

Answer to Kreps Problem 18.2

Suppose that a person has probability p of having a fire. If he has no insurance, his expected utility will be:

$$p\sqrt{10,000} + (1 - p)\sqrt{90,000} = 300 - 200p.$$

If he has full insurance and pays a premium of \$11,600, his expected utility will be

$$\sqrt{90,000 - 11,600} = 280.$$

If he has partial insurance that pays back \$58400 in the event of a fire and that charges a premium of \$5900, then his expected utility will be:

$$p\sqrt{68,400 - 5900} + 1 - p\sqrt{90,000 - 5900} = 250p + 290(1 - p) = 290 - 40p.$$

Part a)

Mr. Reece has $p = .1$. His utility will be 280 if he buys full insurance, 280 if he buys full insurance, and 286 if he buys partial insurance. So he will buy partial insurance.

Mr. Yost has $p = .03$. His utility will be 294 if he buys no insurance, 280 if he buys full insurance, and 288.8 if he buys partial insurance. So he will buy partial insurance.

Part b)

A person will buy no insurance if $300 - 200p > 280$ and $300 - 200p > 290 - 40p$. The first inequality simplifies to $p < 1/10$ and the second inequality simplifies to $p < 1/16$. Therefore a person will buy no insurance if $p < 1/16$.

A person will buy partial insurance if $290 - 40p > 280$ and $290 - 40p > 300 - 200p$. The first inequality is satisfied if $p < 1/4$ and the second is satisfied if $p > 1/16$. So a person will choose partial insurance if $1/16 < p < 1/4$.

A person will buy full insurance if $280 > 290 - 40p$ and $280 > 300 - 200p$. The first inequality is satisfied if $p > 1/4$ and the second is satisfied if $p > 1/10$. Therefore a person will buy full insurance if $p > 1/4$.

Part c)

Many of you fell into the trap of thinking that if the people who currently buy full insurance were to buy partial insurance, the company would make as much profit from them as it does from the people who currently buy partial insurance. This is NOT true. The people who are currently buying full insurance are a much higher risk population. If these guys buy partial insurance, the company will still lose money on them, but not so much as when it sold them full insurance.

The information in this problem is sufficient for us to calculate the average probability of fire for those with full insurance and for those with partial insurance.

Since the fully insured customers pay \$11,600 each and the insurance company loses an average of \$12,400 on each of them, it must be that the expected

payout per fully insured customer is $\$11,600 + 12,400 = 24,000$. Therefore if p is the average risk for these customers, we have $\$80,000p = \$24,000$ and so $p = .3$.

Since the partially insured customers pay $\$5900$ and the company makes an average profit of $\$1228$ per customer, it must be that the average payout per customer is $\$5900 - \$1228 = \$4672$. This means that if p is the average risk for these customers, $\$58,400p = \4672 and so $p = .08$. (Note that this number is between $1/16$ and $1/4$, as required by the answer to the previous question.)

If the firm only sells partial insurance, but maintains the same rates, it will continue to sell partial insurance to the 100,000 people who previously bought partial insurance. Profits received from these people will not change. The 5,000 people who in the past bought full insurance will certainly want partial insurance rather than no insurance. The average payout per person for these 5,000 people will be $.3 * \$58,400 = \$17,520$. The premium received from each of them will be $\$5900$, so the average loss from each of these customers will be $\$17,520 - 5,900 = 11,620$. This is a smaller loss than when the company sold them full insurance. In particular, the company saves an average of $\$12,400 - 11,620 = \780 per person. Since there are 5,000 of these people, the total increase in profits from dropping the full insurance offering is $5,000 \times \$780 = \$3,900,000$. Its total profits will now be $\$60.8 + 3.9 = \64.7 million.