On the Present Value Model in a Cross Section of Stocks

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Abstract: We construct a cross-section of stock prices and their corresponding present values of future cash flows. A regression of present value on the initial stock price should have a slope coefficient equal to 1.0. For short horizons, this is a cross-section version of checking the random walk model and the present value model holds up well. In contrast, using three different samples that go as far back as 1926, the present value model is rejected decisively at moderate and long horizons. We can rule out the possibility that the failure of the present value relationship is due to a misunderstanding of the dividend process. The remaining possibilities are either that agents do not discount very far into the future in a manner consistent with the present value model, or that models of discount rates are too limited to allow the present value model to be a good fit to the data for most firms. We find that the present value works much better, albeit still imperfectly, for larger firms. We also find that stocks that appear on the exchanges for fewer years than longer-lasting stocks deviate even more from the present value model. Our results can be interpreted as a cross-section version of the variance-bounds test, with the result that prices are very much more variable than they ought to be.

JEL classifications: G11, G17

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1. Introduction

According to the canonical model of stock valuation, the price of a stock is the expected present discounted value of future cash flows. Given a discount rate, the ex post present value of a stock is observable and, of course, the ex post present value should equal the expected present value plus an unpredictable error term.

To simplify for a moment, we regress the ex post present values of a cross section of stocks on each stock's initial price at horizons from 1 quarter to 35 years. Figure 1 shows the regression coefficient and 95 percent confidence bands for regressions of log ex post present value on log price, using the CRSP database from 1926-2012. The present value model predicts a coefficient of one at any horizon. Other than at very short horizons of no more than a few quarters, the coefficient is well below 1.0 and the present value model/assumed discount rate model is rejected.

We offer seven contributions:

1. In practice, using the standard present value model to advise a buy-and-hold investor as to how to value a stock leads to large mispricing errors. A 1.0 percent higher stock price is associated with only a roughly 0.6 percent higher present value.
2. The extent of mispricing varies with an investor's horizon. At very short horizons, there is no mispricing. At medium to long term horizons the mispricing is substantial.
3. The extent of mispricing is to some extent explainable by ex ante observable characteristics of the firm. The mispricing of very large cap firms is, while not negligible, much smaller than the mispricing for smaller firms.
4. The extent of mispricing is partially explainable by ex post observable characteristics of the firm, specifically by the length of time the stock is traded. Why this should be observed opens interesting questions, which we do not resolve. Given our estimation strategy, this finding is not a consequence of selection bias.
5. The finance literature has for some time considered whether mispricing or return anomalies are due to errors in cash flow models or errors in discount rate models. We settle one side of the issue: errors in cash flow models are not

the problem, since our use of cash flows is model-free. We suspect this accords with the consensus in the literature.

6. The traditional literature on variance-bounds tests suggests that stock prices are more variable than can be justified by the variability of subsequent cash flows. One interpretation of our results is analogous: the cross-sectional variation of stock prices can be thought of as arising from the variance of the expected present value across different stocks plus a large, pure random noise component.
7. While our methods cannot distinguish between errors in the discount rate models and a failure of forward-looking behavior by market participants, we do show that our empirical findings are not explained by something as simple as random errors in discount rates.

Any empirical test of the present value model necessarily involves three assumptions: (1) that agents who set prices are in fact using expected present value; (2) a model for how agents form expectations of future cash flows; and (3) a model of discount rates. In our approach, the only assumption about cash flow expectations is that the expectations are rational. Therefore, any rejection of the model is not due to a misidentification of a specific cash flow model.

In contrast our estimates do depend on both assumptions (1) and (3), as is the case in most of the literature. At very short horizons, where we find support for the model, the model of discount rates does not much matter since the amount of discounting is very small. At longer horizons, a failure of either assumption (1) or assumption (3) could account for our results. If the problem is with our discount rate models, the error must be one that is correlated with the price of a stock, even after allowing for different discount rates in different size and book-to-market ratio portfolios.

Looking at cross-sections of different horizons, i.e. cross-sections where the terminal date is variously near to or far from the initial date, raises several possibilities. Perhaps agents rationally discount near-term cash flows but do not rationally discount cash flows at longer horizons. While our results are consistent with this hypothesis, we do not have any particular evidence as to whether this is the source of the model's failure. There are other possibilities on which we can shed some light. The most interesting possibility is that our assumed discount rates are wrong, though the regression results are immune to random errors in computing ex post present value. However, since present value is nonlinear in discount rates, using the wrong discount rates can matter. We take special care and use several different models offered by the vast asset pricing literature. None of the models we use "work."

We do find that the model works better for long-living stocks than for stocks which drop off the exchange after a shorter time horizon. Given our data construction methods survivorship bias is not an issue, but long-living stocks differ from shorter-living stocks in other characteristics as well. In principle, the present value model could work well for dividends and not terminal prices or vice versa. Because much more of the present value of the typical stock is due to dividends rather than the terminal price at longer horizons than at shorter horizons, this could help explain the results. In practice, distinguishing between dividend payments and the terminal price does not make an important difference.

Looking at cross-sectional prices offers some econometric advantages, but also introduces some complications. Unlike the many models that center on the price-dividend or price-earnings ratio, we do not need any assumptions about the process for dividends, because actual dividends are observed. Given the cross-section structure, issues of stationarity do not arise. We also have a very large sample, essentially most all exchange-traded American common stocks. The first complication we face is that the composition of available stocks changes with the horizon being considered. So the change in the performance of the model at different horizons could be a composition effect rather than a horizon effect. When we control for composition we find that there are in fact both composition and horizon effects.

The second complication is that a cross-section of prices at a given date might fail our test simply because the market was too optimistic about the future at that particular time. The presence of a single, aggregate error would not seriously refute the model. We control for this possibility by allowing for initial year fixed effects and find them unimportant.

2. Related Literature

Little appears to have been done in examining the relation between present value and prices at different horizons. The closest literature considers return predictability by tests of the random walk model and tests for the predictability of long-horizon returns. The literature is too large to survey here; see chapter 2 of Campbell, Lo and MacKinlay (1997), including the discussion of some of the problems of long-horizon returns tests. While the long-horizon return literature is obviously related to our results, there are some important differences. One interpretation of the finding that long-horizon returns are predictable is that discount rates vary over time in a way that is predictable. Since such predictability might reflect priced risk, the usual long-horizon results are not necessarily a problem for the present value model. In contrast, we use discount rates that are time-of-purchase and portfolio specific. Therefore if the discount rates that we take from the standard literature correctly price risk, our estimate should support the present value model.

The paper that is closest to what we do here is Jung and Shiller (2005); it looks at 49 firms for which very long-horizon data is available and asks whether the dividend-price ratio predicts future cash flows, concluding (p. 226) “there is evidence that individual firm dividend-price ratios predict future dividend growth in the right direction, but no evidence that aggregate dividend-price ratios do.” Vuolteenaho (2002) shows that, for a panel of over 30,000 firm-years, about 75% of the variance of annual stock excess returns can be explained by cash flows. Cohen, Polk and Vuolteenaho (2003) also find future cash flows to be important. Since one interpretation of our findings could be that the present value model fails because discount rates are not well-understood; see Cochrane’s (2011) Presidential Address on discount rates. Our results are also consistent with some of the suggestions in Kasa (forthcoming) on pricing with heterogeneous beliefs.

Since Campbell and Shiller (1988a, 1988b), it is a common practice to decompose log dividend-price ratio into optimally predicted future one-period discount rates and one-period dividend growth rates. The relationship is derived from the log-linearized present value model. The linearized present value model imposes restrictions on a VAR with the dividend-price ratio, dividend growths and other relevant state variables. The present value model does not explain the data well: the model-implied dividend-price ratio does not track the actual one well, and dividend growths are poorly explained.

The rejection of the present value model under the Campbell-Shiller decomposition is ambiguous for three reasons: the Taylor approximation creates bias, though Campbell and Shiller (1988b) show that the bias is not important, the assumptions of the VAR, including lag length, linearity, information set, and stability are questionable, and the choice of discount rates matters.

Campbell (1991) shows that the decomposition can be extended to stock returns: unexpected excess stock returns depend on news about future dividend growth rates, real interest rates and excess stock returns. [1] Campbell and Ammer (1993), and more recently Chen and Zhao (2009), raise the concern that that results derived from VAR are sensitive to the particular specification and list of variables. The relative importance of cash flow (dividend) and discount rate is still an unresolved issue.

Our approach avoids most of the above econometric issues. In particular, we apply the present value model directly without any approximation, do not specify a set of variables that investors rely on for making forecasts, do not assume a specific process for dividends, and do not assume linearity and stability for the model used by investors. In particular, so long as one accepts rational expectations, the failure of the present value model cannot be attributed to a bad model for dividends.

3. Model and Data

3.1 The model

We observe firm i from its first appearance in the CRSP database at time t_i through its last appearance at time T_i. Cash payments on a stock share are the stream of future dividends, $d_{i,\tau}, \tau = t_i + 1, \ldots, T_i$, plus the terminal stock price P_{i,T_i}. For firm i at time t, the term structure of discount rates (which can be both time-varying and firm-dependent) is given by $r_{i,t,\tau},\ \tau = t_i + 1, \ldots, T_i$, where τ indexes the horizon in the term structure. Depending on data availability we consider both the case of time and horizon-

[1] See also Campbell and Mei (1993), Campbell and Vuolteenaho (2004), Campbell, Polk, and Voulteenaho (2009) and Koubouros, Malliaropulos, and Panopoulou (2006), among many others.

constant discount rates, $r_{i,t,\tau} = r_i$ and a full term structure model. For firm i the ex post present value of cash flows is given by

$$V_{i,t_i} = \sum_{\tau=t_i+1}^{T_i} \left(\frac{d_{i,\tau}}{\left(1 + r_{i,t_i,\tau}\right)^{\tau - t_i}} \right) + \frac{P_{i,T_i}}{\left(1 + r_{i,t_i,T_i}\right)^{T_i - t_i}}. \tag{1}$$

Expected present value theory states that the initial price P_{i,t_i} is equal to $E\left(V_{i,t_i}\right)$. While $E\left(V_{i,t}\right)$ is not observable, realized present value, $V_{i,t}$, is. Since realized present value equals expected present value plus a random and unpredictable error, or $V_{i,t} = E\left(V_{i,t}\right) + \varepsilon_{i,t}$, theory plus a simple substitution gives[2]

$$V_{i,t_i} = P_{i,t_i} + \varepsilon_{i,t_i}. \tag{2}$$

If we add to the model the distributional assumption that V_{i,t_i} follows a log-normal distribution, then we have an alternative regression

$$\ln\left(V_{i,t_i}\right) = -\alpha + \ln\left(P_{i,t_i}\right) + \left[\eta_{i,t_i} + (\alpha - \sigma_i^2/2)\right] \tag{3}$$

where $\sigma_i^2 = \mathrm{var}\left(\ln\left(V_{i,t_i}\right) \middle| \ln\left(P_{i,t_i}\right)\right)$, α is the average value of σ_i^2 across firms, and η_{i,t_i} is the unpredictable error $\log\left(V_{i,t_i}\right) - \mathrm{E}\left(\log\left(V_{i,t_i}\right)\right)$.[3]

It may be useful to compare equation (2) or (3) to the widely-used Campbell-Shiller (1988a,b) approximation,

$$\begin{aligned} &\mathrm{E_t}\left[\sum_{j=0}^{T} \rho^j \left((1-\rho)\log d_{t+1+j} - \log\left(r_{t+1+j}\right)\right)\right] + \rho^{T+1}\mathrm{E_t}[\log(p_{t+T+1})] \\ &= \log(p_t) - \frac{k}{1-\rho}(1 - \rho^{T+1}) \end{aligned} \tag{4}$$

where ρ is approximately the average dividend-price ratio and $k \equiv -\log(\rho) - (1-\rho)\log(1/\rho - 1)$.

