Answer to Kreps Problem 18.1

(a) First calculate Drake’s expected utility if he has no insurance. This is

\[ 0.7 \times \sqrt{90,000} + 0.3 \times \sqrt{40,000} = 270. \]

Now calculate his utility if he buys the insurance policy. This is

\[ 0.7 \times \sqrt{80,000} + 0.3 \times \sqrt{60,000} = 271.46 \]

He would buy it.

(b) Let \( P \) be the premium and \( Q \) be the payout if Drake doesn’t get job. Expected payout is \( .3Q \), so we need \( .3Q \leq P \). Efficiency in risk sharing means that Drake would be fully insured, so that his net income is the same no matter what happens. This means that \( Q = 50,000 \). Drake’s certainty equivalent if he gets no insurance is the solution to \( \sqrt{X} = 270 \) or \( X = 270^2 = 72,900 \). If Drake pays \( P \) for full insurance, his income is certain to be \( 90,000 - P \). We need \( 90,000 - P \geq 72,900 \) for Drake to buy the insurance. That is, \( P \leq 17,100 \). Putting these conditions all together, we need \( P \geq 15,000 \) for the insurance company to at least break even and we need \( P \leq 17,100 \) for Drake to buy the insurance.

(c) Who will buy the insurance at a price of $15,000? Expected utility if you buy the insurance is \( \sqrt{75,000} = 273.8 \). Expected utility of no insurance if your probability of getting a job is \( q \) will be \( q \times 300 + (1 - q) \times 200 = 200 + 100q \). Only people with probability \( q \leq .738 \) will buy insurance. Beantown’s customers will be 100 people with \( q = .5 \), 100 with \( q = .6 \) and 100 with \( q = .7 \). Its expected payout per customer will be \( .4 \times \$50,000 = \$20,000 \). Its revenue per customer is only $15,000.

(d) Beantown could charge a premium just over $25,000. This would attract people who have a probability of \( .5 \) of getting a job. If they charged $25,000, expected utility of these people if insured would be \( \sqrt{65,000} = 254.9 \) as compared to the utility of 250 that they would get with no insurance. These people would buy insurance so long as \( \sqrt{90,000 - P} \geq 250 \), which means \( P \leq 90,000 - 62,500 = 27,500 \).

A uniform rate would not allow Beantown to make money and attract customers with higher probability of getting a job than \( .5 \). To attract people with probability \( .6 \) of getting the job, it would have to offer them a price such that \( \sqrt{90,000 - P} \geq 260 \). This means \( 90,000 - P \geq 67,600 \), which means \( P \leq 22,400 \). But to break even the insurance company would need to receive at least \( .45 \times 50,000 = 22,500 \).

(e) Suppose that somebody has probability \( q \) of getting a job. He will want this policy if

\[ q \sqrt{76,000} + (1 - q) \sqrt{56,000} \geq 200 + 100q. \]

This is the case if

\[ 275.6q + 236.6(1 - q) \geq 200 + 100q. \]
Simplifying, we have $61q \leq 36.6$ and $a \leq .6$. So this contract would attract the .5 and .6 types and not the others. Beantown’s expected payout per customer would be $30,000 \times .45 = $13,500, so it would make a profit selling this insurance for $14,000.

(f) Hint: If your probability of getting a job is $q$, and you buy no insurance, your expected utility is $200 + 100q$. If you buy full insurance, your expected utility is 254. If you buy the the 10,000 policy at 3500 is

$$q\sqrt{86,500} + (1 - q)\sqrt{46500} = 215.6 + 78.5q.$$ Make a similar calculation for the other two policies. Each of these is a linear expression. Draw the lines on a graph. Think about how you will tell who will choose each policy.

**Answer to Kreps Problem 18.2**

Suppose that a person has probability $p$ of having a fire. If he has no insurance, his expected utility will be:

$$p\sqrt{10,000} + (1 - p)\sqrt{90,000} = 300 - 200p.$$ If he has full insurance and pays a premium of $11,600, his expected utility will be

$$\sqrt{90,000 - 11,600} = 280.$$ If he has partial insurance that pays back $58400 in the event of a fire and that charges a premium of $5900, then his expected utility will be:

$$p\sqrt{68,400 - 5900} + (1 - p)\sqrt{90,000 - 5900} = 250p + 290(1 - p) = 290 - 40p.$$ 

**Part a)**

Mr. Reece has $p = .1$. His utility will be 280 if he buys full insurance, 280 if he buys full insurance, and 286 if he buys partial insurance. So he will buy partial insurance.

Mr. Yost has $p=.03$. His utility will be 294 if he buys no insurance, 280 if he buys full insurance, and 288.8 if he buys partial insurance. So he will buy no insurance.

**Part b)**

A person will buy no insurance if $300 - 200p > 280$ and $300 - 200p > 290 - 40p$. The first inequality simplifies to $p < 1/10$ and the second inequality simplifies to $p < 1/16$. Therefore a person will buy no insurance if $p < 1/16$.

