both years, so that \( p_1 = p_2 = p \).
(a) If East Frisian Press charges the same price \( p \) for *Microeconomics for the Muddleheaded* in the first and second years, show that the total sales of new copies over the two years are equal to

\[
18,000 - 180p.
\]

**Total sales are** \( D_2(p) + D^k(0.4p_2) = 100(90 - p) + 100(90 - 0.8p)) = 18,000 - 180p \)

(b) Write an expression for East Frisian’s total revenue, net of variable costs, over the first two years as a function of the price \( p \).  
\[
(p - 10)(18,000 - 180p) = 19,800p - 180p^2 - 180,000
\]

(c) Solve for the price \( p \) that maximizes its total revenue net of variable costs over the first two years.  
\[
p = $55.\] At this price, the net cost to students in the first year of buying the text and reselling it is $33. 

The total number of copies sold in the first year will be 5,700. The total number of copies that are resold as used books is 1,100. The total number of copies purchased by students in the second year will be 3,500. (Remember students in the second year know that they cannot resell the book, so they have to pay the full price \( p \) for using it.) The total number of new copies purchased by students in the second year will be 2,400. Total revenue net of variable costs over the two years will be $364,500.

1.6 (2) East Frisian Press is trying to decide whether it would be profitable to produce a new edition of *Microeconomics for the Muddleheaded* after one year rather than after two years. If it produces a new edition after one year, it will destroy the used book market and all copies that are purchased will be new copies. In this case, the number of new copies that will be demanded in each of the two years will be 100(90 − \( p \)), where \( p \) is the price charged. The variable cost of each copy sold remains $10.

(a) Write an expression for the total number of copies sold over the course of two years if the price is \( p \) in each year  
\[
200(90 - p).\] Also, write an expression for total revenue net of variable costs as a function of \( p \). 
\[
200(p - 10)(90 - p).\]
Find the price that maximizes total revenue net of variable costs. 

$50.

The total number of new books sold in the first year would be 4,000, and the total number of books sold in the second year would be 4,000.

east Frisian’s total revenue net of variable costs, if it markets a new edition after one year, will be $320,000.

Would it be more profitable for East Frisian Press to produce a new edition after one year or after two years? After two years. Which would be better for students? (Hint: The answer is not the same for all students.) After two years is better for students who take the course in the first year of publication and plan to sell. After one year is better for the other students.

Suppose that East Frisian Press publishes a new edition only after two years and that demands and costs are as in the previous problems.

Suppose that it sets two different prices \( p_1 \) and \( p_2 \) in the two periods.

Write an expression for the total number of new copies sold at prices \( p_1 \) and \( p_2 \) and show that this number depends on \( p_2 \) but not on \( p_1 \).

\[
100 ((70 - p_1 + .4p_2) + (140 - 1.8p_2) - (70 - p_1 + .4p_2)) = 100(140 - 1.8p_2)
\]

Show that at prices \( p_1 \) and \( p_2 \), the difference between revenues and variable costs is equal to

\[
100 (90p_1 + 108p_2 + 1.4p_1p_2 - p_1^2 - 2.2p_2^2 - 1,800).
\]

This difference is

\[
100(p_1 - p_2)(90 - p_1 + .4p_2) + (p_2 - 10)(180 - 1.8p_2).\]

Expand this expression.
(c) Calculate the prices $p_1$ and $p_2$ that maximize the difference between total revenue and variable costs and hence maximize profits. $p_1 = \$80$, $p_2 = \$50$

(d) If East Frisian Press chooses its profit-maximizing $p_1$ and $p_2$, compare the cost of using Microeconomics for the Muddlehead for a student who buys the book when it is first published and resells it at the end of the first year with the cost for a student who buys the book at the beginning of the second year and then discards it. The former has a net cost of $\$80 - 0.4 \times 50 = \$60$ and the latter has a cost of $\$50$.

1.8 (2) The Silicon Valley company Intoot produces checkwriting software. The program itself, Fasten, sells for $\$50$ and includes a package of checks. Check refill packets for Fasten cost $\$20$ to produce and Intoot sells the checks at cost. Suppose that a consumer purchases Fasten for $\$50$ in period 1 and spends $\$20$ on checks in each subsequent period. Assume for simplicity that the consumer uses the program for an infinite number of periods.

(a) If the interest rate is $r = 0.10$ per period, what is the present value of the stream of payments made by the consumer? (Hint: a stream of payments of $x$ starting next period has a present value of $x/r$.) The total cost of ownership of Fasten is $50 + 20 / 0.10 = \$250$.

(b) Fasten’s competitor produces an equally effective product called Czechwriter. Czechwriter can do everything Fasten can do and vice versa except that Fasten cannot use check refill packets that are sold by anyone other than Fasten. Czechwriter also sells for $\$50$ and sells its checks for $\$20$ per period. A Fasten customer can switch to Czechwriter simply by purchasing the program. This means his switching costs are $\$50$.

(c) Fasten is contemplating raising the price of checks to $\$30$ per period. If so, will its customers switch to Czechwriter? Explain. Yes, the present value of continuing to use Fasten are $\$300$ while the costs of switching to Czechwriter are $\$250$. 

