19. Incentives

In many economic transactions, one party takes actions that affect the welfare of another party, actions that cannot be specified contractually. The actions chosen by the first party are determined by the incentives he or she faces, and the second party may attempt to structure the transaction so that these incentives lead to the choice that the second party prefers. Myriad examples exist; perhaps the most prominent is so-called pay for performance. This chapter explores some basic ideas about incentives. It begins with a fundamental tension between motivation and efficient risk sharing and goes on to a number of variations and extensions, including incentives for groups, in dynamic settings, and for screening.

- An insurance company sells a policy that insures a factory against fire and wonders whether the factory owner, now that she has insurance, will take care not to leave oily rags lying around.

- A company that manufactures large capital equipment employs salespersons who deal with the firm’s clients. The sales effort takes place outside the view of top management of the manufacturer, which wonders whether its salespersons exert themselves in selling the product.

- A day laborer is hired to remove tree trunks from a field. His employer wonders whether, at the end of the day, she will find a large number of stumps removed or only a few, accompanied by tales of hard, rocky soil and a dull axe.

- The loan officer at a commercial bank, considering whether to make a loan to an entrepreneur, wonders whether the entrepreneur will be prudent with the funds provided or will gamble with the funds, hoping for a big win and a spectacularly successful initial public offering.

- Five lawyers form a partnership with the intention of splitting equally the profits of the partnership. Each wonders whether this arrangement would mean that all five spend more time on the golf course and less in the office.

These are examples of moral hazard, situations in which the actions of one party affects the welfare of others, where the interests of the parties diverge to some extent, and the actions chosen by the first party are not completely controllable. As these examples indicate, the range of contexts in which moral hazards can be found is wide.
The solution to problems of moral hazard is, broadly speaking, incentives. A variety of motivators or incentives can work on individuals and groups, including

- Intrinsic motivators, such as pride in a job well done.
- Reliance on norms of appropriate behavior, including the norm of keeping promises.
- The desire to elicit positive reciprocal actions from one’s immediate trading partner(s) or to avoid future punishment or bad behavior.
- The desire to acquire and maintain a general reputation for good behavior, because such a reputation provides future benefits.
- For employees and contractors, the desire not to be fired.
- The desire not to be sued for breach of promise or contract.
- The prospect of promotion, desirable assignments, or lucrative future opportunities.
- Direct financial incentives based on measures of performance, such as sales commissions and other forms of pay for performance.

Intrinsic motivation and norms of behavior are discussed in Chapter 25. Chapters 22 through 24 discuss reciprocity, reputation, and the lure of future opportunities. This chapter focuses on direct financial incentives.

19.1. A Basic Trade-off: Risk Sharing versus Motivation

There are, potentially, two simple solutions to problems of incentives. The first is to determine what choice of action is desirable and fix this contractually. In the case of a factory owner, the insurance contract specifies that no insurance is paid if the owner does not store oily rags safely. A salesperson is provided no compensation if he does not exert himself to a prespecified level. The entrepreneur loses control of her business to the bank if she does not take prudent decisions. The problem with each of these is that it may be impossible, ex post, to verify that the contractual terms were met. It might be possible to monitor compliance with a contractual provision on the storage of flammable items, with spot checks. But how can a salesperson’s level of effort be monitored? How can the level of prudence of an entrepreneur be measured? Even if these things can be monitored or measured, can this be done in a manner that a court could enforce? A commercial
banker who loans money to an entrepreneur may know that decisions made by the entrepreneur were imprudent, but can a judge or jury in a civil trial be convinced of this? The problem, generally, is that often the desired "inputs" cannot be adequately measured or monitored or, if they can be, cannot be made part of an enforceable contract.

This takes us to the second potential simple solution. Construct an arrangement that puts the onus entirely on the party choosing the action. Do not offer fire insurance to the factory owner, so he bears fully the consequences of not storing oily rags carefully. Have the salesperson bear fully the impact of his effort decisions, by giving him a payment that equals (on the margin) the full effect of whether he makes the sale. Have the entrepreneur bear completely the consequences of her choice, by making a loan that guarantees a fixed amount of repayment to the bank.

The problem with these is that they imply no risk sharing between the parties involved. If the marginal profit from a sale is $10 million and a sale is uncertain even if the salesperson tries his best, will he be willing to bear that much risk in his level of compensation? Will the entrepreneur be willing to bear all the financial risk of her venture? (Given laws on limited liability, is it even legally possible to achieve this?) The problem is perhaps clearest in the insurance context: If the solution to moral hazard in an insurance context is not to offer any insurance, then the insurance business is dramatically curtailed.

(Another problem is associated with this second solution: What if more than one party takes an action subject to moral hazard? Suppose the salesperson must choose a level of effort to exert and, simultaneously, the firm must choose an uncontractable level of after-sales service to provide. The ability of the salesperson to make repeat sales depends on the firm's level of after-sales service and his own efforts. If he bears fully the consequences of his sales levels, the firm has no incentive to provide good after-sales service. But if the firm bears fully the consequences of the sales level, the salesperson loses incentive to exert himself in making sales. This situation of simultaneous moral hazard occurs in any situation where outcomes are influenced by the decisions of multiple parties, such as in the context of a legal partnership, and we take it up later in the chapter.)

A fundamental trade-off concerning incentives involves the confluence of three factors:

1. The desired actions cannot be specified contractually, because of problems of measurement, monitoring, or enforceability.

2. Even if the desired actions are taken, there is uncertainty about the consequences.
3. Loading the full consequences on the party taking the action is undesirable, because of the economic benefits of risk sharing among the parties. To share risk efficiently typically means shielding the party choosing the action from at least some consequences of that choice, but doing so lowers the level of incentives. The trick is to balance risk sharing and motivation.

An Example: Salesperson Compensation
A simple model of salesperson compensation illustrates these ideas. Suppose that you employ a salesperson who is going to try to make a particular sale for you. If the person makes the sale, you will earn a profit of $60,000. If the person does not make the sale, you earn $0. These figures do not include wages you pay the salesperson.

This salesperson must decide on a level of effort to devote to selling for you. He can kill himself, he can work hard, he can try but not hard, or he can loaf. His level of effort affects the probability that he makes the sale. If he kills himself, he will make the sale with probability 0.5. If he works hard, the probability that he will make the sale is 0.4. If he tries but not hard, he makes the sale with probability 0.25. And if he loaf, he will make the sale with probability 0.05.

This salesperson has a utility function that depends on his wages and the amount of effort he puts into making the sale. If he is paid a wage of $w$, his utility is

$$\sqrt{w} - \text{disutility of effort},$$

where his disutility of effort is 40 if he kills himself, 20 if he works hard, 10 if he tries but not hard, and 0 if he loaf. If the salesperson faces uncertainty in his wages, this utility function is used to compute expected utility.