The left-hand sides of equations (2) and (3) are analogous to the realized value of the left-hand side of equation (4), and equation (4) shows the 1.0 slope coefficient that we look for. In other words, our estimating equation is not terribly different from a direct

[2] This is not a statement about causality: we do not argue that initial "causes" future present value. The equation is a statistical relationship saying that the actual value is the expected value plus an expectation error.
[3] The initial stock price may be correlated with the firm-specific variance term in (3). We address this potential endogeneity problem in Section 6.

application of Campbell-Shiller. The Campbell-Shiller formulation does involve an approximation error. Our equation (2) does not involve an approximation error and our equation (3), which like the Campbell-Shiller version is written in logs, involves an approximation error only to the extent that the log normal distribution is an approximation. However, Campbell-Shiller approximation error is believed to be of relatively little importance, although perhaps somewhat more important at longer horizons (Campbell and Shiller (1988a)). Our calculation of the realized left-hand side uses actual dividends, and therefore is not subject to error attributable to errors in a dividend growth model. Like estimates using the Campbell-Shiller we do rely on model-based estimates of the discount rates.

Collecting values on stock prices and their subsequent cash flows, equation (2) or (3) can be estimated by a cross-sectional regression. The present value model implies that the coefficient on P_{i,t_i} or $\ln\left(P_{i,t_i}\right)$ should equal 1.0 and that no other variable known at time t_i should enter the regression. Since the nominal price of a share is rather arbitrary, i.e. if the number of shares outstanding were ten-fold higher the decimal point in the share price would move left one place, we focus on the log model, where the slope coefficient has an easy interpretation in terms of percentage changes. The log model does have the disadvantage that heteroskedasticity puts an omitted variable, σ_i, in the error term. We check below for this potential omitted variable bias and find it unimportant in practice.

The model has a special interpretation if we truncate horizons after one period, $T_i = t_i + 1$. Realized present value is simply $(P_{i,t_i+1} + \ d_{i,t_i+1})/(1 + r_{i,t_i})$. Since a single dividend and a single period discount are both negligible, equations (2) and (3) amount to cross-sectional versions of the standard random walk model. We know that at horizons of more than a day, the random walk model gives a good approximation to the data (Campbell, Lo and MacKinlay (1997)). So a quick check on our model is that the slope coefficient should be very close to one for a one-quarter horizon. It is (see results below).

3.2. Sample Selection

We begin with the universe of common stocks of nonfinancial firms traded on the NYSE, AMEX, and NASDAQ exchanges as listed in the CRSP database from 1926 through 2012. A firm is defined as a unique CRSP permanent number (PERMNO). Initial and terminal dates for each firm are chosen by selecting the longest span of data uninterrupted by missing data.[4] Our largest sample includes almost 19,000 firms.

Computing discount rates, as described below, requires auxiliary information. For this reason, we use three overlapping samples determined by data availability. Summary statistics are given in Table 1. The mean observation span in each sample is between 10 and 12 years. The maximum span varies from 58 to 87 years. The median initial date is between 1985 and 1989 and the median terminal date is between 1997 and 1999. The three samples are:

1) *CRSP, 1926-2012*: This is the largest sample available from the CRSP database, with approximately 19,000 firms.

2) *CRSP-COMPUSTAT, 1926-2012*: We augment the CRSP 1926-2012 sample by adding book equity data. The book equity data from 1926 through 1954 is as described in Davis, Fama and French (2000)[5]. Book equity data from 1955 onwards is from CompuStat. Since the book equity data is not available for all firms, the number of firms reduced by approximately 20 percent.[6] Neither the mean nor the maximum span changes appreciably.

3) *CRSP-COMPUSTAT, 1955-2012*: Our most sophisticated discount rate model uses macroeconomic data available only from 1955. In our third sample we use the same observations as in the second sample, but restricted to 1955 and later. The number of firms is slightly smaller. The mean observed span is reduced by about one year, while the maximum observed span is 27 years shorter.

Figure 2 describes the number of firms in each sample at various observation spans. The samples are large for relatively long spans. For example, at 35 years the first sample has 1116 observations and the third sample has 677. At 55 years the first sample still has 307 observations and the third sample has 53.

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[4] We observe prices on the last trading day of each quarter and cumulate ordinary dividends through the quarter. (Following Jung and Shiller (2005) we exclude non-ordinary dividends.) Fourteen firms are dropped from the sample due to their unusually large stock prices (after stock split adjustments). With the unusual adjustments, these firms have initial price above the top 0.1% among all firms. Dropping these firms does not significantly affect the performance of the expected present value model.

[5] The data are available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_historical_be_data.html .

[6] We drop firms with negative *initial* book equity data, and those firms only account for a negligible amount of the difference in sample size between 1) and 2).

Table 2 gives a sense of the importance of dividends in calculating the present value.[7] At a horizon of 20 quarters, less than 10% of present value is due to dividends. As horizon increases, dividends become more important. At a horizon of 140 quarters, over 70% of present value is due to dividends. For firms that have paid dividends at some point in the sample the percentages are (of course) higher, especially at the shorter horizons. The remainder of the present value is accounted for by the terminal stock price.

3.3. Calculating discount rates

Different discount rates are used for each sample, based on data availability. For the first sample, sort all firms observed in a given year into five market value (firm size) quintiles and then assign each firm to a quintile based on the firm's market value in its initial observation year. For the second sample, we rank by both firm size and book-to-market (B/M) ratio into 25 groups. We then use the time-constant, average, value-weighted realized returns for these groups over the period 1926-2012 as the discount rate, following the procedure given in Fama and French (1993).[8] For example, if a stock is in the lowest 20% in firm size in the year 1926, the average realized return of that group is used to discount the dividends and terminal stock price for that firm. Notice that the discount rates vary across groups but not over time. In other words, each firm is assigned to one of 5 (firm size quintiles), or 25 (size quintiles crossed with book-to-market ratio quintiles), discount rates based on its size class in the initial year of observation.

Greater data availability over the third, more recent sample allows us to use a more sophisticated method for calculating the discount rates. We follow Ang and Liu (2004). In each period, the Ang and Liu model produces a term structure of discount rates for valuing future cash flows. The model assumes time-varying risk premiums and betas. First, we take the 25 book-to-market and size sorted portfolios over the period 1955-2012. Next, for each portfolio, we estimate a VAR for dividend growth, time-varying beta, 1-year zero-coupon risk-free rate, CPI inflation, the Lettau and Ludvigson

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[7] We do not consider stock repurchases. Data on repurchases are available from COMPUSTAT but only at an annual frequency and only for after 1971. We cannot discount them as ordinary dividends within our quarterly framework. Since repurchasing stock started to become a common practice only since the 1980s (see Fama and French (2001)), and that we do not seem to find different results for stocks from the early cohorts implies that incorporating repurchases is unlikely to change the results substantially.

[8] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html We rank the firms by size or B/M ratio before dropping firms with less than 2 quarters in the sample, and as a result the grouping for the firms in our sample is the same as that defined in French's database.

(2002) "*cay*" factor (the residual from a cointegrating regression between log consumption, asset wealth, and labor income), and the portfolio's average payout ratio. All variables are quarterly. Dividend growth is from the Fama and French dataset (see footnote 3), and time-varying beta is estimated by rolling regression as in equation (26) in Ang and Liu. The risk-free rate (1-month T-Bill rate is from the CRSP riskfree series, and inflation (quarterly change of end of quarter headline CPI) is CPIAUCNS from the St. Louis Federal Reserve Economic Data (FRED). The *cay* factor is from Ludvigson's website.[9] Payout ratio is calculated by the average ratio of dividends to earnings for all firms in each group over time. Since some of these variables are not available before 1955, this method is not applicable to the other two samples.

For each VAR estimated, we apply equations (18)-(20) in Ang and Liu to derive the term structure of discount rates. For each of the 25 portfolios, we obtain quarterly values of the discount rate for discounting cash flows from any period in the future. In other words, we estimate a discount rate term structure at each date for each portfolio. Within the same group, the discount rates used are different if two firms have different initial dates, reflecting the different information available to Ang-Liu discounters making purchases at different dates. The initial date for a firm is the first quarter for which firm data is observed within a year for which financial statement variables are available. A given firm is permanently assigned to one of the 25 portfolios based on this initial year. After matching each firm with its appropriate term structure of discount rates for its initial quarter, we calculate the present value (all observed future dividends and terminal stock price) of each firm.

Figure 3 plots the average term structure across horizons for the 1955-2012 period of Ang-Liu discount rates. We show average term structures for two different groups of firms: the smallest stocks with the highest book-to-market ratio ("risky" stocks) and the largest stocks with the lowest book-to-market ratio ("safe" stocks). There are notable changes in the discount rates at different horizons, especially between 1 quarter and 10 years. Discount rates are generally higher for riskier portfolio groups. Figure 4 plots the

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[9] http://www.econ.nyu.edu/user/ludvigsons/data.htm

5-year and 24-year discount rates over the period 1955-2012 for the same groups in Figure 3.[10]

Note that the discount rate calculations assume agents are discounting with information that was not available to them at the time they priced stocks. Our calculations are as if agents not only had access to ex post returns, but they had also read Fama and French in 1926 and Ang and Liu in 1955. Using ex post returns in this way might give the present value model an "unfair" advantage. (Consider the results if one used the ex post internal rate of return for each stock.) Given our results, any unfair advantage does not seem to have helped much.

4. Basic Results

We begin with the basic results for the universe of firms in each sample. The expected present value model is rejected. We then break down the universe of firms in order to track down the source of the failure. In this section we focus on elements that would be known to investors ex ante. Specifically, we look at the effect of the investor's investment horizon and the differential outcome for large cap stocks.

Basic results for the universe of firms in each sample are given in Table 3, first for the log specification and then for the linear specification. Regressions for all the three samples show that the present value model is decisively rejected. The slope coefficient for the log specification is around 0.6 for all the three samples, so for each 1% increase in initial price there is only a 0.6% increase in present value.

The second panel shows that the linear model fails completely. Present value and initial price are essentially not correlated in levels. Each dollar increase in initial price predicts less than ten cents of increase in present value. To give the best chance to the present value model, we focus on the log version.

Table 3 provides results in which we follow firms for their entire CRSP history. Consider instead the perspective of an investor with a shorter horizon, an investor who plans to cash out his position after 1 quarter, or after 2 quarters, etc. Figure 5a gives the

[10] The Ang-Liu method allows for negative discount rates when interest rate is near zero. It applies to under 3% of firms and less than 1% of firm-quarters, mostly firms with large size or low B/M ratio that either start very early (1955-1958) or very late (2009-2012) in the sample. Omitting firms that start in those periods do not change the results.

slope coefficients for each sample for horizons from 1 quarter through 35 years. (The solid line reprises the slope estimates shown in Figure 1.) More than a few quarters out, the coefficients are very far from 1.0.

We know that the random walk model gives a good empirical representation of stock behavior at one-period, i.e. quarterly, horizons. How far out does one have to go before the representation does not work well? Figure 5b provides a magnified view of the short-horizon results from Figure 5a. The answer is not very far. Even by 10 quarters, the slope coefficients fall below three-quarters. If the first message of the paper is that the present value model does not work, the second message is that the present value model does work well—or at least much better—at very short horizons.

The finding that horizon matters admits of three possible explanations. First, perhaps investors are relatively myopic and act "rationally" with respect to near-term cash flows but pay insufficient attention to cash flows further out in the future. Second, our understanding of the term structure of discount rates might be incorrect with the discount rates being accurate for near-term cash flows but less accurate for more distant periods. Note though that for the third sample we use Ang-Liu discount rates. These are quite sensitive to the discount horizon, but the longer horizon results shown in Figure 5 are not noticeably better for the third sample than for the two samples where we use time-constant and horizon-constant discount rates.

The traditional variance-bounds literature argues that stock price exhibit excess volatility over time, relative to the present value of cash flows. (Leroy and Porter (1981) and Shiller (1981), for example.) Our findings can be given an analogous, cross-sectional, interpretation. Excess cross-sectional variation of stock prices explains our finding that the slope coefficients in the medium- to long-run regressions are persistently below 1.0. Suppose that the cross-sectional variance of expected present value is $\mathrm{var}(\mathrm{E}(V)) = \sigma_{EV}^2$ and that the stock price equals expected present value plus a purely random excess variability component u, $P = \mathrm{E}(V) + u, u \sim (0, \sigma_u^2)$. We can back out the noise-to-signal ratio as $\frac{\sigma_u^2}{\sigma_{EV}^2} \approx \frac{1-\hat{\beta}}{\hat{\beta}}$, where $\hat{\beta}$ is the estimated slope coefficient. If we take 0.6 as a representative of the long-run slope coefficient, this suggests that the excess cross-sectional random error is two-thirds the size of the cross-sectional variation in stock prices.