(d) Fasten contemplates raising the price of checks to $22 per period. Will its customers switch? No. The present value of continuing to use Fasten are $220 while the present value of switching to Czechwriter is $250.

(e) At what price for checks will Fasten’s customers just be indifferent to switching? (Hint: Let $x$ be this amount. Compare the present value of staying with Fasten with the present value of switching to Czechwriter.) Solve the equation $x/0.10 = 250$ to find $x = 25$.

(f) If it charges the highest price that it can without making its customers switch, what profit does Fasten make on checks from each of its customers per period? $5$. What is the present value of the profit per customer that Fasten gets if it sets the price of checks equal to the number determined in the last question? $PV = 5/0.10 = 50$. How does this compare to the customer switching cost? It is the same.

(g) Suppose now that the cost of switching also involves several hours of data conversion that the consumer values at $100. The total cost of switching is the cost of the new program plus the data conversion cost which is $150.

(h) Making allowances for the cost of data conversion, what is the highest price that Intoot can charge for its checks? Solve $x/0.10 = 250 + 100$ for $x = 35$. What is the present value of profit from this price? $150$. How does this compare to total switching costs? It is the same.

(i) Suppose that someone writes a computer program that eliminates the cost of converting data and makes this program available for free. Suppose that Intoot continues to price its check refill packages at $25. A new customer is contemplating buying Fasten at a price of $50 and paying $25 per period for checks, versus paying $50 for Czechwriter and paying $20 for checks. If the functionality of the software is identical, which will the consumer buy? Czechwriter.
(j) Intoot decides to distribute a coupon that offers a discount of $50 off of the regular purchase price. What price would it have to set to make consumers indifferent between purchasing Fasten and Czechwriter?

Solve $50 + 25/.10 - d = 50 + 20/.10$ to find that the discount should be $50.$

(k) Suppose that consumers are shortsighted and only look at the cost of the software itself, neglecting the cost of the checks. Which program would they buy if Intoot offered this coupon? Fasten. How might Czechwriter respond to the Fasten offer? Issue its own coupon for $50 and raise the price of its checks to $35.$

1.9 (2) Sol Microsystems has recently invented a new language, Guava, which runs on a proprietary chip, the Guavachip. The chip can only be used to run Guava, and Guava can only run on the Guavachip. Sol estimates that if it sells the chip for a price $p_c$ and the language for a price $p_g$, the demand for the chip-language system will be

$$x = 120 - (p_c + p_g).$$

(a) Sol initially sets up two independent subsidiaries, one to produce the chip and one to produce the language. Each of the subsidiaries will price its product so as to maximize its profits, while assuming that a change in its own price will not affect the pricing decision of the other subsidiary. Assume that marginal costs are negligible for each company. If the price of the language is set at $p_g$, the chip company’s profit function (neglecting fixed costs) is

$$\pi_c = [120 - p_c - p_g]p_c$$

(b) Differentiate this profit function with respect to $p_c$ and set the result equal to zero to calculate the optimal choice of $p_c$ as a function of $p_g$.

$$p_c = 120 - 2p_g.$$  

(c) Now consider the language subsidiary’s pricing decision. The optimal choice of $p_g$ as a function of $p_c$ is

$$p_g = 120 - 2p_c.$$  

(d) Solving these two equations in two unknowns, we find that $p_c = 40$ and $p_g = 40$, so that $p_c + p_g = 80$. 
(e) Sol Microsystems decides that the independent subsidiary system is cumbersome, so it sets up Guava Computing which sells a bundled system consisting of the chip and the language. Let \( p \) be the price of the bundle. Guava Computing’s profit function is \( \[120 - p\]p \)

(f) Differentiate this profit with respect to \( p \) and set the resulting expression to zero to determine \( p = 60 \)

(g) Compare the prices charged by the integrated system and the separate subsidiaries. Which is lower? Integrated system. Which is better for consumers? Integrated system. Which makes more profit? Integrated system.

1.10 (2) South Belgium Press produces the academic journal Nanoconomics, which has a loyal following among short microeconomists, and Gigaeconomics, a journal for tall macroeconomists. It offers a license for the electronic version of each journal to university libraries at a subscription cost per journal of \$1,000 per year. The 200 top universities all subscribe to both journals, each paying \$2,000 per year to South Belgium. By revealed preference, their willingness to pay for each journal is at least \$1,000.

(a) In an attempt to lower costs, universities decide to form pairs, with one member of each pair subscribing to Nanoconomics and one member of each pair subscribing to Gigaeconomics. They agree to use interlibrary loan to share the other journal. Since the copies are electronic, there is no incremental cost to doing this. Under this pairing scheme, how many subscriptions of each journal will South Belgium sell? 100.

(b) In order to stem the revenue hemorrhage, South Belgium raises the price of each journal. Assuming library preferences and budgets haven’t changed, how high can they set this price? They can raise the price to \$2,000, since libraries have already indicated that they are willing to pay this much for the pair of journals.

(c) How does library expenditure and South Belgium’s revenue compare to those of the previous regime? They remain the same.
(d) If there were a cost of interlibrary loan, how would your answer change? Assuming they still bought both journals, libraries would be worse off since they would have to pay the transactions cost for interlibrary loan.