The salesperson's best alternative to working for you is a job in which, for the length of time this sales call will take, he will make $10,000 with no disutility of effort. So to get this salesperson to work for you, you must give him a contract where his expected utility is at least $\sqrt{10,000} = 100$. You are risk neutral. You want to maximize your net expected profit from this venture, net of wages you pay to the salesperson.

Before going further, I should comment on the assumption that you are risk neutral, while the salesperson is risk averse. This assumption is typically made in economic models of incentives, because it simplifies the analysis. But the assumption is more than a matter of modeling convenience. If we think of "you" in this story as a sizeable, publicly held firm, with shares widely dispersed among many shareholders, then it is reasonable to suppose
19. Incentives

that “you” are risk neutral (although see Section 17.3 and financial market theory for an important caveat). On the other hand, the salesperson is likely to bear entirely the risk in his own compensation, for which he is risk averse.

With this assumption in place, we come to an easy first conclusion. Suppose that you can contractually fix the level of effort the salesperson expends. That is, you are able to write and enforce a contract that reads, in essence, The salesperson will exert effort level $A$ and be paid $X$ if a sale is made and $Y$ if not. Since the amount of money to be shared between you is random, you are risk neutral, and the salesperson is risk averse, efficient risk sharing would dictate that $X$ should equal $Y$. The salesperson should be shielded entirely from risk.

To determine $A$ and $X = Y$, reason as follows:

- If you wish the salesperson to put in no effort at all, pay him a wage $w$ sufficient to get him to work at this level of effort; that is, $w$ must satisfy

$$\sqrt{w} - 0 \geq 100, \text{ or } w \geq $10,000.$$

The 0 on the left-hand side of the first inequality represents the disutility of loafing. (In these constraints, I assume that, if the utility and later the expected utility of the salesperson equals his next best alternative, then he will choose to work for you. If this is not so, then we have to sweeten the salary a little bit.) If you pay this person $10,000, your net profit is $(0.05)(60,000) - 10,000 = -$7000; that is, you wind up losing money.

- If you wish the salesperson to try but not too hard, pay him a wage $w$ sufficient to get him to work at this level of effort; that is, $w$ must satisfy

$$\sqrt{w} - 10 \geq 100, \text{ or } w \geq $12,100.$$

The 10 on the left-hand side of the first inequality represents disutility of trying but not too hard. If you pay this person $12,100, your expected net profit is $(0.25)(60,000) - 12,100 = $2900.

- If you wish the salesperson to work hard, pay him a wage $w$ sufficient to get him to work hard, which is

$$\sqrt{w} - 20 \geq 100, \text{ or } w \geq $14,400,$$

where the 20 on the left-hand side of the first inequality represents the disutility of working hard. If you pay this person $14,400, your expected net profit is $(0.4)(60,000) - 14,400 = $9600.
And if you wish the salesperson to kill himself, pay him a wage \( w \) that is sufficient to get him to kill himself, which is

\[
\sqrt{w} - 40 \geq 100, \text{ or } w \geq 19600.
\]

(Your turn: Why is 40 on the left-hand side of the first inequality?) If you pay this person $19,600, your expected net profit is \((0.5)(60,000) - 19,600 = 10,400\). So the optimal thing to do is to write a contract in which you agree to pay this person $19,600 in return for a killing level of work—\( A \) is “killing level of effort” and \( X = Y = 19,600 \); for which you net $10,400.

Now suppose that you cannot fix the salesperson’s level of effort contractually, whether because you cannot observe his level of effort or this cannot be made a part of an enforceable contract. What do you do?

If the salesperson were risk neutral, the second easy solution could be used: Put the onus on the salesperson to take the right action by loading the consequence of the salesperson’s actions squarely on his shoulders. In effect, you would give the salesperson the following sort of contract:

The salesperson has the right to choose his preferred effort level. If he makes a sale, he keeps the full marginal profit he generates, $60,000. And to have this opportunity, he pays the firm (you) a fixed amount \( Z \), regardless of whether he eventually makes the sale or not.

If the salesperson were risk neutral, this would be a fine solution. He can bear the risk as well as anyone, so put all the risk—and all the consequences of his effort choice—on him. But he is not risk neutral, and so at least some risk sharing is a good idea. Indeed, because he is risk averse and you are risk neutral, efficient risk sharing, taken on its own, means you should bear all the risk.

That does not work either. If he bears no risk (if his level of compensation is the same whether he makes a sale or not) what incentive does he have to put in any effort? The answer, at least within the model, is none. And we know already that if he expends minimal effort, giving a 0.05 probability of a sale, your net profit, after paying him what it takes to get him to take the job, leaves you with a negative net expected profit.

I can hear the protests: “What about pride in doing a good job?” “What about the salesperson’s desire to make the sale just for the self-satisfaction this generates?” “What about extracting a promise of hard work from him, a promise he is likely to keep if he is at all the sort of person you want your company to deal with?” “What about the salesperson’s concern to keep his
job in your sales department?" "What about his desire to become a district sales manager?" "What about his concern for his general reputation as a salesperson?"

All these are good questions. They point out that motivating a salesperson, and more generally motivating anyone who faces a moral hazard, involves a vast complex of factors. We must evaluate these other factors in real-life examples to see how powerful they are. Perhaps they are powerful enough that, in a particular context, efficient risk sharing can continue without causing the salesperson to loaf.

But, in this chapter, the focus is on extrinsic, explicit, and formal monetary incentives, so we assume all these away in our model. And, in that case, the conclusion we just reached (efficient risk sharing means no bonus means no effort) is correct. To motivate the salesperson, we have to give him some incentive to try, which means making the amount he is paid if a sale occurs greater than what he makes if he comes back without a sale. But we do not want to go too far in this direction, since that means loading too much risk on him. It is a matter of trading off incentives against risk sharing, trying to find the right balance.

We do not, in this chapter, push all the way to the optimal solution in this model. You can do that on your own, in Problem 19.1. But we start you toward the solution, with a bit of trial and error:

Suppose you can write a contract for the salesperson that specifies that he gets a base wage of $9500, regardless of whether a sale is made or not, and in addition a bonus of $15,000 if he makes a sale. On these terms, would he take the job? If so, what level of effort would he put in? What would be your net expected profit?

He has the following five choices:

1. If he chooses not to work for you at all, he nets utility 100 in his next best alternative.

2. If he chooses to work for you and loaf, he receives income $9500 with probability 0.95 and $24,500 with probability 0.05. He has zero disutility of effort, so his overall expected utility is \((0.95)\sqrt{9500} + (0.05)\sqrt{24,500} - 0 = (0.95)(97.468) + (0.05)(156.525) - 0 = 100.421\).

3. If he chooses to work for you and tries but not too hard, he receives income $9500 with probability 0.75 and $24,500 with probability 0.25. He has disutility of effort equal to 10, so his overall expected utility is \((0.75)\sqrt{9500} + (0.25)\sqrt{24,500} - 10 = (0.75)(97.468) + (0.25)(156.525) - 10 = 102.232\).
4. If he chooses to work for you and try hard, he receives income $9500 with probability 0.6 and $24,500 with probability 0.4. He has disutility of effort equal to 20, so his overall expected utility is $(0.6)\sqrt{9500} + (0.4)\sqrt{24,500} - 20 = (0.6)(97.468) + (0.4)(156.525) - 20 = 101.091$.