4.1. Random errors in the discount rate

Using correct discount rates matters a lot for discounting distant cash flows but matters little for near term cash flows. This suggests a third explanation for the longer horizon results: small errors in discount rates exist but matter much more at long horizons. However, this explanation almost certainly does not account for the observed results. Random errors in the dependent variable do not constitute an errors-in-variables problem. The intuition is that increasing errors in measured present value generates heteroskedasticity but should not bias the regression coefficients. Because errors in the discount rate cause nonlinear errors in V, the actual situation is a little more complicated than intuition would suggest.

To check for bias, we conducted a Monte Carlo investigation as follows. First, we generate expected future values for $n = 10{,}000$ firms as $V_i = 1.01^H + v_{1i},\ v_{1i} \sim N(0,1)$, for horizons $H = 1, \ldots, 240$ quarters. Prices are then set at discounted expected future values, $P_i = EFV_i/1.01^H$, so the standard model is true in the generated data. Realized future value is expected future value plus a random error, $FV_i = EFV_i + v_{2i}, v_{2i} \sim N(0,1)$. Finally we have our simulated econometrician use error-ridden discount rates drawn from a uniform distribution $r_i \sim U\left[.01 - \frac{u}{2}, .01 + \frac{u}{2}\right]$, $u \in \{0, 0.001, 0.005\}$, and to compute ex post present values $V_i = FV_i/(1 + r_i)^H$. In the Monte Carlo we repeat the exercise 1,000 times and collect mean regression coefficients for each horizon.

Figure 6 shows the mean slope coefficients for 1,000 Monte Carlo repetitions. The solid line shows estimates using the correct discount rate and, unsurprisingly, the estimates center on 1.0. The dashed line draws discount rates from a range centered on 0.01 with a width of 0.001, which we think of as small but not trivial errors in the discount rate. Here too, the estimated slope coefficients are quite close to 1.0. The dotted line allows much larger discount rate errors taken from a range centered again on 0.01 but with a width of 0.005, which we think of as relatively large. With these larger discount rate errors, simulated regression coefficients do deviate from 1.0 at long horizons, but they are higher than 1.0. In other words the random errors in discount rates increase longer-horizon slope coefficients, in contrast to the empirical finding of decreasing slope coefficients.

4.2. Very large cap stocks

While our results for the universe of stocks lean very strongly against the present value model, we find rather different results when we look at very large cap stocks. We restricted the sample to firms with a capitalization greater than 0.3 percent of total market cap in the year the firm is first observed (less than 1% percent of the firms in the sample.)[11] Figure 7 shows the slope coefficients from the log regressions for the three samples. While the regression estimates are below 1.0, they are not much below one. Indeed, for the most recent sample with the most sophisticated discount rate model, the coefficients are not significantly different from one. While this in part reflects large standard errors due to small sample sizes, the slope coefficients for these very large cap stocks are closer to 1.0 than was true for the universe of stocks. We conclude that the present value model works better, and perhaps works well, for very large cap stocks.

This finding raises the question of how far the better performance extends. In Table 4 we run separate regressions for five size portfolios for the first sample and for 5×5 size and book-to-market ratio portfolios for the second and third samples. While a few coefficients are insignificantly different from 1.0, most remain far from 1.0. We conclude that the better performance of the model for large cap stocks is confined to very large cap stocks.

5. The Present value Model And Ex Post Information

The previous two sections present results that are based on ex ante information on the right-hand side: log initial stock price, an investor's forecast horizon, and the initial ranking in size and book-to-market groups. In this section we look at ex post observable factors associated with the poor performance of the model. An ex post observable factor which predicts a deviation of the present value model is not a reason for rejecting the model, since ex post factors are not in the investor's information set. Nonetheless, looking at these factors may further our understanding of when the present value model works relatively well versus relatively poorly.

5.1. The present value model and sample length

[11] We choose these firms based on the initial size and we do not include firms that become large in the future (which is only known ex post).

The results in the previous section emphasize the importance of the discount horizon. In practice, changes in the horizon conflate two factors. When we add one quarter to the horizon, we add one more quarter of dividends to the cash flow and postpone the terminal price date by one quarter. However, we also drop firms from the sample that do not survive through the additional quarter. Here we separate the effects of horizon (which is ex ante observable) from the effects of ex post sample length.

Note that a firm dropping from the sample after a given span of observations is not the usual form of sample selection. Dropping from the sample may be either a positive, negative, or neutral event. A successful firm may be bought out, or a firm may go bankrupt, or a share identified with a given PERMNO may be refinanced with a different issue of equal value. But even if a stock surviving only for a short period is correlated with a negative event at the terminal date, this does not induce classical selection bias when using the entire sample. After all, half the stocks remain in the sample for more than the median sample length. So at the time of investment, an agent will expect half of stocks to leave the sample "early," and half the stocks to leave the sample "late." While one group will be overpriced ex post and the other group will be underpriced ex post, the universe should be correctly priced ex ante.

In order to separate the effects of sample length from horizon, we look at the performance of the model for the same firms at different horizons by varying the terminal period. We first divide firms that are in the sample for 21-40 quarters, 41-80 quarters, and over 80 quarters. The three groups are of similar sizes. We then estimate the present value model (3) from 1 to 20-quarter horizons for the first group, 1 to 40-quarter horizons for the second group, and 1 to 80-quarter horizons for the third group. In other words, within a group the sample is the same at all horizons. Since the White standard errors are usually negligible, we only report the point estimates to avoid clustering the graphs too much. The sample size for each group is reported in the graph.

Figure 8 shows the results for all firms that are in the sample for at least 21 quarters. At the very short horizons of a few quarters, the coefficients are close to 1, consistent with a random walk. For all three samples, for all three groups, it remains true that the random walk model is rejected at longer horizons. However, the slope coefficient declines noticeably less for those firms which remain in the sample for more than 20 years. We see little difference between the coefficients for the 5-to-10-years group and the 10-20-years group.

The CRSP dataset provides information on why the firms leave the sample. We define three groups of firms: the first, or truncated, group includes firms that are "terminated" just because they hit the end of the sample period (end of 2012);[12] the second, or merged, group includes firms that are merged and become a different firm;[13] the third, or "troubled", group includes firms that leave the sample due to insufficient number of shareholders, insufficient capital, bankruptcy, violation of financial guidelines and other problems.[14] These three groups constitute over 90% of the firms that are in the sample for at least 21 quarters. We plot the coefficients for the three groups in Figures 9 to 11.

For the truncated sample, the results are quite similar to the full sample in Figure 7, except that the coefficients for the second group are much lower than those for the first group. Note that since all these firms survive through the end of the sample, the differences in sample length reflect the year in which the firm entered the CRSP sample. The overall pattern is similar for the merged firms in Figure 10.

In contrast, the performance of the model deteriorates quickly as horizon increases for all "troubled" firms regardless of the sample length. The three lines are much closer in Figure 11. Even among long-living firms, the coefficient quickly drops to around 0.6 at the horizon of a few years.

6. Robustness Tests

Suppose we said that investors in 1926 had on average under (or over) estimated the value of subsequent cash flows. Even if this were true for a large sample of stocks, it might not be an interesting rejection of the present value model if it simply means that there was a single aggregate error. We want to be sure that our results are not due simply to aggregate errors. In the third panel of Table 3 we present results for the regression with initial year dummies instead of a single constant term. The year dummies can account for possible time fixed effects in the stock market. Other than improving the fit, controlling

[12] Firms that have a delisting code of 100.
[13] Firms that have delisting code of a value larger than or equal to 200 and below 300.
[14] Firms that have a delisting code of 550, 551, 552, 560, 561, 574, 580 or 584.

for time fixed effects does not seem to affect the performance of the model. We have also estimated the model by allowing the log initial price to interact with the year dummies. The joint tests for the null of all coefficients equal to 1 and for the null of the average of all coefficients equal to 1 are both strongly rejected.

In the fourth panel of Table 3, only firms that have paid dividends at some point are included. More than half of the firms are dropped, and most of them are from the recent periods (see the "disappearing dividends" phenomenon discussed in Fama and French (2001)). The performance of the model improves: the in-sample fit is higher and the coefficient is closer to one. For all the three samples we still strongly reject the model. The poor performance of the model is not explained by the inclusion of firms that never pay dividends.

As mentioned in Section 2, we divide firms into 5 size groups and 5 book-to-market groups (except for the first sample, which only contains size). To account for the different levels of risk that firms are exposed to, discount rates are on average higher for "riskier" (small or with high book-to-market ratio) firms. Here we test if the performance of the present value model depends on these two factors. Unlike dividends and terminal price, notice that these two factors are ex ante publicly available information at the initial date. Table 4 presents results with group dummies and interactions of log initial price with group dummies.

In the first sample, where only size information is available, there does not seem to be any relationship between firm size and the performance of the present value model. Once book-to-market ratio is accounted for in the second and third samples, coefficients for larger firms are closer to 1 on average, though not by much. There is also no clear relationship between book-to-market ratio and the slope coefficient.

In principle, equation (3) suffers from an omitted variable problem is σ_i^2 in the error term is correlated in the cross section with $\log(P_i)$. While the issue does not arise in the linear model, the log model may be preferred as being more consistent with the literature. We can make an approximate check for omitted variable bias as follows. If $\hat{\beta}$ is the regression coefficient in equation (3), so under the null $\beta = 1$, then omitted variable bias would give us $\mathrm{E}(\hat{\beta}) = \beta - \lambda$, where λ is the population regression coefficient from a regression on σ_i^2 on $\log(P_i)$. It follows from the properties of the log-normal distribution that $\sigma_i^2 = \log\left(1 + \frac{\operatorname{var} \varepsilon_i}{P_i^2}\right)$. While we do not observe var ε_i, under the null we do observe $\varepsilon_i = V_i - P_i$. We approximate σ_i^2 with $\hat{\sigma}_i^2 = \log\left(1 + \frac{\varepsilon_i^2}{P_i^2}\right)$ and regress $\hat{\sigma}_i^2$ on $\log(P_i)$. The regression coefficients $\hat{\lambda}$ (and White standard errors) for the three samples are -0.039 (0.004), -0.074 (0.006), and -0.037 (0.005), respectively. These results suggest that the omitted variable bias is negligible and that the estimated slope coefficients in the log regressions are upward biased, so accounting for omitted variable bias slightly reinforces our earlier conclusions

7. Conclusion

The goal of this paper is to evaluate the performance of the present value model. Doing the best we can in allowing for time-varying and firm-specific discount rates, we find that the present value model fails by an economically and statistically significant margin. On average, each 1% increase in the initial price predicts less than 0.6% increase in the ex post present value. The point estimate is usually several standard errors away from the theoretical value of 1. In other words, while the model does not say that stocks are mispriced relative to expected present value on average (the regression necessarily goes through the mean of the data), we find that high priced stocks turn out to be over-valued and low priced stocks turn out to be under-valued.

At short horizons the model works well. At long horizons the model works very poorly. We do find that the model works quite a bit better for very large capitalization firms, but past that performance is poor for all size and book-to-market quintiles. The model works somewhat better for firms observed for long spans than for firms observed for short spans, but even for long-span firms the model does not work well.

Nothing we do distinguishes between investors simply not being rational, expected present value discounters and the possibility that we have inadequate models for discount rates. Note, however, that use of the more sophisticated Ang-Liu rates does not noticeably improve model performance. Further, the Monte Carlo we offer suggests that the failure to find appropriate slope coefficients cannot be attributed to random, small errors in discount rate models.

Whatever the explanation of the rejection, we are left with the distressing conclusion that the standard model of stock pricing is difficult to usefully apply in practice.

