Answer to Kreps Problem 18.1

(a) First calculate Drake’s expected utility if he has no insurance. This is

\[ 0.7 \times \sqrt{90,000} + 0.3 \times \sqrt{40,000} = 270. \]

Now calculate his utility if he buys the insurance policy. This is

\[ 0.7 \times \sqrt{80,000} + 0.3 \times \sqrt{60,000} = 271.46 \]

He would buy it.

(b) Let \( P \) be the premium and \( Q \) be the payout if Drake doesn’t get job. Expected payout is \( 0.3Q \), so we need \( 0.3Q \leq P \). Efficiency in risk sharing means that Drake would be fully insured, so that his net income is the same no matter what happens. This means that \( Q = 50,000 \). Drake’s certainty equivalent if he gets no insurance is the solution to \( \sqrt{X} = 270 \) or \( X = 270^2 = 72,900 \). If Drake pays \( P \) for full insurance, his income is certain to be \( 90,000 - P \). We need \( 90,000 - P \geq 72,900 \) for Drake to buy the insurance. That is, \( P \leq 17,100 \). Putting these conditions all together, we need \( P \geq 15,000 \) for the insurance company to at least break even and we need \( P \leq 17,100 \) for Drake to buy the insurance.

(c) Who will buy the insurance at a price of \$15,000\? Expected utility if you buy the insurance is \( \sqrt{75,000} = 273.8 \). Expected utility of no insurance if your probability of getting a job is \( q \) will be \( q \times 300 + (1 - q) \times 200 = 200 + 100q \). Only people with probability \( q \leq 0.738 \) will buy insurance. Beantown’s customers will be 100 people with \( q = 0.5 \), 100 with \( q = 0.6 \) and 100 with \( q = 0.7 \). Its expected payout per customer will be \( 0.4 \times 50,000 = 20,000 \). Its revenue per customer is only \$15,000.

(d) Beantown could charge a premium just over \$25,000. This would attract people who have a probability of \( 0.5 \) of getting a job. If they charged \$25,000, expected utility of these people if insured would be \( \sqrt{65,000} = 254.9 \) as compared to the utility of 250 that they would get with no insurance. These people would buy insurance so long as \( \sqrt{90,000 - P} \geq 250 \) which means \( P \leq 90,000 - 62,500 = 27,500 \).

A uniform rate would not allow Beantown to make money and attract customers with higher probability of getting a job than \( 0.5 \). To attract people with probability \( 0.6 \) of getting the job, it would have to offer them a price such that \( \sqrt{90,000 - P} \geq 260 \). This means \( 90,000 - P \geq 67,600 \), which means \( P \leq 22,400 \). But to break even the insurance company would need to receive at least \( 0.45 \times 50,000 = 22,500 \).

(e) Suppose that somebody has probability \( q \) of getting a job. He will want this policy if

\[ q \sqrt{76,000} + (1 - q) \sqrt{56,000} \geq 200 + 100q. \]

This is the case if

\[ 275.6q + 236.6(1 - q) \geq 200 + 100q. \]
Simplifying, we have $61q \leq 36.6$ and $a \leq 0.6$. So this contract would attract the .5 and .6 types and not the others. Beantown’s expected payout per customer would be $30,000 \times 0.45 = $13,500, so it would make a profit selling this insurance for $14,000.

(f) Hint: If your probability of getting a job is $q$, and you buy no insurance, your expected utility is $200 + 100q$. If you buy full insurance, your expected utility is 254. If you buy the the 10,000 policy at 3500 is

$$q\sqrt{86,500} + (1-q)\sqrt{46500} = 215.6 + 78.5q.$$ 

Make a similar calculation for the other two policies. Each of these is a linear expression. Draw the lines on a graph. Think about how you will tell who will choose each policy.