5. If he chooses to work for you and kill himself, he receives income $9500 with probability 0.5 and $24,500 with probability 0.5. He has disutility of effort equal to 40, so his overall expected utility is $(0.5)\sqrt{9500} + (0.5)\sqrt{24,500} - 40 = (0.5)(97.468) + (0.5)(156.525) - 40 = 86.996$.

Among his five options, taking the job and trying but not too hard maximizes his expected utility (net of the disutility of effort). So we conclude that this is what he would do, if offered this incentive contract. Therefore, your expected profit, net of the cost of his wages, is

$$(60,000)(0.25) - 9500 - (15,000)(0.25) = 1750;$$

this is your expected gross profits, less his base wage, less the bonus times the probability that he earns the bonus.

That is not bad, but can you do better? To help answer this question, we use the spreadsheet CHAP19, depicted in Figure 19.1. Here is what it does: The base wage and bonus payment are placed in cells B1 and B2. Then the spreadsheet computes the gross (of the disutility of effort) levels of utility if a sale is made and if not; and for each of the four effort choices, it finds the

![Figure 19.1. A spreadsheet for analyzing different incentive contracts.](image-url)

This spreadsheet, sheet 1 of CHAP19, computes for a given base wage and bonus level the salesperson's expected utilities and the employer's expected net profit for each level of effort. Of course, the salesperson chooses whichever effort level gives him the highest expected utility, subject to the constraint that he must have a net expected utility of 100 to take the job at all.
net expected utility (EU), net of the disutility of effort, as well as the expected net profit, net of the salesperson’s expected compensation, for that effort level. So, in Figure 19.1, we see the analysis we did for the contract with a base wage of $9500 and a bonus of $15,000: The highest EU for the salesperson is from trying but not hard; this beats the salesperson’s reservation level of utility 100 and so is the salesperson’s choice. With this compensation offer, you (the employer) have an expected net profit of $1750.

Note that, with the base wage and bonus levels of Figure 19.1, if the salesperson chooses to kill himself, the expected net profit for you rises to $13,000. But the nature of the moral hazard problem here is that you do not choose the effort level, the salesperson does. And in this model, he chooses his effort level (or whether to work for you at all) based on the incentives you put before him. With the contract of Figure 19.1, his choice is to try but not hard, and the only way you can induce a higher level of effort from him, intuition tells us, is to give him a bigger bonus if he makes the sale.

Will it be worthwhile for you to induce more effort from him? With the spreadsheet in hand, we can play with the base wage and bonus to find out. Suppose, for instance, you increase his bonus to $20,000. See Figure 19.2. Now his choice is to work hard. And having induced this level of effort, your expected net profit rises to $6500.

![Figure 19.2. Getting the salesperson to work hard with a somewhat larger bonus. By increasing the bonus to $20,000, the salesperson is motivated to try hard, which improves the employer’s expected net profit.](image)

Note that the salesperson’s expected utility in the arrangement in Figure 19.2 is 107.18. You need to give him an expected net utility of only 100 to get him to work, so you are paying him more than his next best outside alternative. You can probably increase expected net profit by taking some wages away from him, and I tried to do this by decreasing his base wage. As I do this, I have to be careful that he continues to choose to work hard,
19.1. A Basic Trade-off: Risk Sharing versus Motivation

but this did not seem to be much of a problem; I found that cutting the base wage to $8,000 kept him working and choosing to work hard, and it pushed your expected net profit up to $8,000. The numbers are shown in Figure 19.3.

The scheme in Figure 19.3 looks pretty good, but there is still some room for maneuver. Note that working hard gives the salesperson an expected utility of 100.6, while the next best alternative level of effort (trying but not hard) gives utility 98.915. We continue to get the desired incentive effect (motivating hard work) and do better in terms of risk sharing if the bonus is decreased a bit. While we do this, we must add a bit to the base wage, to keep his net expected utility at or above the reservation utility level of 100. Figure 19.4 shows the best I was able to do with this, by hunting around: With a base wage of $8700 and a bonus of $17,000, the salesperson is just barely willing to work for the firm (100.089 vs. 100 for his next best alternative) and just barely prefers working hard to trying but not hard (100.089 vs. 100.033). This gives an expected net profit of $8500.

There is one final thing to try. Expected net profit is better still if we get the salesperson to kill himself. Indeed, we know that this is the effort level to shoot for if we could directly monitor and contract on effort level. Can we increase the bonus enough that this is his choice, leaving us with a higher expected net profit? I found that getting him to choose to kill himself requires an enormous bonus. The best I could do by hunting is in Figure 19.5: a base wage of $1530 and a bonus of $56,500, which means an expected profit of $220. It looks like the optimal incentive scheme will not involve motivating him to kill himself.
19. Incentives

Figure 19.4. Fine-tuning the incentive scheme of Figure 19.3. By moving in the direction of better risk sharing (a lower bonus) while keeping hard work the choice of the salesperson (which includes meeting the reservation constraint that hard work gives an EU at 100 or above), expected net profit rises to $8500.

Figure 19.5. Getting the salesperson to kill himself. A very large bonus is required to motivate the salesperson to kill himself, leading to a relatively small net profit. This is the best I could do in terms of net profit while motivating him to kill himself.

All I am doing in these exercises is hunting with my spreadsheet, trying to find successively better schemes, where better is measured by the effective expected net profit of the firm. I apply two general procedures in doing this: I try to see what is the best I can do in terms of expected net profit if I give the salesperson incentives to take each of the four actions. (Actually, I could give up on loafing from the start, since I know that loafing means an expected loss.) And for each level of effort in turn, I try to fine-tune the incentive scheme so that (1) the salesperson is brought as close to his reservation utility of 100 as possible (no need to give him extra surplus) and (2) he is shielded from risk to the maximum extent possible, consistent with keeping him at the effort level I am working at. I cannot be sure, since I am
just hunting heuristically, that the scheme in Figure 19.4 is optimal. But I bet that it is pretty close.

You may be wondering why I hunt heuristically. Why not just use Solver? I will not take you through all the details, but with logical variables, the spreadsheet can be set up so that Solver can be employed. (Find the maximal EU, and set up a logical variable for each action that equals 1 if the action achieves the maximum and 0 otherwise, being careful about ties—not pleasant, but doable.) But Solver doesn’t do well in general with maximization problems that involve logical variables; this might not work.

There is a better way to have Solver solve this problem. Indeed, this problem, viewed properly, can even be solved by hand. Problem 19.1 takes you through the drill, where you’ll discover that the absolute best that can be done in this problem is to set a base wage of $8711 and a bonus payment of $16,890 (rounded to the nearest dollar), leading to an expected net profit of $8533. Figure 19.4 is within $33 of the optimum.