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Table 1: Summary Statistics of The Three Samples of Publicly Listed Stocks

	CRSP 1926-2012	CRSP-COMPUSTAT 1926-2012	CRSP-COMPUSTAT 1955-2012
Number of Firms	18969	15527	14921
Number of Firm-Quarters	845126	734868	641494
Mean Sample Length (quarters)	44.55	47.32	42.99
Maximum Sample Length (quarters)	348	348	232
Median Initial Year	1986	1988	1989
Median Terminal Year	1997	1999	1999
% of firms paying dividends	35.71	36.37	34.31

Note: Refer to Section 2 and the Appendix of the paper for the sample selection procedures.

Table 2: Average Proportion of Present Value Accounted for By Dividends

Horizon	CRSP 1926-2012		CRSP-COMPUSTAT 1926-2012		CRSP-COMPUSTAT 1955-2012	
	All Firms	Dividend-Paying Firms	All Firms	Dividend-Paying Firms	All Firms	Dividend-Paying Firms
20 Quarters	0.071	0.144	0.072	0.144	0.058	0.122
60 Quarters	0.290	0.396	0.293	0.392	0.293	0.406
100 Quarters	0.534	0.598	0.535	0.592	0.528	0.599
140 Quarters	0.747	0.767	0.739	0.758	0.703	0.729

Note: Refer to Section 2 and the Appendix of the paper for the sample selection procedures.

Table 3: Present Value Regression Results

	CRSP 1926-2012	CRSP-COMPUSTAT 1926-2012	CRSP-COMPUSTAT 1955-2012
	Log		
Initial Log Price	0.594	0.586	0.652
	(0.011)	(0.012)	(0.014)
Adjusted R^2	0.170	0.176	0.173
	Linear		
Initial Price	0.058	0.015	0.010
	(0.043)	(0.005)	(0.004)
Adjusted R^2	0.043	0.008	0.002
	Log, With Year Dummies		
Initial Log Price	0.599	0.607	0.628
	(0.012)	(0.013)	(0.015)
Adjusted R^2	0.247	0.243	0.258
# of Obs	18869	15527	14921
	Log, Dividend-Paying Firms		
Initial Log Price	0.742	0.689	0.820
	(0.012)	(0.014)	(0.016)
Adjusted R^2	0.434	0.412	0.404
# of Obs	6773	5647	5119
	Log, Initial Market Share > 0.3%		
Initial Log Price	0.896	0.858	0.920
	(0.037)	(0.041)	(0.083)
Adjusted R^2	0.839	0.832	0.525
# of Obs	133	125	87

Note: White heteroskedasticity-consistent standard errors in parenthesis. The dependent variable is the present value calculated by discounting future dividends and terminal (the last available) stock price, with the discount rate depending on the firm's size and/or the book-to-market ratio. The independent variable is the initial stock price, defined as the stock price in the first year in which the firm's size and book-to-market ratio are known. Constant terms are not reported.

Table 4: The Role of *Ex Ante* Information – Firm Size and Book-to-Market Ratio

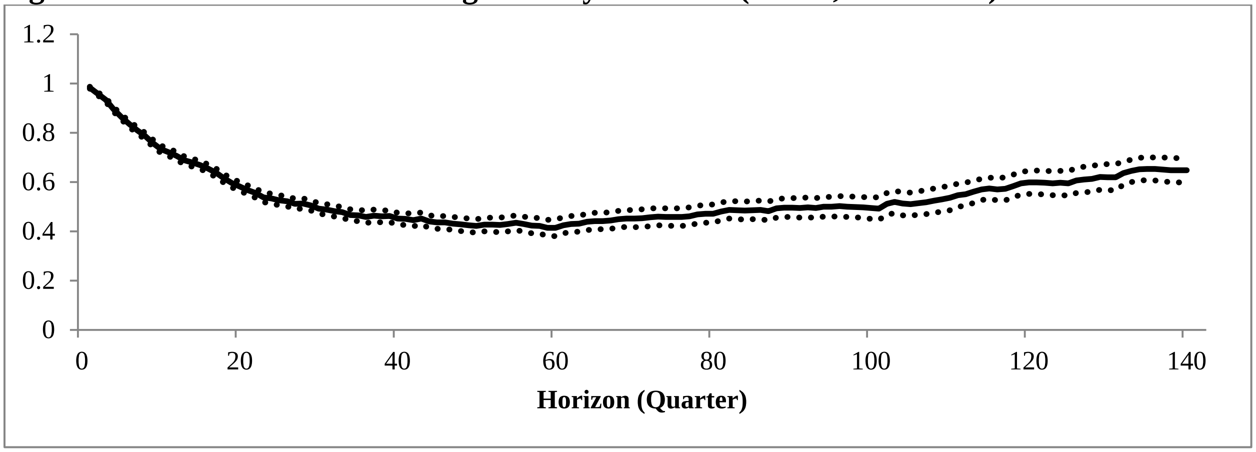
Dummy Variable Interacting with Initial Log Price	CRSP 1926-2012	Dummy Variable Interacting with Initial Log Price		CRSP-COMPUSTAT 1926-2012	CRSP-COMPUSTAT 1955-2012
Size Group 1 (Bottom 20%)	0.545 (0.024)	Size Group 1(Bottom 20%)	B/M Group 1 (Bottom 20%)	0.454 (0.047)	0.492 (0.051)
			B/M Group 2	0.379 (0.042)	0.450 (0.044)
			B/M Group 3	0.381 (0.065)	0.387 (0.063)
			B/M Group 4	0.542 (0.053)	0.543 (0.051)
			B/M Group 5(Top 20%)	0.567 (0.051)	0.600 (0.044)
Size Group 2	0.526 (0.024)	Size Group 2	B/M Group 1 (Bottom 20%)	0.381 (0.038)	0.355 (0.040)
			B/M Group 2	0.409 (0.047)	0.438 (0.051)
			B/M Group 3	0.401 (0.065)	0.402 (0.069)
			B/M Group 4	0.353 (0.085)	0.380 (0.090)
			B/M Group 5 (Top 20%)	0.542 (0.069)	0.651 (0.066)

Table 4: The Role of *Ex Ante* Information – Firm Size and Book-to-Market Ratio (continued)

			B/M Group 1 (Bottom 20%)	0.370 (0.037)	0.415 (0.044)
			B/M Group 2	0.249 (0.059)	0.359 (0.074)
Size Group 3	0.394 (0.029)	Size Group 3	B/M Group 3	0.408 (0.074)	0.532 (0.082)
			B/M Group 4	0.420 (0.090)	0.525 (0.100)
			B/M Group 5 (Top 20%)	0.405 (0.113)	0.532 (0.107)
			B/M Group 1 (Bottom 20%)	0.281 (0.037)	0.318 (0.046)
			B/M Group 2	0.332 (0.065)	0.403 (0.070)
Size Group 4	0.297 (0.027)	Size Group 4	B/M Group 3	0.355 (0.092)	0.271 (0.078)
			B/M Group 4	0.692 (0.079)	0.817 (0.103)
			B/M Group 5 (Top 20%)	0.700 (0.108)	0.899 (0.093)
			B/M Group 1 (Bottom 20%)	0.434 (0.042)	0.386 (0.045)
			B/M Group 2	0.547 (0.096)	0.044 (0.101)
Size Group 5 (Top 20%)	0.365 (0.036)	Size Group 5(Top 20%)	B/M Group 3	0.930 (0.077)	0.848 (0.098)
			B/M Group 4	0.602 (0.167)	0.341 (0.199)
			B/M Group 5 (Top 20%)	0.691 (0.099)	0.516 (0.079)
Adjusted R^2	0.228			0.285	0.337
# of Obs	18969			15527	14921

Note: White heteroskedasticity-consistent standard errors in parenthesis. The dependent variable is the log present value calculated by discounting future dividends and terminal (the last available) stock price, with the discount rate depending on the firm's size and book-to-market ratio. The independent variable is the initial stock price, defined as the stock price in the first year in which the firm's size and book-to-market ratio are known. Constant terms are not reported.

Figure 1: Coefficient on Initial Log Price by Horizons (CRSP, 1926-2012)



Note: At which horizon from 1 to 140 quarters, we calculate the present value for firms available for that horizon and regress its log on log initial price. As horizon increases, the number of firms drops. White standard errors are used for the 95% confidence intervals.

Figure 2: Number of Firms By Minimum Number of Quarters in The Sample

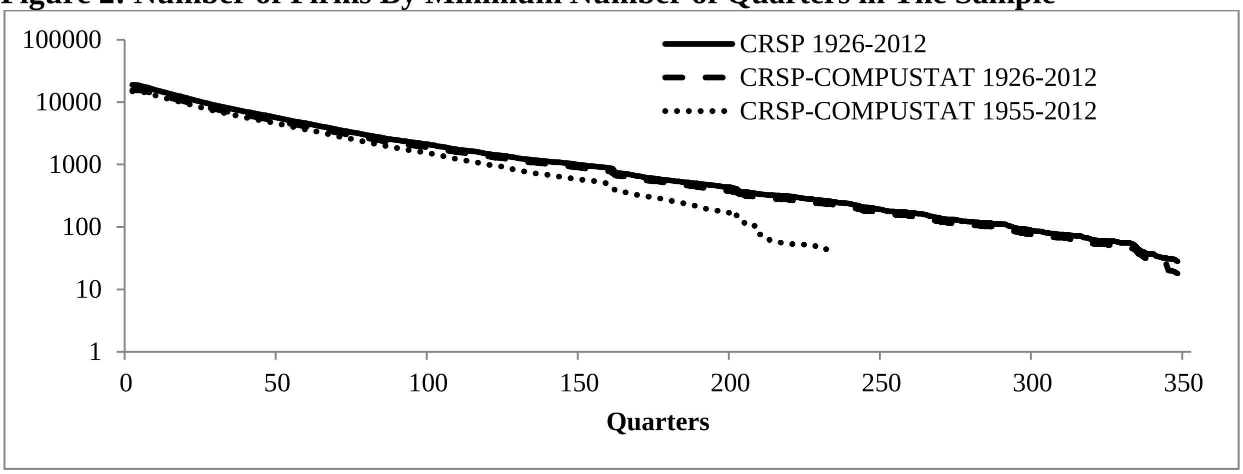
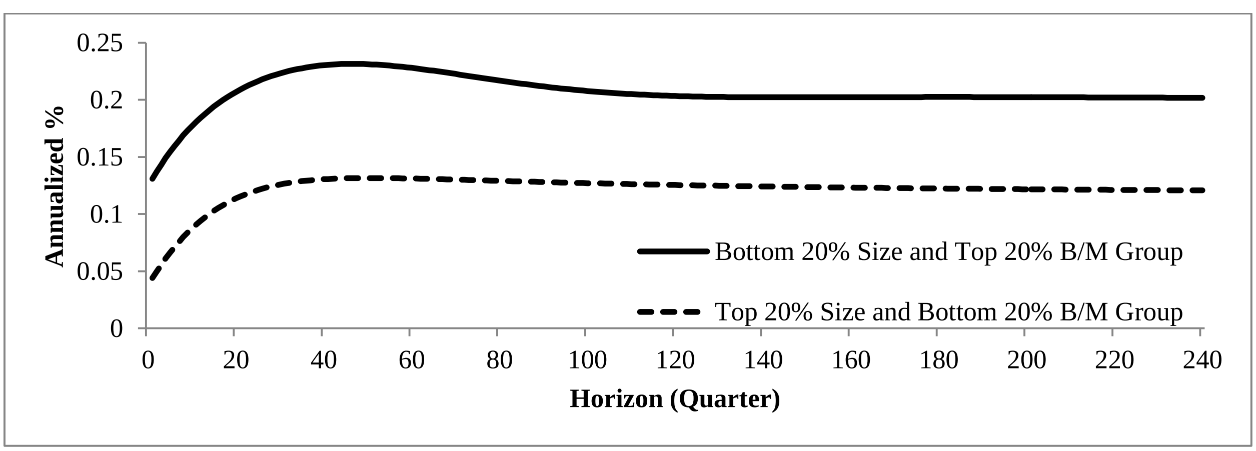


Figure 3: Average Term Structure of Discount Rates



Note: See section 2.3 for the method of estimation for discount rates.