A person will buy partial insurance if $290 - 40p > 280$ and $290 - 40p > 300 - 200p$. The first inequality is satisfied if $p < 1/4$ and the second is satisfied if $p > 1/16$. So a person will choose partial insurance if $1/16 < p < 1/4$. 

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A person will buy full insurance if $280 > 290 - 40p$ and $280 > 300 - 200p$. The first inequality is satisfied if $p > 1/4$ and the second is satisfied if $p > 1/10$. Therefore a person will buy full insurance if $p > 1/4$.

**Part c)**

Many of you fell into the trap of thinking that if the people who currently buy full insurance were to buy partial insurance, the company would make as much profit from them as it does from the people who currently buy partial insurance. This is NOT true. The people who are currently buying full insurance are a much higher risk population. If these guys buy partial insurance, the company will still lose money on them, but not so much as when it sold them full insurance.

The information in this problem is sufficient for us to calculate the average probability of fire for those with full insurance and for those with partial insurance.

Since the fully insured customers pay $11,600 each and the insurance company loses an average of $12,400 on each of them, it must be that the expected payout per fully insured customer is $11,600 + 12,400 = 24,000$. Therefore if $p$ is the average risk for these customers, we have $80,000p = 24,000$ and so $p = 3/128$.

Since the partially insured customers pay $5900 and the company makes an average profit of $1228 per customer, it must be that the average payout per customer is $5900 - 1228 = 4672$. This means that if $p$ is the average risk for these customers, $58,400p = 4672$ and so $p = .08$. (Note that this number is between 1/16 and 1/4, as required by the answer to the previous question.)

If the firm only sells partial insurance, but maintains the same rates, it will continue to sell partial insurance to the 100,000 people who previously bought partial insurance. Profits received from these people will not change. The 5,000 people who in the past bought full insurance will certainly want partial insurance rather than no insurance. The average payout per person for these 5,000 people will be $.3*58,400 = 17,520$. The premium received from each of them will be $5900$, so the average loss from each of these customers will be $17,520 − 5,900 = 11,620$. This is a smaller loss than when the company sold them full insurance. In particular, the company saves an average of $12,400 - 11,620 = $780 per person. Since there are 5,000 of these people, the total increase in profits from dropping the full insurance offering is $5,000 \times 780 = 3,900,000$. Its total profits will now be $60.8 + 3.9 = 64.7$ million.

**Answer to Kreps Problem 19.2**

A) With no insurance, the owner’s utility if he does not take due care is:

$$0.05 \sqrt{1,000,000} + 0.95 \sqrt{9,000,000} = 2900$$

and if he takes due care his utility is

$$0.01 \sqrt{1,000,000} + 0.99 \sqrt{9,000,000} - 50 = 2930.$$  

He is better off taking due care.
B) If he can contractually specify due care, the insurance company could sell insurance contingent on due care for any price $P$ such that

$$\sqrt{9,000,000 - P - 50} \geq 2930$$

The left side is expected utility of buying full insurance and taking due care. The right side is expected utility of not buying insurance and taking due care. The owner will buy the insurance for any price less than $P$ where $P$ solves the equation above. The solution for this equation is $P = 119,600$.

C) With this policy, it will pay owner to take no care. The most that the company can charge is $P$ such that

$$\sqrt{9,000,000 - P - 50} = 2930.$$

The solution is $P = 415,100$. The insurance company’s expected payout will be $.05 \times 8,000,000 = 400,000$. So it would make a profit of 15,100.

D) Let $P$ be the premium charged and $D$ be the deductible. For this policy to be accepted by the buyer, it must be that the buyer is at least as well off with the policy as he would be by having no insurance and taking due care. For this policy to induce the buyer to take due care, it must be that the buyer’s expected utility is at least as high if he takes due care as if he does not. Let us solve for $P$ and $D$ so that he is just indifferent about buying the policy or not and also just indifferent about whether to take due care. This gives us two equations.

$$0.01 \sqrt{9,000,000 - P - D} + 0.99 \sqrt{9,000,000 - P - 50} = 2930.$$  
$$0.05 \sqrt{9,000,000 - P - D} + 0.99 \sqrt{9,000,000 - P} = 2930.$$  

To solve these equations, let $A = \sqrt{9,000,000 - P - D}$ and $B = \sqrt{9,000,000 - P}$. Then these are two linear equations:

$$0.01A + 0.99B = 2980$$  
$$0.05A + 0.95B = 2930.$$  

When I solve this, I get $B = 2992.50$ and $A = 1742.50$. Then I solve the equations $A = \sqrt{9,000,000 - P - D}$ and $B = \sqrt{9,000,000 - P}$ by squaring each side to find that $9,000,000 - P = 2992.5^2$, so that $P - 9,000,000 - 2992.5^2 = 44,943.5$ and $9,000,000 - P - D = 1742.5^2$ and so $D = 9,000,000 - P - 1742.5^2 = 5,918,750$.

In this case the company’s expected payoff is $.01(8,000,000 - 5918750) = 20,812$. Its profits will be the premium minus expected payoff, which is 44,943.5 – 20812 = 24,131.5.