The technique described in Problem 19.1 is probably worth learning on conceptual grounds, and it is a favorite for exam questions on this topic. But it is not very practical, since you rarely know with precision the utility function of salespersons who work for you. What is more important is the qualitative insight generated by hunting for the answer heuristically:

The basic trade-off in this problem is efficient risk sharing versus motivation. To motivate higher levels of effort, you have to pay a higher bonus. But as you pay a higher bonus, you load more and more risk on the salesperson, because the range of his incomes increases, bad for risk sharing. The optimal solution involves a compromise of these two opposing forces. In particular, you want a high-enough bonus to motivate the level of effort you target. But you want just enough motivation so this is true and no more. And you want to fine-tune the scheme so that the salesperson’s utility of working for you just beats his next best alternative.

Of course, this is just an example. But, in all sorts of problems in which individuals must be given incentives to take desired actions, incentives based on noisy measures of the actions the individual does take, we see the same basic trade-off. To motivate the desired action, the individual has to be given more compensation in circumstances that indicate that the desired action was taken. If the individual cannot entirely control the outcome, this subjects him or her to risk, which typically compromises efficient risk sharing.

To take another example, suppose an insurance company ensures an individual’s home against fire. The chance of a fire depends on how much care the individual takes, although even the most careful individual can face
a risk of fire. Efficient risk sharing, assuming the homeowner is risk averse and the insurance company is risk neutral, means insuring the homeowner entirely against any loss; that is, the insurance company bears entirely the risk of fire, the homeowner is completely compensated for any loss. But, if taking care is costly for the homeowner and she is entirely insured against loss, she has no incentive to take care. The insurance company, to motivate her to take care, must "reward" her if there is no fire; that is, she must be better off if there is no fire than if there is. Typically, this is done by partial insurance or, equivalently, by insuring the house but with a substantial deductible. The prospect of losing the deductible if there is a fire motivates the homeowner to take care. But it necessarily means less than perfectly efficient risk sharing. For a worked out example of this, similar to the salesperson compensation problem, see Problem 19.2.

19.2. Sundry Comments, Qualifications, Extensions, and Variations

The salesperson compensation problem of the previous section and the fire insurance and venture capital models provided in Problems 19.2 and 19.3 are very good exercises for firming up one basic trade-off concerning incentives: efficient risk sharing versus motivation. But this trade-off is far from the end of the story. Incentives and motivation are complex and subtle phenomena, about which entire books have been written. To complement the basic insight from the previous section, this section has a selection from the most important comments, qualifications, extensions, and variations.

Continuous Effort Choice, Continuous Outcomes

In the model just presented, the salesperson is limited to one of four possible levels of effort and there are only two possible outcomes: A sale is made or it is not. A richer, more realistic model would allow for a continuous effort choice variable and permit more outcomes, such as the level of the sale made. With such a model, we could address more easily the following questions:

- Is it better for the employer to have a salesperson who is more risk averse or one who is less? How does the salesperson's level of risk aversion affect the level of effort put forth?

- How does the level of a salesperson's disutility of effort enter into the compensation scheme he or she is offered? If we compare two salespersons, one of whom is more effort averse than the other, should the second
be given stronger incentives or weaker? Does the employer prefer the less effort-averse salesperson?

- In general, should you aim for effort levels greater than or less than the optimal level if the effort level can be enforced contractually? In other words, do problems of moral hazard mean that effort levels go up or that they go down?

- How does the level of noise affect things like the employer’s expected net profit and the effort level by the agent?

In general, these questions have no simple answers. Even the question that seems most obvious—surely when incentives are a problem, the firm should aim for a lower level of effort—is not true in all cases. Optimal incentive contracts can be fairly wild things. The reason for this, if you can stand the math-speak, is that the key to optimal incentives is to reward the agent highly for outcomes whose likelihood, if he takes the effort level you desire, is relatively higher than the likelihood of that outcome if he takes his next best alternative level of effort. Since likelihoods (conditional probabilities of outcomes, conditioned on the agent’s choice of effort level) can jump around a lot, so do optimal incentive contracts.

Still, it is possible to put together specific model formulations with continuous levels of effort and continuous outcomes, in which these questions can be answered. If you are interested, you can find one such model formulation and analysis in Appendix C of J. Baron and D. Kreps, *Strategic Human Resources* (New York: J. Wiley & Sons, 1999). I do not repeat that analysis here because it is several degrees more difficult than the model given previously, but if you can tolerate some difficult math, it is worth trying.

**Robustness**

Finding the optimal incentive scheme for a particular model, as done in the problems, is an interesting exercise in mathematics and logic. But do not put too much credence into such analyses, or rather, do not attach too much significance to the precise results of the optimization exercise. These analyses are predicated on the parties involved being very sophisticated about the environment they are in. For instance, the parties are assumed to understand fully the impact the employee’s effort choice has on the distribution of observables, such as whether a sale is made or not. The employer is assumed to know perfectly the utility function of the employee, including the employee’s precise attitude toward risk and level of aversion to effort.

These are nice assumptions when you are playing with a model. To be more precise, they are not terrible assumptions when your objective is to
understand, say, the trade-off between motivation and risk sharing. But for real-life application, they are pretty silly as assumptions go. Real incentive schemes, at least those that are effective, are relatively simple and robust to perturbations in the basic data of the situation, because the parties involved lack the data necessary to fine-tune things.

This is not to say that incentive systems are never fine-tuned to the individuals who are meant to be motivated by them. So-called management-by-objectives (MBO) systems are based on the notion that incentives should be tailored to the specific situations of specific employees. But, even with MBO, when the incentives for a given employee are set by the boss, it is doubtful that the boss knows everything necessary to solve for the specific employee’s optimal incentive scheme.

Instead, real-life incentive design looks a lot like the hunting we did with the spreadsheet, but without the instant feedback the computer gave us. Firms experiment with different incentive schemes, seeing what sorts of behavior result, and to the extent that they try to fine-tune, it is very much trial and error, blended with past experiences with similar situations.

**Screening**

Taking this a step further, do not lose sight of the fact that incentive schemes can serve several purposes simultaneously. They motivate individual employees, of course, but they also act as a screen for desirable employees.

In some cases, the two purposes push incentives in the same direction. Recall, from Problem 18.7, the case of the real estate network RE/MAX. RE/MAX offers its agent-employees a compensation scheme that is atypical of the industry: Individual agents retain 100% of the commissions they generate and pay a fixed fee for clerical and informational services and for the reputation gained by working for RE/MAX. This compensation scheme attracts able and aggressive agents, which gives RE/MAX a reputation for such agents, a reputation on which it is able to capitalize by charging its agents high fees for the services it provides. Agents in turn pay those fees because, by being RE/MAX agents, they are instantly branded as able and aggressive. This is a screening story. But at the same time, this sort of compensation scheme, which provides very little insurance for the individual agent, motivates each agent to work very hard, which is right in line with what RE/MAX desires.