Figure 4a: 20-Quarter and 120-Quarter Discount Rates for the Bottom 20% Size and Top 20% B/M Ratio Group

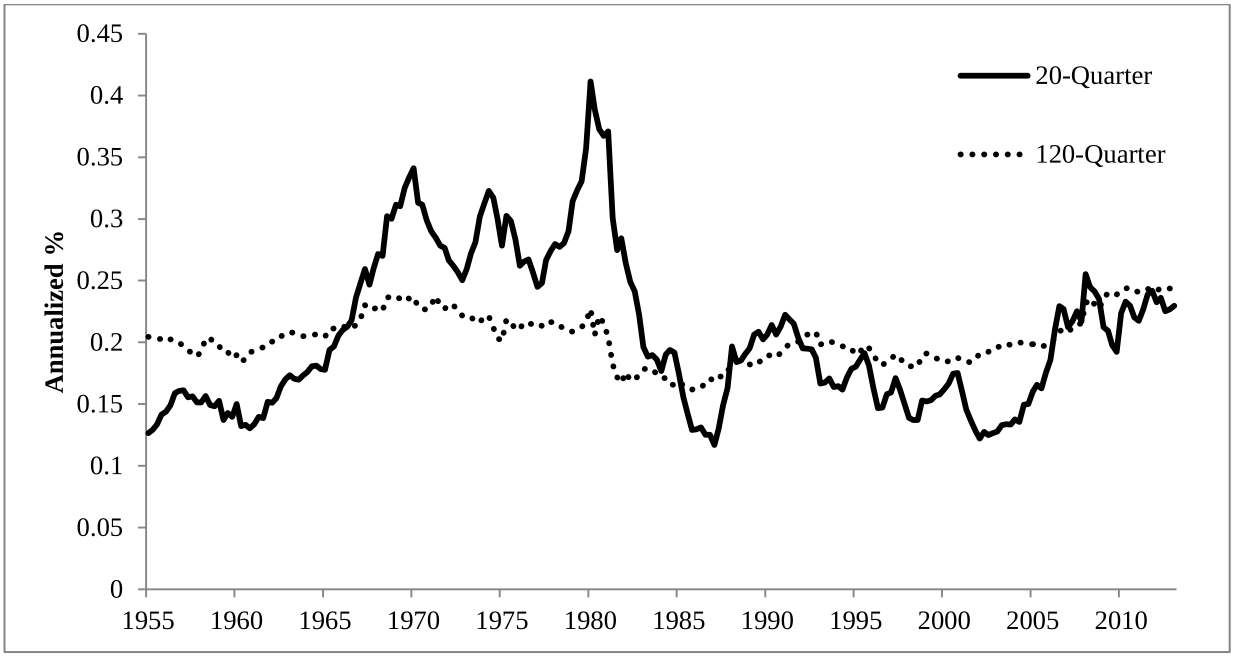
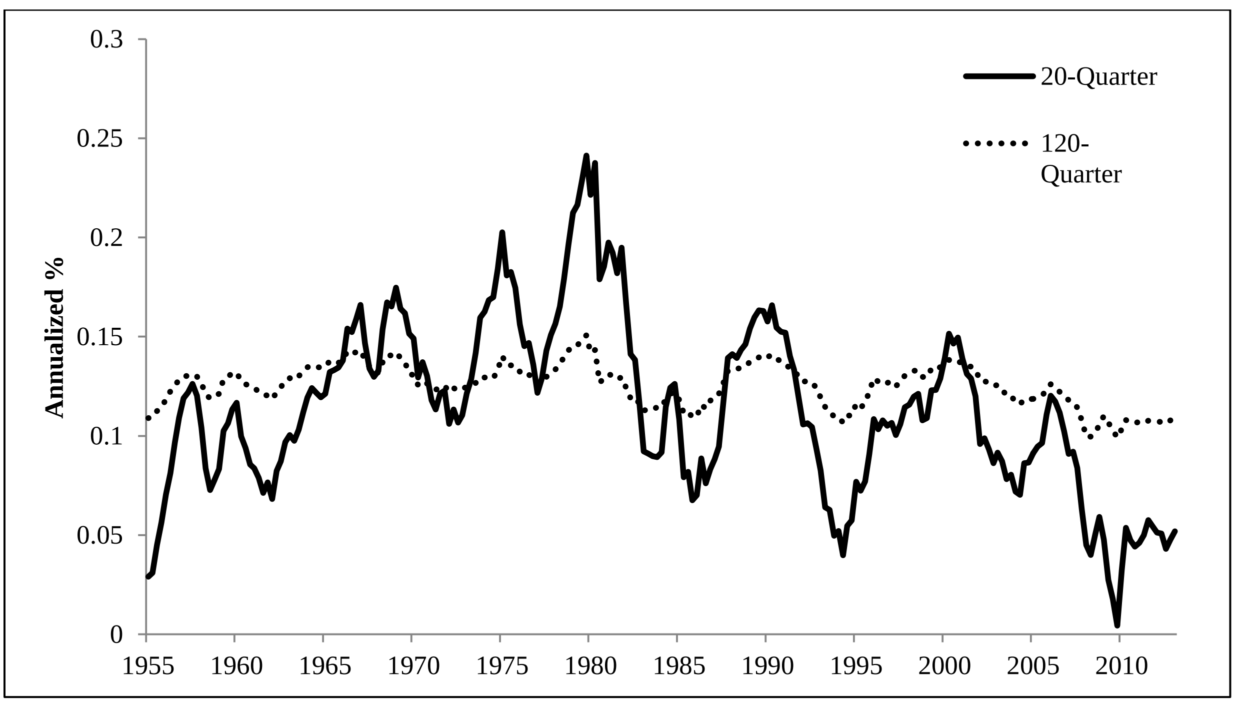
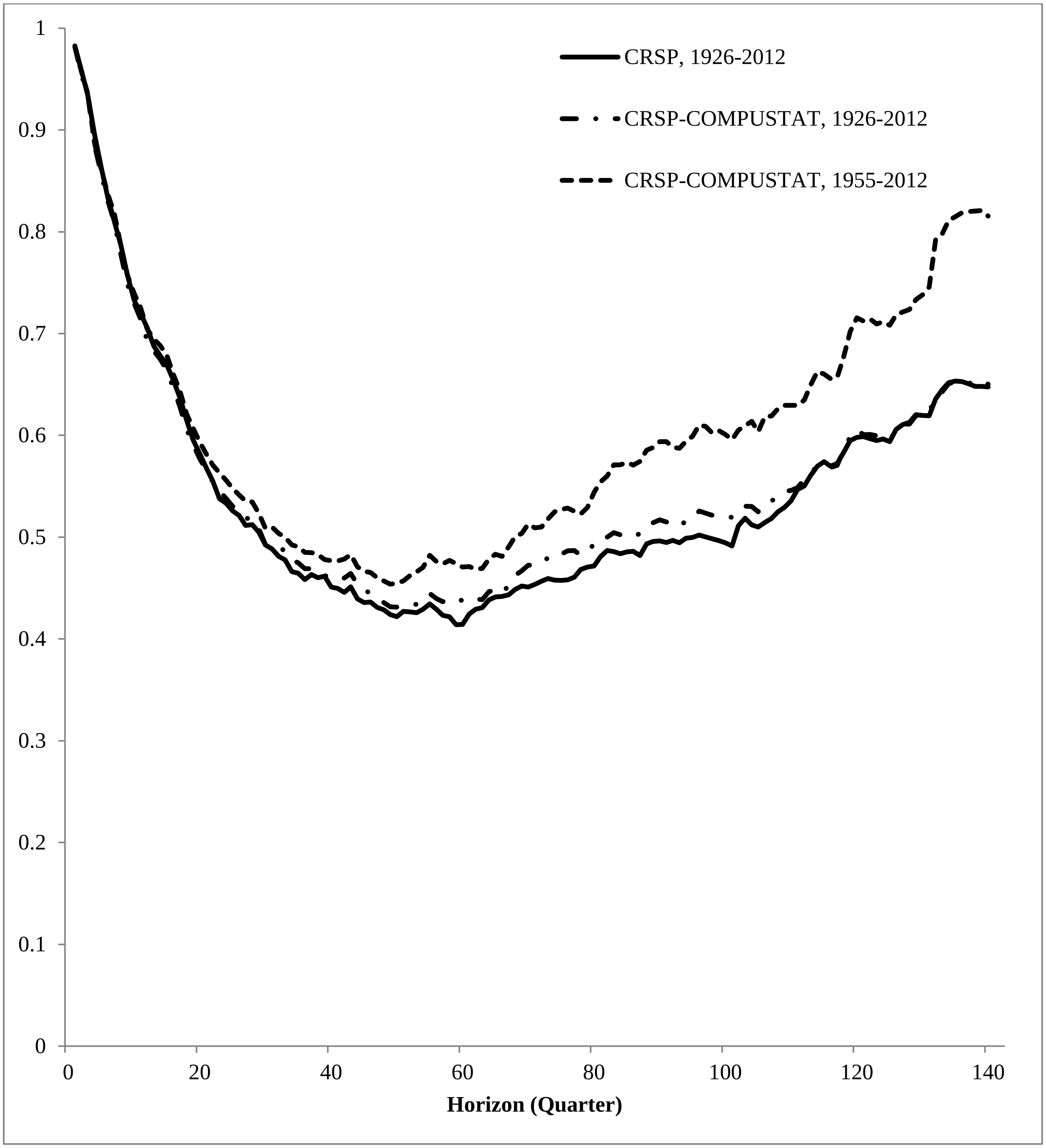


Figure 4b: 20-Quarter and 120-Quarter Discount Rates for the Top 20% Size and Bottom 20% BM Ratio Group



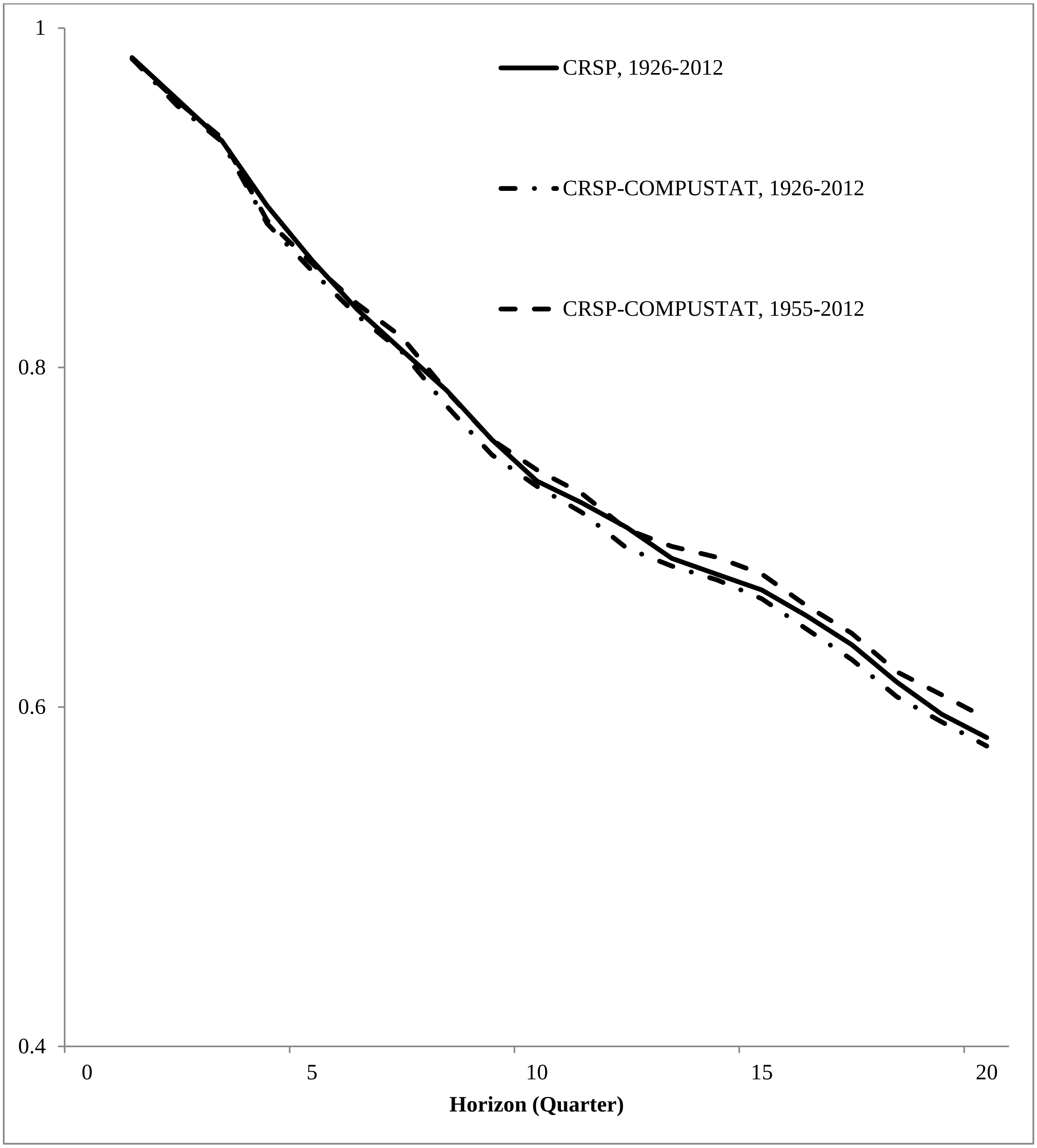
Note: See section 2.3 for the method of estimation for discount rates.

Figure 5a: Coefficient on Initial Log Price by Horizons



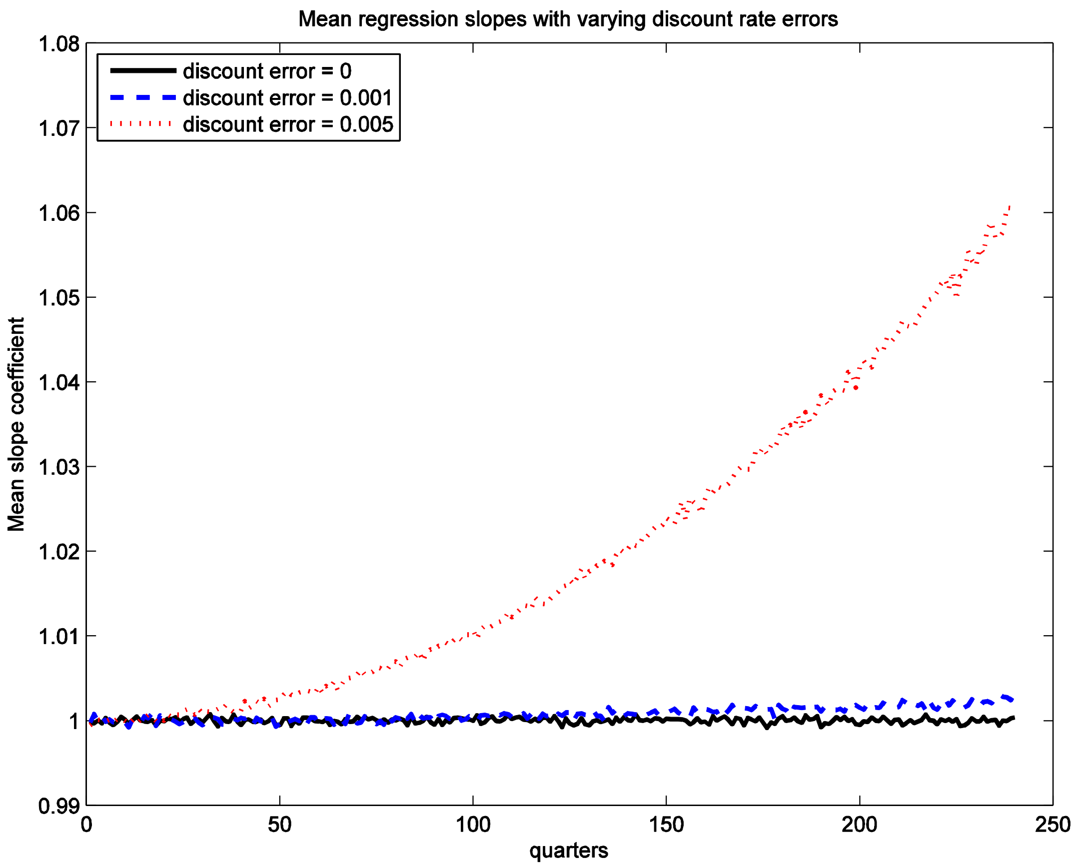
Note: At each horizon from 1 to 140 quarters, we calculate the present value for firms available for that horizon and regress its log on log initial price. As horizon increases, the number of firms drops.

Figure 5b: Coefficient on Initial Log Price for Horizons At or Below 20 Quarters



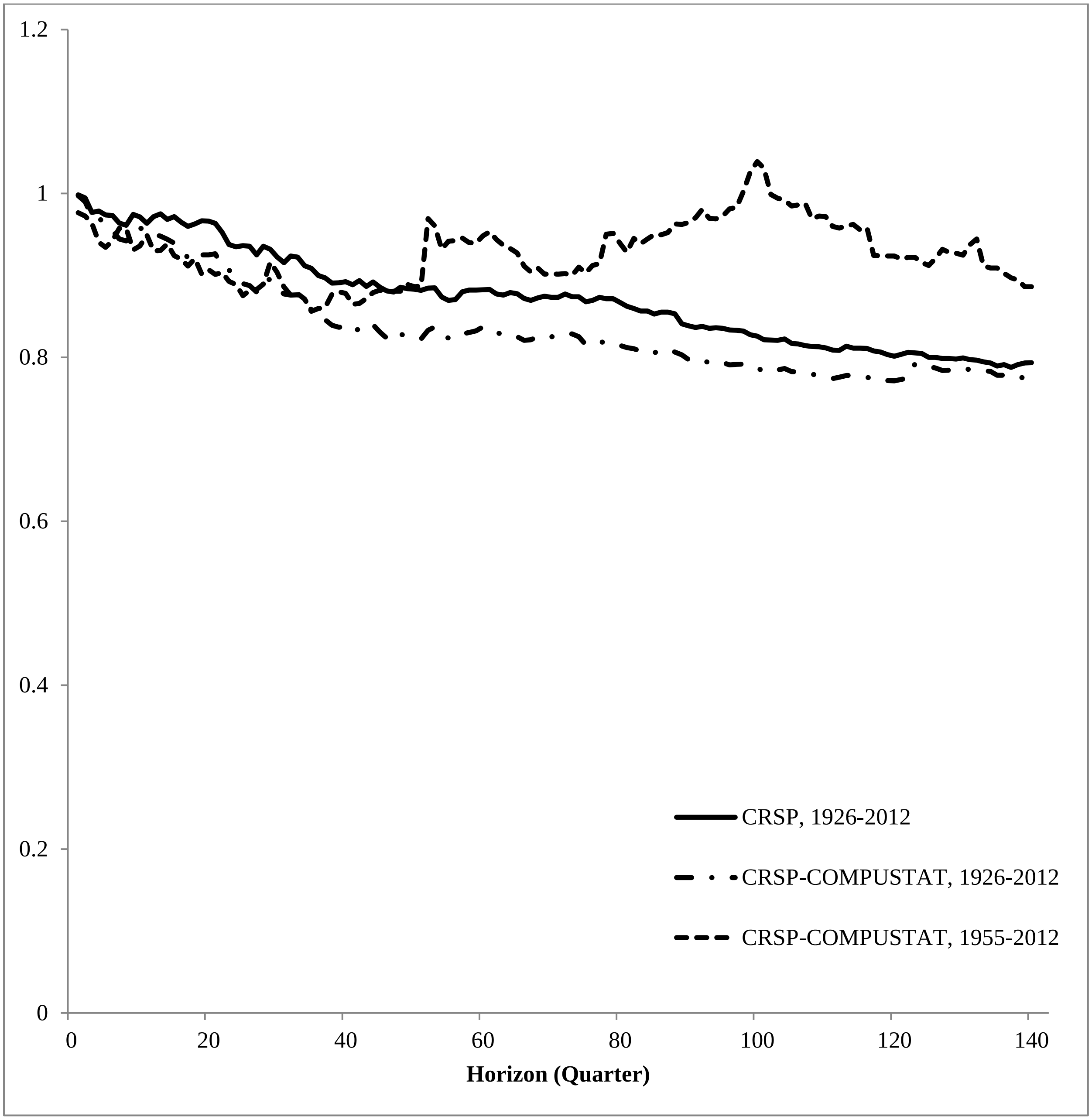
Note: At each horizon from 1 to 20 quarters, we calculate the present value for firms available for that horizon and regress its log on log initial price. As horizon increases, the number of firms drops.

Figure 6: Monte Carlo Mean Slope Coefficients



Note: Please refer to Section 3.

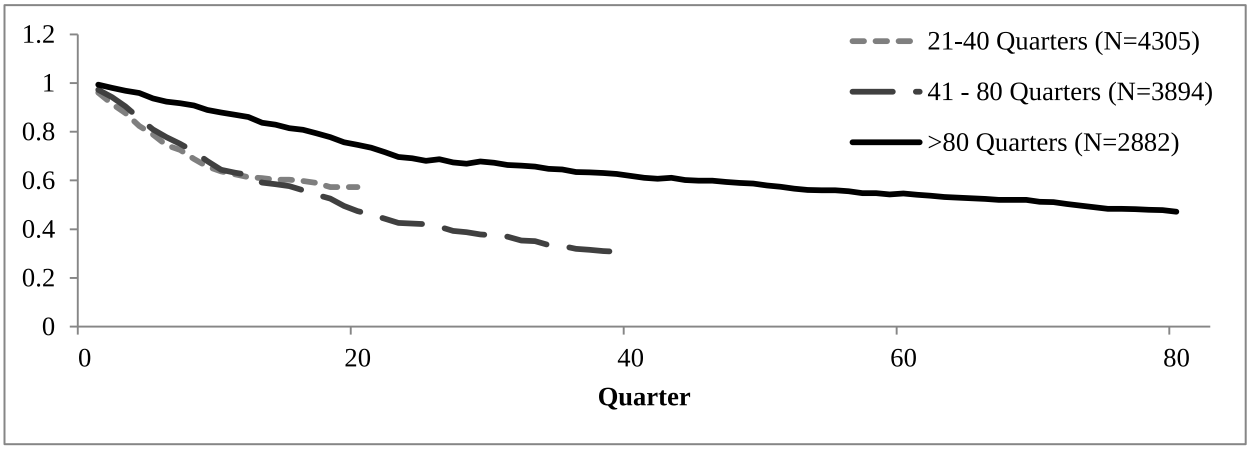
Figure 7: Coefficient on Initial Log Price by Horizons (Firms with Market Share >0.3%)



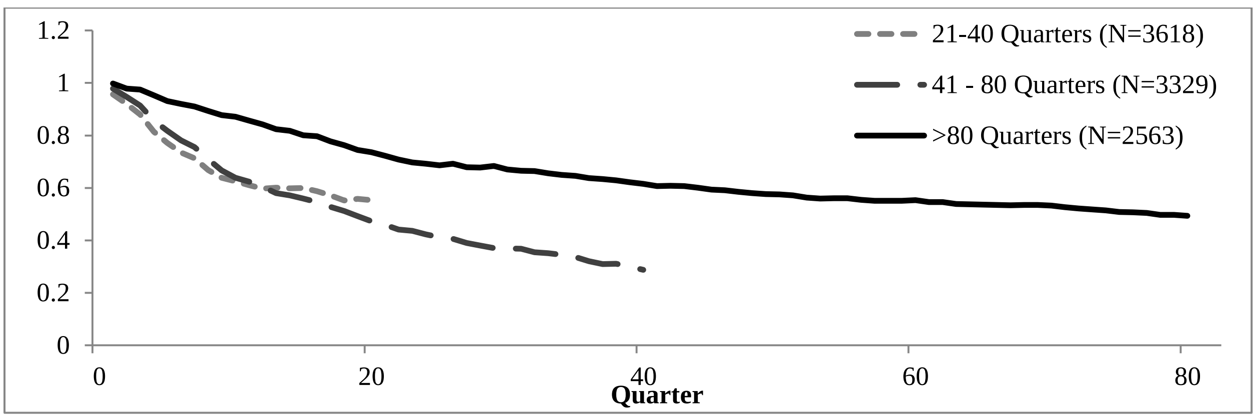
Note: We restrict the sample to firms that have initial market share larger than 0.3%. At which horizon from 1 to 140 quarters, we calculate the present value for firms available for that horizon and regress that on initial price. As horizon increases, the number of firms drops. 95% confidence intervals are calculated from White standard errors.