Sometimes screening and incentive effects run in contrary directions. For instance, engineers in high-tech firms in Silicon Valley, during the boom years of the late 1990s, were often motivated by the lure of the big payoff. To motivate an engineer to work hard, high-tech firms had to offer compensation schemes that rewarded outstanding performance. To the extent that
But for these firms, engineers must take risks in order to be successful. However, firms adopting this form of incentive scheme for engineers found that they were attracting engineers who were after the big payoff and willing to take risks to get there. This had drawbacks: A driver of success in Silicon Valley was the ability to retain key technical employees in a labor market with extraordinarily high rates of worker mobility. So, to the extent that a firm attracted engineers who would gamble for a big payoff, it attracted key technical employees who were inclined to leave when the next startup came along, offering a chance at a super payoff in the form of stock options and the like. As long as a firm retained its engineers, they were motivated to perform as desired, but the same incentives led to a workforce more likely to turn over, a highly undesirable characteristic.

Hewlett-Packard (H-P), which, at least until recently, has been one of the most successful firms in Silicon Valley, motivated its engineers differently. Instead of motivating engineers with the prospects of a big payoff, H-P rewarded engineers with the freedom and encouragement to pursue engineering projects and puzzles of interest to the engineer. By setting up its incentive system this way, H-P attracted engineers who were turned on more by technical puzzles and the freedom to innovate than by the chance of a big payoff. Turnover rates among technical staff at H-P were a good deal lower than the local industry average, which meant success overall.

My point is this: Calculating the optimal incentive scheme for a given employee is all well and good, but when incentive schemes screen as well as motivate, which is practically all the time, do not ignore screening by focusing your attention solely on the motivation half of the story.

### Incentives Should Not Be Based on Irrelevant Noise

Imagine a firm that wishes to provide incentives for the manager of one of its plants. Imagine a very simple situation in which the plant manager’s sole responsibility is to produce a given amount of a product as cheaply as possible. Suppose that quality of output is not an issue, nor the development of human resources at the manager’s plant, nor any one of a host of other issues that afflict plant managers beyond unit cost. Unit cost is measurable ex post, and it is partially responsive to the manager’s efforts in running the factory. But random factors creep in, such as the quality of raw materials and the weather, which cause unit cost to be a noisy measure of the plant manager’s efforts.

Once the product is made, it is sold at a price the market will bear. The profit level of the firm depends on the unit cost of production but also on the state of the economy, the actions of the firm’s competitors, and so forth.
To motivate the manager, the firm considers two sorts of incentive schemes. One would tie the manager’s compensation to the measured unit cost of production at the plant. The second would tie the manager’s compensation to the firm’s overall level of profit. Which of these is better?

If we assume the manager is vastly less able to bear risk than the firm—an entirely sensible assumption when the firm in question is financed by widely spread, publicly traded equity—then on economic grounds, the first scheme is very likely to be better. On pure-risk sharing grounds, the firm wants to shield its vastly more risk-averse employee from risk wherever it can. The firm may have to compromise on this, to motivate the employee, by tying compensation to measures of employee performance not fully controlled by the employee. But tying the employee’s compensation to uncertain variables that have nothing to do with the employee’s level of effort simply loads unnecessary risk on the employee. (As we see in Chapter 25, however, there is more to this story than these economic considerations.)

**Tournaments and Benchmarking**

Return to the problem of motivating a salesperson. Why does the salesperson’s level of effort not completely determine the outcome on any given sales call, measured, say, in terms of the dollar value of sales made on this call? Any number of factors, together with the salesperson’s level of effort, influence the level of sales. Some factors are idiosyncratic to this particular sales call, things having to do with the particular customer to whom the sale is being made. But other factors outside the control of the salesperson are likely to influence similarly the level of sales achieved by other salespersons working for this firm; factors such as the quality of the product, the state of the general economy, the efforts of competitors to sell a rival product, and so on. If a salesperson does not make a sale or sells only a pittance, the product could be no good, the economy sluggish, or rivals cut the price of a similar good. But, if one of these factors is largely to blame for a poor result, it is more likely that other salespersons working for the firm do not do well on their sales calls. So the best evidence available that the particular salesperson (she) is trying hard may be that she books a large level of sales when other salespersons do poorly, and the best evidence that she is loafing may be that she fails when others do well. Note well, this may be the best evidence, but it need not be conclusive: Other idiosyncratic factors are at work on any single sales call.

Thus, a firm with multiple salespersons may be able to evaluate better the performance of each by comparing how each did with the others’ results in the same time period; that is, the firm uses comparative evaluation. The firm might, for example, give its salespersons a base wage, then a bonus to
19.2. Sundry Comments, Qualifications, Extensions, and Variations

Schemes of promotion to top seller in a given time period, or a bonus to the top 25%, or whatever. Schemes that tie compensation to purely ordinal rankings are called tournament incentive schemes. Relative to incentive schemes that tie the individual’s pay entirely to how the individual does, tournament incentive schemes and schemes that use both absolute and relative performance data can do better, because they allow the firm to reduce the effective risk faced by the individual; the only risk left—if all salespersons are equally able or even if the abilities of each is known to others—is idiosyncratic risk related to the customers called on by that salesperson.

A number of problems arise with these incentive schemes: (1) They give individual salespersons incentive to collude against the firm; if all hold back on their effort, then each looks fairly good “in comparison” with the overall average. (2) They sometimes promote unhealthy competition inside the firm; salespersons spend some of their time trying to affect adversely the performance of their peers or fail to render assistance to one another that would improve the firm’s profit. Of course, it is unlikely that you will face both problems for the same two salespersons at the same time; either they are colluding to hold down on the overall level of effort or each is spiking the efforts of the other. The standard ways to control these two problems is to isolate the employees in the comparison set and put a large number of employees in the set: The first precludes collusion by precluding communication and the second both makes collusion harder and gives smaller rewards to pernicious actions directed against the efforts of one or a few others in the comparison set.

Another way to control these problems is for Firm A to compare the performance of its salespersons with the performance of some external group, say, the salespersons of Firms B and C. Terminology in these matters is imprecise, but this is often referred to as benchmarking. For example, Firm A might benchmark its growth in sales revenue with that of several competitors, rewarding salespersons who do better than the “industry benchmark.” This controls the twin problems of collusion and pernicious activities because:

1. It is much less likely that the salespersons in firm A will collude with those in Firms B and C, because more people must be in the collusive scheme and they have less opportunity to talk, scheme, and otherwise interact.
2. Firm A is probably less concerned if its salespersons act uncooperatively with the salespersons of its rivals.

On the other hand, the disadvantage of benchmarking against salespersons of another firm is that the comparison controls for fewer common
factors. If we compare the sales results of two individuals selling the same product, we control for the quality of the product they sell, the prices charged by rivals, and the general reputation of the firm's products. When we benchmark the sales results in Firm A against what Firms B and C achieve, we no longer control for these things.