Figure 8: Coefficient on Log Initial Price over Different Horizons by Sample Length

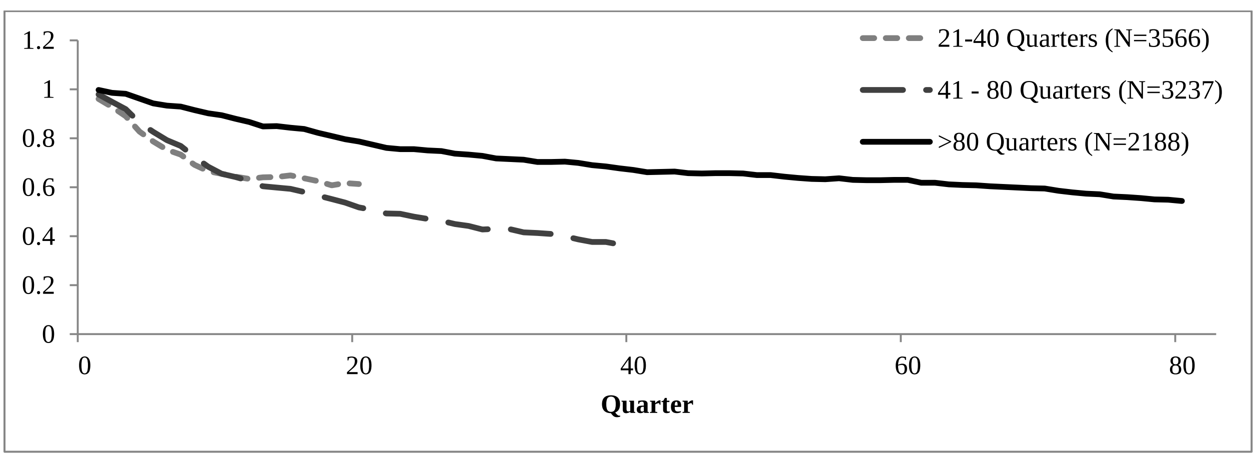
A. CRSP, 1926-2012



B. CRSP-COMPUSTAT, 1926-2012



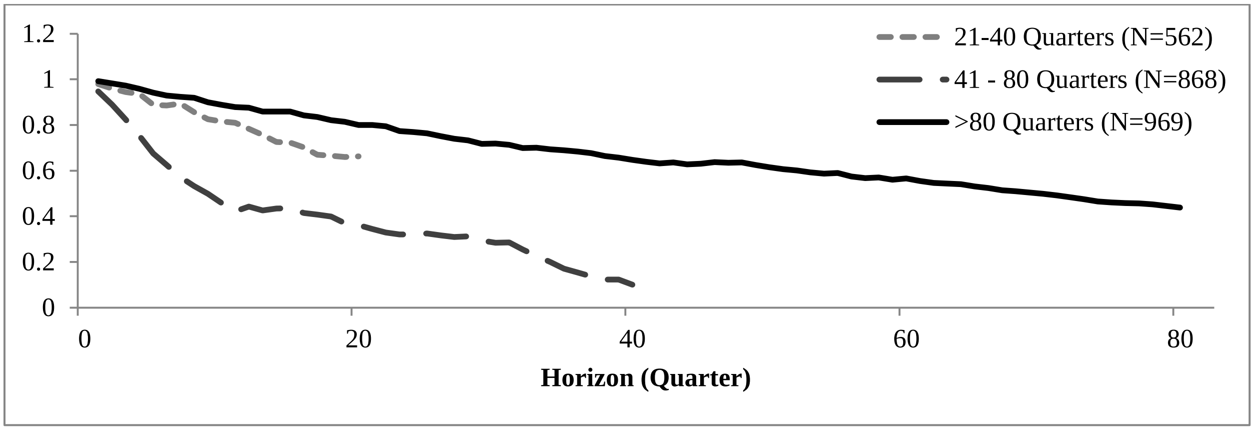
C. CRSP-COMPUSTAT, 1955-2012



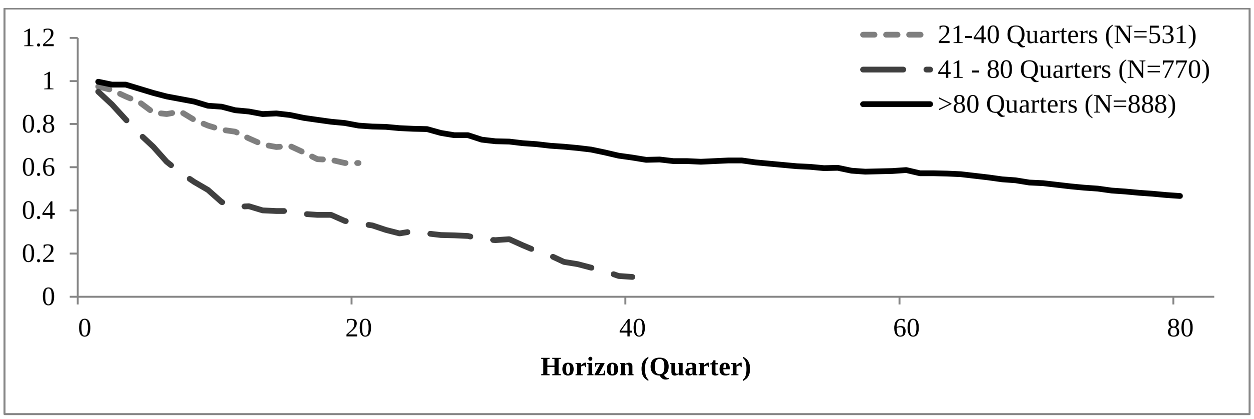
Note: We consider firms that belong to three groups: those that are in the sample for 21 to 40 quarters, for 41 to 80 quarters, and for above 80 quarters. For the first group, we run the regression from 1 to 20-quarter horizon; for the second group, we run the regression from 1 to 40-quarter horizon; for the third group, we run the regression from 1 to 80-quarter horizon.

Figure 9: Coefficient on Log Initial Price over Different Horizons by Sample Length (Truncated Firms)

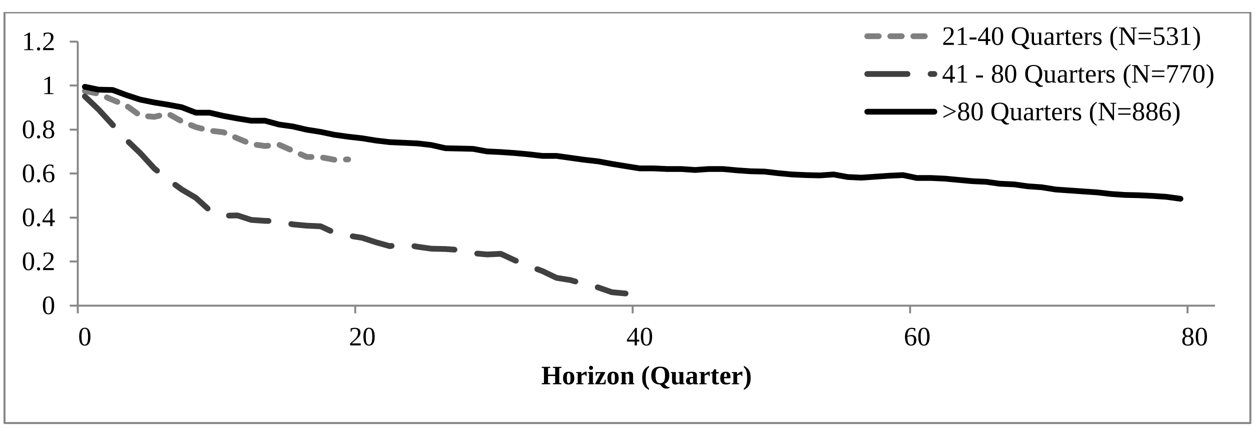
A. CRSP, 1926-2012



B. CRSP-COMPUSTAT, 1926-2012



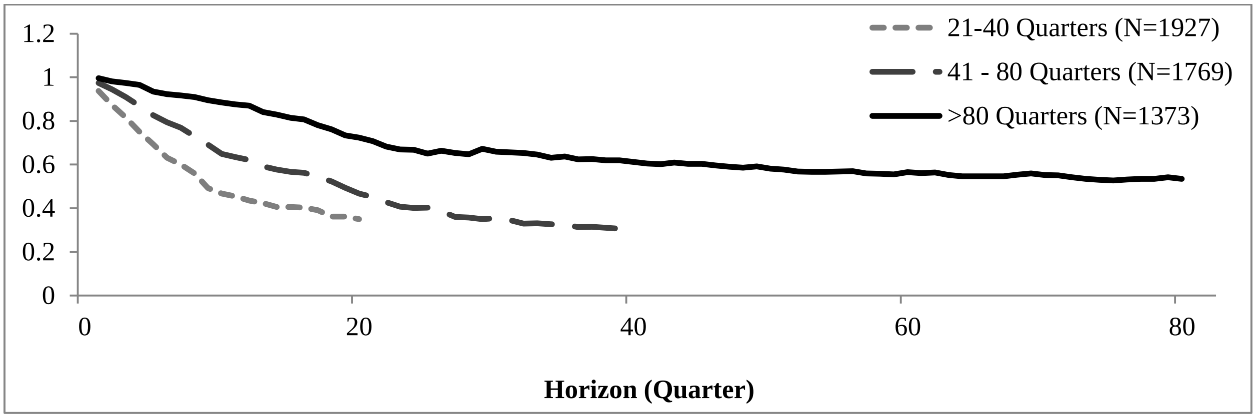
C. CRSP-COMPUSTAT, 1955-2012



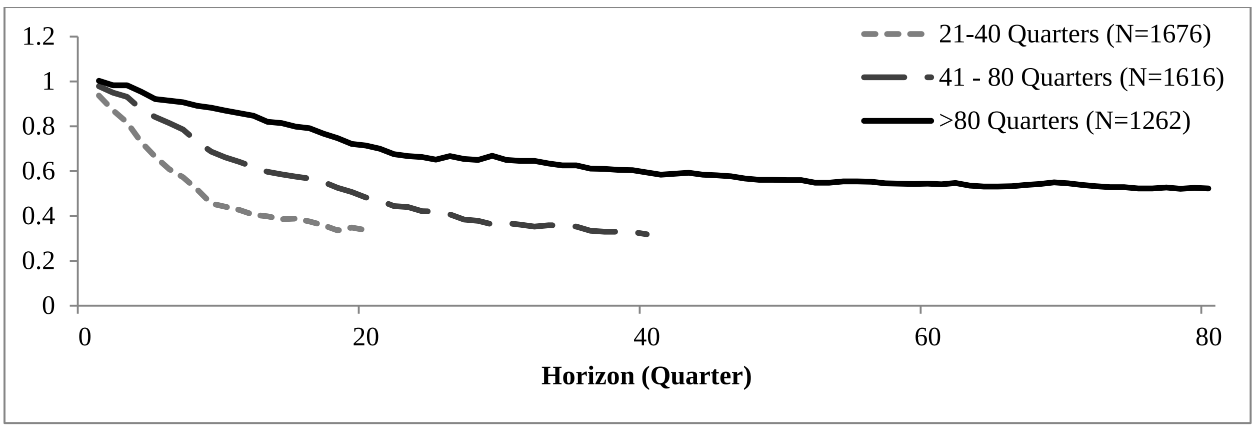
Note: We consider firms that belong to three groups: those that are in the sample for 21 to 40 quarters, for 41 to 80 quarters, and for above 80 quarters. For the first group, we run the regression from 1 to 20-quarter horizon; for the second group, we run the regression from 1 to 40-quarter horizon; for the third group, we run the regression from 1 to 80-quarter horizon.