A third problem with tournament schemes, especially those that reward only the top performance, is that the prize must be increasingly large as the number of people in the tournament grows. If 500 salespersons all compete for a free dinner, they are unlikely to pay it much attention; for a 500-person tournament, it may take a week in the Bahamas to see much effect. To deal with this, it is often more effective to provide rewards to the top 10% of performers, rather than the top one (or two or three) individuals.

Related to this is the problem that arises when different participants in the tournament have different skill levels. If salesperson X is better than Y and Z, and all of them know it, or if X has a better client list, and all three know that, giving a prize to whomever sells the most may not have much of an impact. It may even have a negative impact, if Y and Z resent that they have little chance to win. Beyond this psychological risk, Y and Z may see increased effort as pointless, since X is bound to win. And if X anticipates that this is how Y and Z see matters, then X need not try very hard to win. In theory, handicapping the results can help here. But, handicapping, if done even somewhat subjectively, invites corruption; for instance, Z takes the boss to lunch, hoping to persuade her to favor him in the handicap scheme. If handicapping is done on a historical basis (X has won in the previous three quarters, so to win he must outperform Y and Z by more than 20% this quarter), then pernicious dynamic effects intrude; if X knows she will be penalized next period for doing well now, her incentives to perform well in this period are lessened. There are other important dynamic problems with tournaments; see the general discussion of dynamic effects in incentive schemes that follows.

Group Incentives and Internal Monitoring
At the other extreme from tournaments are situations in which incentives are tied to group performance. Each worker is a member of a work group, and the bonus the worker receives depends on how the group performs. The main negative associated with group incentive schemes is the free-rider problem: Because each individual has proportionately little impact on the final outcome, each has less motivation to try hard. Despite this, group incentive schemes are often used, for a variety of reasons:

- The production process may not be conducive to the measurement of
anything other than group output. When production is on a continuous flow process or an assembly line, no single worker has much control over the level of output.

- Rewarding individual output may cause workers to look only to their own level of output, without regard to how others are doing. In situations where workers can assist each other to the benefit of the total outcome, some reward for offering assistance should be offered. Tying compensation to the level of output of the group promotes helping efforts within the group.

- Small groups have a great advantage in self-monitoring, especially when the group works together in close proximity to one another. By rewarding at the level of the group, the firm promotes monitoring of members of the group by other members.

At the risk of overdoing it, let me develop the third point further. Providing group-based incentive schemes, especially when the group is small, can be advantageous when three factors come together: good measures of the quality of group performance are available, members of the group can monitor each other's individual effort levels easily and accurately, and groups have at their disposal the means and the inclination to enforce a healthy group norm for hard work. The means can include the ability to punish slackers, either immediately or in future dealings, and the ability to enforce social sanctions on slackers. The inclination is trickier; one needs to watch out for groups that adopt a norm in which no one works hard, groups that have dysfunctional social relations, and groups that may scapegoat individual members.

**Incentives in Dynamic Settings**

Most incentive problems arise in settings that are more dynamic than the very stark, static setting of a single period of sales. Salespersons deal with not one customer but many, and they deal with their customers sequentially. Imagine, say, a tournament scheme in which the top salesperson over the period of a month gets an especially large bonus. If salespersons are able to keep track of how everyone is doing through the month, and if after the second week or so, one salesperson has built up a big lead, then that salesperson may coast, as may others; the others recognize that if they get close to the leader, the leader will speed up, while the leader is willing to sit on her lead. Everyone slows down, just what the firm does not want in terms of the incentives it provides. On the other hand, if the contest is close as the month comes to an end, those in or near the lead may take actions not
In the interest of the firm, such as making sales to clients who are likely to default on their orders or their payments, simply to win the contest.

Dynamic effects arise as well when incentives are based on the performance of the individual only. Suppose I promise you a reward if you are able to sell $20 million or more over some fiscal year. Suppose that three-quarters of the way through the year, you have sold $21 million. You can safely coast for the rest of the year. Or, suppose that, three-quarters of the way through the year, you have sold $7 million. Now you “coast” for a different reason: There is no chance you will meet your goals so why struggle? The incentive effects of a bonus-if-hurdle-met scheme kick in largely during the early stages of the period and then very powerfully at the end, if the goal is in sight but not yet met. Even here there are problems: What will you do to make one final sale in the fiscal year if, with two workdays left to go in the year, your sales for the year sit at $19.8 million?

The effects in the previous paragraph result from the combination of two factors: The salesperson can monitor the likelihood of meeting the goal as the year progresses, and the reward scheme is discontinuous in the sense that there is a big jump in compensation if a particular hurdle level is met. The problems caused by these factors can be controlled to some extent—even to large extent—if the reward scheme is continuous; for instance, if the bonus is a flat percentage of sales generated for the entire period, or if the salesperson gets, say, 1% of sales for any sales over a $5 million total, rising to 2% for incremental sales beyond a $10 million total.

Another worry is, Where do target levels or hurdles come from? Often they are based on how the individual did the previous year; for instance, a salesperson (he) earns a bonus for improving his sales level from the previous year by, say, 5%. This sort of goal-setting procedure is subject to a host of problems, associated with the rubric of the ratchet effect.

1. The salesperson has an absolute disincentive to beat last year’s sales figure by any more than 5%, since doing so will only make it harder for the next year.
2. Three-quarters of the way through the year, the salesperson who realizes that the 5% improvement will not be met has an incentive to go into the tank for the remainder of the year, to make the next year easier.
3. The salesperson who has a really good year acquires an incentive to get another job, because the hurdle for next year will be very high.

On the other side of this story, when individuals interact through time, we can use time and the promise (threat) of good (bad) treatment in the future to provide incentives. Executives in a corporation may need no incentive
to work hard early in their career other than their career aspirations; there may be no need to tie their immediate compensation to their immediate performance.

**Other-Than-Effort Incentives and Multitask Jobs**

The notion that people must be motivated to work hard sometimes is relevant, for instance, for day laborers hired to pull tree stumps. In many cases of interest, however, it is not relevant at all. The issue is not motivating people to work hard but motivating them to direct their efforts in a manner that is best for the organization and to avoid motivating efforts that are ultimately counterproductive. For instance, in a piece-rate compensation system, a fixed amount, the piece rate, is paid for each unit produced. This motivates speed but not quality. Where quality is easily measured, it is easy to repair matters: The piece-rate is paid only for high-quality output, or the employee must rework (without additional compensation) defective units.

But what if quality is hard to measure? For instance, in the provision of a service, the number of clients served is easy to measure, but it is often much harder to obtain reliable statistics on whether the service provided was appropriate, clients were treated respectfully, and so forth. In such cases, and in all jobs that mix easy-to-measure and harder-to-measure tasks, a problem arises: If compensation is tied strongly to the easy-to-measure aspects of performance but not to the harder-to-measure aspects, the employee is motivated to ignore the harder-to-measure aspects. (In a simple piece-rate system, employees ignore quality to maximize quantity.) But to tie compensation strongly to both the easy-to-measure and the harder-to-measure aspects of performance is either expensive, if it is costly to measure the harder-to-measure aspects, or introduces risk into compensation levels, if the harder-to-measure aspects involve imprecise or noisy measures. The latter means a lot of risk inefficiently loaded on the employee. The only way out is to go easy on direct incentives altogether and deal with loafers by other means, such as with appeals to professional pride and so forth.

**Stock Options for Top Managers**

Top managers have jobs that are especially multitask in character, so that providing them with good direct incentives is especially tricky. On grounds that the value of shareholder equity is a good proxy for what is good for the organization, compensation for top managers is sometimes tied to the price of the firm’s equity, often through the granting of stock options, which allow top managers to buy shares in the company at a fixed price.

Stock options given to executives have been controversial, in terms of how they should be reported in a firm’s financial accounting statements.
and for the level of income they provide top managers. But their basic raison d'être has been pretty noncontroversial. They are meant to tie the compensation of top management to the fortunes of the enterprise, thus aligning the interests of top managers with those of equity holders. Stock options are used, the story goes, because the market price of equity most accurately captures how the enterprise is doing; market prices capture the value of the firm better than any formula because market prices are set by savvy investors interested in predicting the future value of the firm.

This is a nice story, but as events in 2002 have shown, the assumption on which this story is based (that the market price of equity reflects the true value of the firm) may be flawed. The market price of a firm’s equity necessarily reflects the information that equity markets possess, and to the extent that top management can manipulate financial statements, top management may be able to manipulate, at least for a while, the price of equity. Thus stock options given to top management may contain the seeds of a classic multitask incentives problem or, worse, a problem of maligned incentives. Top management, provided with options keyed to the market price of the firm’s equity, are motivated to inflate that market price. Top management can, as originally intended, serve that incentive by improving the economic fortunes of the firm. But they can also serve that incentive by engaging in accounting practices that paint a rosier picture of the firm’s prospects than is the truth, hoping to cash in the options and get their money before their misleading accounting practices have been discovered.

Psychological and Social Effects

This brings us to a brief closing comment about incentives and moral hazard problems, at least as discussed in this chapter. Except for some remarks near the beginning about intrinsic motivation and the like, the approach in this chapter has been resolutely economic in nature. We assume that, to motivate an individual, tangible incentives have to be put in place. Without those incentives, individuals loaf or otherwise take actions that serve their private agendas exclusively.

If you view them through the lens of social psychology and organizational sociology, you can get a very different take on many of these issues. Individuals are not so resolutely self-centered or selfish as economic models assume. They are sometimes motivated by an intrinsic pride in what they do. They can internalize the welfare of the organization for which they work. They avoid behavior they consider unethical. Moreover, and very important, the degree to which they are motivated by pride in what they do, by the fortunes of their organization, and by ethical considerations can be negatively affected by the extent to which they are subject to explicit, extrinsic incentive
Executive Summary

- When a party to a transaction takes an action that affects the value of the transaction to the other side, a potential moral hazard exists: Will the action chosen by the first party be to the detriment of the second party? Means for dealing with problems of moral hazard include good will and honesty on the part of the first party, contractual enforcement of an agreed-to action, observability of the action chosen and a means by which the second party can reward the first if the action chosen is "correct" (see Chapter 22), general observability of the action chosen and a desire on the part of the first party to preserve that party's reputation (see Chapter 23), and direct and explicit incentives, based on some observable measure or signal of the action chosen.

- In many problems in which explicit incentive schemes are used, the individual subject to the incentive scheme does not fully control the observable measure or signal on which the incentive is based. Thus, to employ the incentive scheme subjects the individual to risk, which gives rise to a basic trade-off: Efficient risk sharing (usually) mandates shielding the individual from this risk, but unless some risk is imposed, there is no motivation.

- Beyond this basic trade-off of risk sharing (and shielding) versus motivation, incentives give rise to a host of complications and extensions. Incentive schemes should be robust to the characteristics of the individual being motivated and the situation in which motivation is being applied. Incentive schemes often screen as well as motivate. Tournament schemes and benchmarking can, if used carefully, help control the risk to which the motivated individual is subject. In some instances, group-based incentive schemes can be employed. Dynamic aspects of incentive schemes (where the choices of the first individual are taken over time) should be carefully considered. Most incentive schemes are found in situations where the individual being motivated has a multi-dimensional choice of "effort."
The impact of a given incentive scheme on the full choice of the individual—not only how hard to work but on what tasks in particular—must be carefully considered. Explicit incentive schemes have social psychological and sociological effects that should not be ignored (see Chapter 25).

Problems

19.1 This problem takes you, step by step, to the precise solution of the salesperson compensation problem of Section 19.1. In fact, we go through this twice, once using spreadsheets and Solver and then doing it by hand.

The key to both ways of solving the problem is to break the problem into pieces. First, for each of the four effort levels, you answer the question: What is the cheapest (in terms of expected compensation) way to motivate the employee to take that effort level? Then, once you have the answers to this question, combine your answers to find the optimal incentive contract.

So, to begin, take the spreadsheet CHAP19. For reference sake, Figure 19.1 is reproduced here as Figure 19.6. Pick one of the four levels of effort; say, tries but not hard. Ask Solver to maximize cell F9 (the firm’s expected net profit if the salesperson tries but not hard) varying cells B1 and B2 (the base wage and bonus), subject to the constraints that E9 (the net expected utility from this level of effort) is at least as large as the three other levels of expected utility and the reservation utility of 100 (in cell E12). By telling Solver to keep E9 larger than E8, E10, E11, and E12, you effectively restrict Solver to incentive schemes (base wage and bonus pairs) that lead to tries but not hard as the level of effort chosen.

![Figure 19.6. A spreadsheet for analyzing different incentive contracts. This is sheet 1 of the spreadsheet CHAP19.](image)
The result is shown in Figure 19.7: The best scheme has a base wage of $9506 and a bonus of $12,250, for an expected net profit of $2431.

![Figure 19.7](image)

Figure 19.7. Finding the cheapest way to induce tries, but not hard. We ask Solver to maximize cell F9, varying B1 and B2, subject to the constraints that E9 is at least as large as E8, E10, E11, and E12. This is the answer it provides.

I said we would find the expected-compensation-minimizing scheme for inducing tries but not hard, then instructed Solver to maximize the expected net profit for that choice of effort. Recognize that, fixing the action, the two are the same: The expected net profit is just the probability of a sale given the chosen action (0.25 in this case) times the gross value to the firm of a sale ($60,000), less the expected compensation to the salesperson. In other words, since the gross contribution to the firm of this action is (0.25)($60,000) = $15,000, when Solver reports that the maximized expected net profit is $1750, it is telling you that the minimum expected compensation to get that effort level is $15,000 - $2431 = $12,569. Just to check this computation: The expected level of compensation is the base wage of $9506, plus the bonus of $12,250 paid with probability 0.25, or an expected $1750, for a total expected compensation of $12,568.50.