Figure 10: Coefficient on Log Initial Price over Different Horizons by Sample Length (Merged Firms)

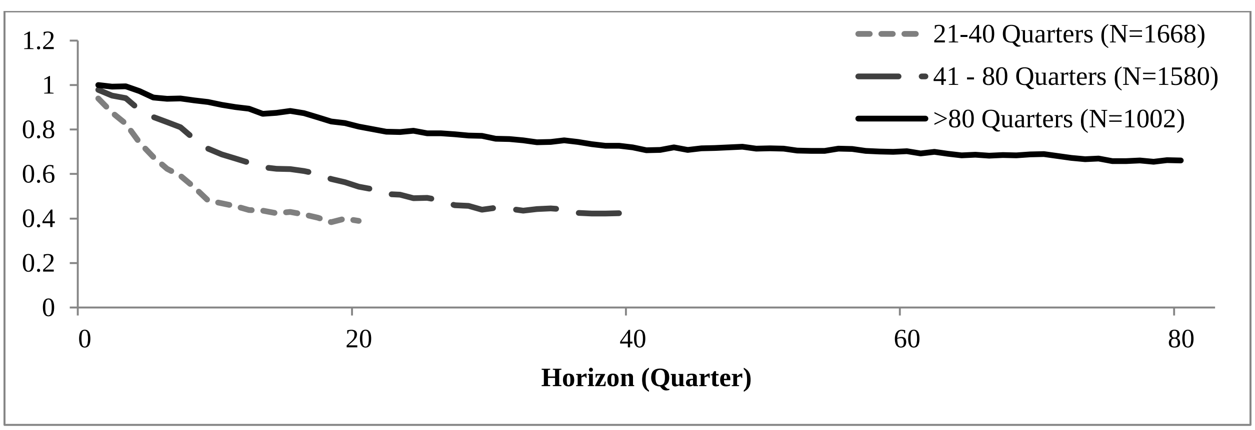
A. CRSP, 1926-2012



B. CRSP-COMPUSTAT, 1926-2012



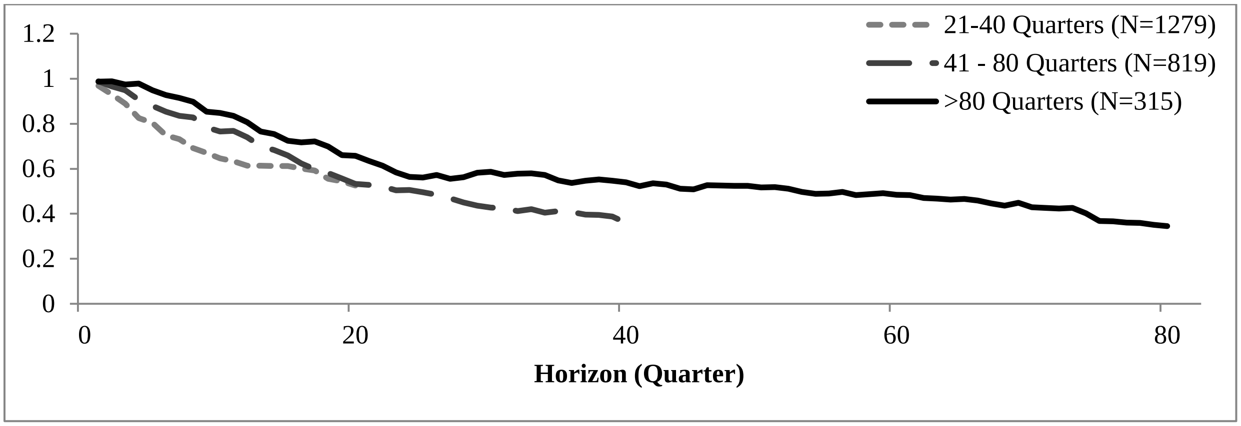
C. CRSP-COMPUSTAT, 1955-2012



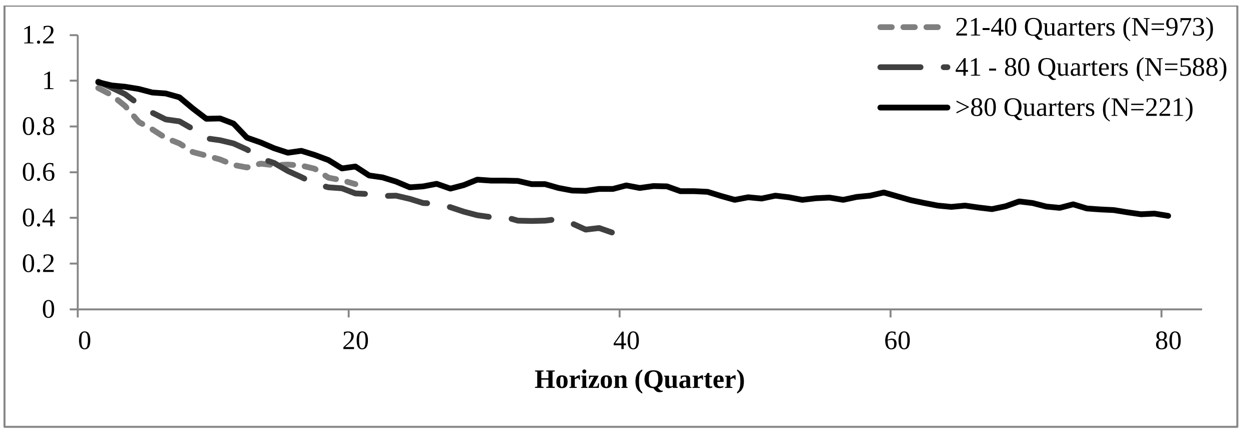
Note: We consider firms that belong to three groups: those that are in the sample for 21 to 40 quarters, for 41 to 80 quarters, and for above 80 quarters. For the first group, we run the regression from 1 to 20-quarter horizon; for the second group, we run the regression from 1 to 40-quarter horizon; for the third group, we run the regression from 1 to 80-quarter horizon.

Figure 11: Coefficient on Log Initial Price over Different Horizons by Sample Length ("Troubled" Firms)

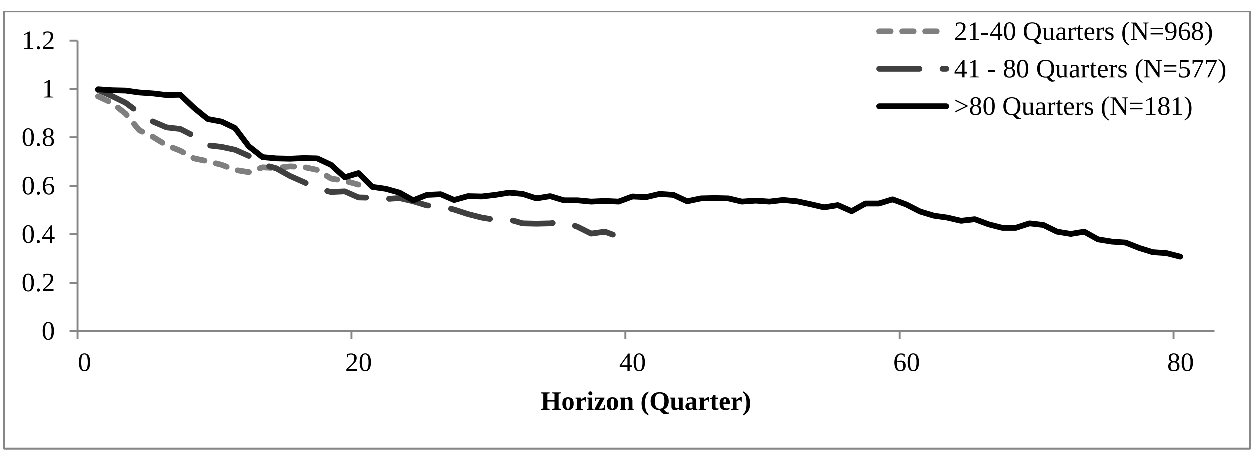
A. CRSP, 1926-2012



B. CRSP-COMPUSTAT, 1926-2012



C. CRSP-COMPUSTAT, 1955-2012



Note: We consider firms that belong to three groups: those that are in the sample for 21 to 40 quarters, for 41 to 80 quarters, and for above 80 quarters. For the first group, we run the regression from 1 to 20-quarter horizon; for the second group, we run the regression from 1 to 40-quarter horizon; for the third group, we run the regression from 1 to 80-quarter horizon.

Data Appendix – not for publication

In this appendix we describe how we construct the data from the original CRSP files.

1. Keep stocks that are traded on NYSE, AMEX or NASDAQ.

2. Keep common stocks with no special status (share code (shrcd) being either 10 or 11).

3. Drop financial firms (first digit of the Standard Industrial Classification Code (siccd) being 6).

4. Drop observations (not firms) with missing stock price.

5. Include only ordinary dividends (first digit of the Distribution Code (distcd) being 1).

6. Adjust stock price and shares outstanding by the factors cfacpr and cfacshr. Price is divided by cfacpr and the number of shares is multiplied by cfacshr.

7. Drop 14 outliers with unusual stock splits that made their adjusted stock price very large.

8. Drop a small number of firms with book value denominated in Canadian dollars for the second and third samples.

9. Drop firms that never have book equity data for the second and third samples.

10. Rank firms by size in June of each year, and rank firms by book-to-market ratio in December in the previous year (following Fama and French (1993)).

11. Convert the data into quarterly frequency.

12. Define stock price to be the closing price on the last trading day of each quarter.

13. If gaps (i.e., missing quarters or years) are found for a firm, eliminate all observations before the last gap observed and keep only the data afterwards.

14. If a firm is first observed in a quarter with no group (size and/or book-to-market ratio) information, search and use the information from the next 5 quarters. If group information is still not found, drop the first quarter and forward until group information is found.

15. Keep only firms that exist in the sample for at least 2 quarters.

Appendix on the log-normal approximation – not for publication

Derivation of equation (3) is straightforward. Suppose V is distributed log-normal, $V \sim \ln N(\mu, \sigma^2)$. We have $\mathrm{E}(V) = e^{\mu + \frac{\sigma^2}{2}}$, $\mathrm{E}(\log V) = \mu$, and $\mathrm{var}(\log V) = \sigma^2$. Under rational expectations it must be true that $\mathrm{E}(V) = P = e^{\mu + \frac{\sigma^2}{2}}$ or $\log P = \mu + \frac{\sigma^2}{2}$. As always we can write

$$\log V_i = \mathrm{E}(\log V_i) + \eta_i$$

Substituting for $\mathrm{E}(\log V_i) = \mu_i = -\frac{\bar{\sigma}_i^2}{2} + \log P_i$

$$\log V_i = -\frac{\sigma_i^2}{2} + \log P_i + \eta_i$$

which, becomes equation (3) when we move the unobserved, firm-specific variance into the error term.

$$\log V_i = -\frac{\bar{\sigma}^2}{2} + \log P_i + \left[\eta_i + \left(\frac{\bar{\sigma}^2}{2} - \frac{\sigma_i^2}{2}\right)\right] \qquad (3)$$

Our heteroskedasticity-bias robustness test on the general property of the log normal that if $V_i \sim \ln N(\mu_i, \sigma_i^2)$ and $\varepsilon_i = V_i - P_i$, then $\sigma_i^2 = \log\left(1 + \frac{\mathrm{var}(\varepsilon_i)}{P_i^2}\right)$. This gives us an estimate of σ_i^2,

$$\hat{\sigma}_i^2 \approx \ln\left(1 + \frac{\varepsilon_i^2}{P_i^2}\right) \approx \sigma_i^2 + e_i$$

To check for correlation between $\log P_i$ and the unobserved firm-specific variance we regress the calculated values for $\hat{\sigma}_i^2$ on $\log P_i$.