Now redo this for the other three levels of effort. Then combine the results to get the answer. (See the solution in the Student's Companion, if what I want you to do here is not obvious.)

As for doing this by hand, let me write $B$ for the base wage and $X$ for the base wage plus bonus; that is, $X$ is the sum of $B$ and the bonus. And let me write $b$ for the square root of $B$ and $x$ for the square root of $X$. In other words, $b$ and $x$ are the gross levels of utility the salesperson gets, if he does not make or makes a sale, respectively.
We answer by hand the question Solver just answered: What is the cheapest way (in expected compensation) to induce the effort level *tries but not hard*? If the salesperson chooses to try but not hard, the chances of a sale are 0.25, and the disutility of effort is 10, so his expected utility is $0.25x + 0.75b - 10$. If we want the salesperson to try but not hard, this has to be at least as large as 100 (to induce work) and at least as large as the levels of net expected utility for the other three levels of effort, or $0.05x + 0.95b$, $0.4x + 0.6b - 20$, and $0.5x + 0.5b - 40$. We not only want the first of these expected utilities to be larger than 100 and larger than the other three, but we want $B$ and $X$ (or $b$ and $x$) to be such that the expected compensation cost is as low as possible.

The discussion in the chapter suggests that at the optimizing values of $B$ and $X$ (or $b$ and $x$), the salesperson gets no more expected utility than he requires to accept the job:

$$0.25x + 0.75b - 10 = 100.$$

And the bonus should just barely induce *try but not hard*. The least incentive to induce this is where the expected utility from *trying but not hard* is equal to the expected utility of the next lower level of effort, *loafing*. So we have the equation

$$0.25x + 0.75b - 10 = 0.05x + 0.95b - 0.$$

(In theory, the left-hand side of this equation should be greater than or equal to the net expected utilities from the three other possible effort level choices. Why is this one the binding constraint? See the solution in the Student’s Companion for further discussion of this point.) This is two equations in two unknowns, so we can solve for $x$ and for $b$. Do the algebra, and you’ll come up with $b = 97.5$ and $x = 147.5$, and therefore, $B$, the base wage, is $97.5^2 = 9506.25$, while the bonus is $X - B$, or $147.5^2 - 97.5^2 = 12,250$. Compare this answer with what Solver told us.

Now redo this (by hand) for the other three levels of effort, combine your results, and you have the answer (by hand).

If you feel you know what is going on, try to finish off this problem both with the spreadsheet and by hand. If you are not quite there yet, consult the solution in the Student’s Companion, then try Problems 19.2 and 19.3, which are solved the same way, although the contexts are insurance and venture capital.
19.2 As an insurance underwriter, you have been asked to write a policy that insures a factory against loss by fire for a period of 1 year. If the factory has a fire, it will be a total loss of $8 million. The owner of the factory is an expected utility maximizer, with (gross) utility function $\sqrt{x + 1 \text{ million}}$, where $x$ is the value of the factory at year's end; that is, $x = 8$ million if there is no fire and $x = 0$ if there is a fire. Your insurance company is risk neutral.

The chance of a fire depends on whether the owner of the factory takes due care. If he does not, the chance of a fire over the 1-year period is 0.05. If he does take due care, the chance of a fire over the 1-year period is 0.01. To take due care is psychologically wearing on the owner and lowers his expected utilities by 50. That is, if the factory owner’s overall utility depends on both $x$ and on his decision whether to take due care or not, with utilities $B$ and $b$ as large where $x$ is equal to $8$ million or $0$ million, we have

$$U(x, \text{no due care}) = \sqrt{x + 1 \text{ million}} - 0 \quad \text{and} \quad U(x, \text{due care}) = \sqrt{x + 1 \text{ million}} - 50.$$ 

(a) If the factory owner cannot get insurance for the building, will he choose to take due care or not? What will be his overall expected utility?

(b) The insurance company wishes to maximize its expected profit from writing insurance for this factory owner. If it could contractually specify the level of care taken by the factory owner, what policy or contract would it write? (When the factory owner takes insurance, $x$ in his utility function is adjusted down by the amount of any premiums he pays and up by any indemnification he receives from the insurance company in the event of fire.)

(c) Suppose the insurance company cannot contractually specify the level of care taken by the factory owner. Suppose as well that, in the pursuit of efficient risk sharing, the insurance company is determined to insure the building fully; that is, it will write a policy that pays the owner $8$ million in the event of a fire. What is the best (profit-maximizing) policy of this sort the insurance company can write?

(d) The insurance company decides to investigate insurance policies with a deductible. That is, it will charge a premium $P$ and, in the event of a fire, reimburse the factory owner $8$ million less some prespecified deductible amount. What is the best (profit-maximizing) policy of this sort that the insurance company can write? Please do this by hand and not using Excel.

(e) Suppose that taking due care does not involve a “psychological cost” that lowers gross utility but instead comes at a dollar cost of, say, $100,000.
That is, in the factory owner's utility function $\sqrt{x+1}$ million, $x$ includes the value of the building, less any premiums paid for insurance, plus any indemnification from the insurance company in the event of fire, less $100,000 if the owner takes due care. This change in formulation makes the problem harder to solve (by hand). Why? Do not grind through the numbers unless you want practice using spreadsheets; simply redo the four previous parts of the problem with this reformulation, until you get to calculations that are too hard to do by hand.

19.3 An entrepreneur has a venture that will make either $100 million or $0. The chance that this venture will make $100 million depends on the effort level expended by the entrepreneur: If she tries hard, the chance of the $100 million outcome is 0.1. If she does not try hard, the chance of this outcome is 0.02. This entrepreneur is risk averse, with utility function

$$\sqrt{x} - \text{disutility of effort},$$

where the disutility of effort is 0 if the entrepreneur does not try hard and 500 if she does.

(a) Assuming this entrepreneur bears all the risk of this venture, will she try hard or not? What will be her expected utility, net of the disutility of effort (if any)?

(b) A risk-neutral venture capitalist is prepared to support this venture. Specifically, the venture capitalist will pay the entrepreneur a base amount $B$ up front, in return for which the venture capitalist will retain $X$ out of the $100 million the venture generates, if the venture succeeds. Assuming this venture capitalist is the entrepreneur's only alternative to going it alone (doing whatever you determined the answer was in part a), and assuming the venture capitalist can make part of his contract with the entrepreneur a specification of her effort level, what is the optimal contract of this sort for the venture capitalist to write? What will be the venture capitalist's net expected monetary value with this contract?

(c) Unhappily, the venture capitalist cannot contractually specify the effort level of the entrepreneur. If the venture capitalist wishes to motivate the entrepreneur to try hard, he must do this with the terms $B$ and $X$ in the contract he provides. What is the best contract for the venture capitalist to offer the entrepreneur, assuming that if the entrepreneur does not accept this contract, she is stuck going it alone on this venture?